Broadband squeezed light from phase-locked single-mode sub-Poissonian lasers

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(Received 11 March 2008; published 15 May 2008)

We consider a sub-Poissonian single-mode laser with external synchronization and analyze its applicability to the problems of quantum information. Using Heisenberg-Langevin theory, we calculate the quadrature variances of the field emitted by this laser. It is shown that such systems can demonstrate strong quadrature squeezing. Taking into account that the emitted field is temporally multimode, the application of such sources to multichannel quantum teleportation and dense coding protocols is discussed.

DOI: 10.1103/PhysRevA.77.052316 PACS number(s): 03.67.Hk, 42.50.−p

I. INTRODUCTION

The concept of multimode squeezing [1] has turned out to be very productive for both quantum optics and quantum information. It is justified by recent advances in quantum holographic teleportation, ghost imaging [2], and quantum dense coding [3]. These schemes were based on the use of traveling-wave optical parametric amplifiers (OPAs) with spatially multimode structures. The efficiency of these schemes turned out to be essentially enhanced in comparison with the strongly single-mode models since many spatial modes ensure the multichannel parallelism in the information transfer. In the cited works, a temporal spectrum has not been taken into account because of a too wide spectral range of OPA radiation. Obviously, if systems with similar spatial structure and a reasonably restricted temporal spectrum could be found, then the efficiency of the protocols could be made even higher.

From this point of view, single-mode lasers seem to be rather promising. Indeed, on the one hand, there is a possibility to construct from the single-mode laser light a spatially multimode structure. For example, one could build an array of many lasers that would be able to generate spatial mode structure not worse (maybe even better) than in the case of the OPA. On the other hand, the role of the temporal spectrum (the spectral mode width) will be essential too. In realistic lasers, the linewidth of a mode is similar to the spectral capacity of realistic detectors, i.e., the information encoded in spectral components can be used effectively. At the same time, this width is not small (from units of MHz up to hundreds of GHz). Thus one can expect that this source of squeezed light can be more effective than the OPA, since both the spatial and temporal factors start to play a role.

Below we are going to consider only the temporal aspects of squeezed light from a single-mode laser. We will show that certain obstacles restricting the direct use of these systems in quantum information can be overcome. The most important problem is the phase diffusion of a free-running laser. In quantum information, it is extremely important to have the possibility to follow the so-called squeezed quadrature component. However, it is impossible for traditional lasers due to the phase diffusion, which results in random rotation of the squeezing ellipse. A possible solution would be to introduce a synchronizing mechanism that would allow for phase-diffusion suppression. However, it is *a priori* not clear whether such synchronization would be compatible with the quantum features of the sub-Poissonian laser. Detailed investigation of this problem is the aim of the present paper.

The paper is organized as follows. In Sec. II, the Heisenberg-Langevin theory of the phase-locked sub-Poissonian laser is developed. It is assumed that the laser is synchronized by an external field in the coherent state.

In order to preserve the quantum features of the laser field, we have to limit the power of the synchronizing field. Otherwise this field will impose its own Poissonian photon statistics to the laser mode. It is very clear that too low power is unable to ensure effective phase locking, but we will demonstrate that this restriction is still compatible with an effective laser synchronization. At the end of the section, we calculate the spectral densities of the quadrature components of the intracavity field and express the observed spectral variances via these values. These results will then be used to evaluate the capacity of the specific information channels.

In Secs. III and IV, we consider two information schemes, namely quantum dense coding and quantum teleportation, with use of sub-Poissonian single-mode lasers as the sources of the multimode squeezed light.

II. THEORY OF PHASE-LOCKED SINGLE-MODE SUB-POISSONIAN LASERS

The very first theory of the sub-Poissonian laser was developed in Ref. [4]. The theory was constructed on the basis of the well-known quantum Lamb-Scully approach Ref. [5] in terms of the master equation for the field density matrix. Later on, this idea was realized experimentally by Yamamoto *et al.* [6] in semiconductor lasers. Now the best results in reduction of shot noises are reached in low-dimensional semiconductor lasers [the so-called vertical-cavity surface-emitting lasers (VCSELs)].

The limited applicability of sub-Poissonian lasers for quantum information is due to phase diffusion. The random walking of the radiation phase makes it impossible to follow the squeezed quadrature components. One of the possibilities to suppress the phase diffusion is to use an external electromagnetic field in a coherent state to lock the laser phase. A similar approach was elaborated on in Ref. [7], where the problem was considered in terms of the master equation for the Wigner distribution. Although the main attention there

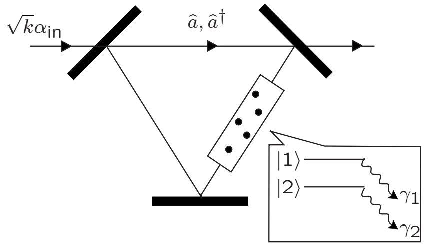

FIG. 1. Phase-locked single-mode laser.

was paid to the role of feedback, some of the results are important in the context of the present discussions.

There are two reasons why we want to discuss the theory of the phase-locked laser once more. First of all, we would like to have a theory that could be suitable for semiconductor lasers, because it is precisely these systems that seem to be the most promising for quantum optics and quantum information. As is well known, the approach based on the master equation and developed in Ref. [7] is suitable only for the gas medium. Here we are going to develop the approach based on the Heisenberg-Langevin equations. Moreover, the Heisenberg approach turns out to be most convenient for application of the theory in the information schemes.

Furthermore, by using the external field in the coherent state for the phase-locking, we have to remember that the power of the field cannot be too high. Otherwise the coherent statistics will be imposed on the laser field, i.e., the quantum properties of the laser field will be destroyed. This important question has not been addressed in the previous consideration [7], but it is extremely important to find out whether this requirement does not contradict the effective locking of the laser phase.

A. Physical model and the Heisenberg-Langevin equations

First, we address a physical model of a laser upon which we base our discussions (see Fig. 1). The principal components of the model have properties similar to that of the standard discussions of the Heisenberg-Langevin laser theory [8,9,11]. It is assumed that the laser high-Q cavity supports only a single mode, which is described by the creation and annihilation operators $\hat{a}^{\dagger}$ and $\hat{a}$, obeying the canonic commutation relation $[\hat{a},\hat{a}^{\dagger}]=1$. Furthermore, one of the mirrors forming the cavity is partially transparent, allowing the laser light to leave the cavity for the photodetection.

It is assumed that the active laser medium consists of independent two-level atoms, with $|1\rangle$ and $|2\rangle$ being the upper and lower lasing levels, respectively. For the sake of simplicity, the atomic transition between these levels is assumed to be exactly resonant to the cavity mode. The lifetimes of the atomic states are determined by the spontaneous decay rates γ_1 and γ_2. From the quantum optical point of view, the best relation is $\gamma_1 \ll \gamma_2$ [4].

Having this model at hand, lasing is described by the following Heisenberg-Langevin equations:

$$\dot{\hat{a}} = -\frac{\kappa}{2}(\hat{a} - a_{in}) + g\hat{P} + \hat{F}_a, \quad (1)$$

$$\dot{\hat{P}} = -\gamma_{\perp}\hat{P} + g(\hat{N}_1 - \hat{N}_2)\hat{a} + \hat{F}_p, \quad (2)$$

$$\dot{\hat{N}}_1 = R - \gamma_1\hat{N}_1 - g(\hat{a}^{\dagger}\hat{P} + \hat{a}\hat{P}^{\dagger}) + \hat{F}_1, \quad (3)$$

$$\dot{\hat{N}}_2 = -\gamma_2\hat{N}_2 + g(\hat{a}^{\dagger}\hat{P} + \hat{a}\hat{P}^{\dagger}) + \hat{F}_2. \quad (4)$$

Here $\hat{a}$ and $\hat{a}^{\dagger}$ are the above-mentioned photon annihilation and creation operators, $\hat{P}$ is the slowly varying collective atomic polarization on the laser transition, and $\hat{N}_1$ and $\hat{N}_2$ are the operators of the populations of the corresponding atomic states. The coefficients in the equations are as follows: the spectral mode width κ, the atom-field coupling constant g, the rate of spontaneous escape γ_1 and γ_2, the rate of transverse decay $\gamma_{\perp}$, and the rate of an incoherent pump of the upper laser state R.

The inhomogeneous term in the first equation $\kappa a_{in}/2$, where

$$a_{in} = \sqrt{n_{in}}e^{i\varphi_{in}}, \quad (5)$$

ensures a coherent excitation of the laser mode by the external classical field with the intracavity amplitude a_{in}.

Apart from the presence of a_{in}, the system of equations (1)–(4) is exactly the same as in Refs. [8,9,11]. Note that it is possible to take into account adiabatically slow phase motion of the synchronizing field assuming $\varphi_{in}=\varphi_{in}(t)$. It is enough to require that the linewidth connected with the phase diffusion is much narrower than κ, which is always satisfied in practice and especially for the VCSELs.

The other inhomogeneous terms $\hat{F}$ in Eqs. (1)–(4) represent the noise processes in the laser system. They appear due to the interaction of the cavity field and the atoms with their own independent Markovian reservoirs, in both cases being the continuum vacuum modes. These operators possess zero average values, and, as can be shown by direct substitution or via the Einstein relation, the only nonzero pair correlation functions read

$$\langle\hat{F}_a(t)\hat{F}_a^{\dagger}(t')\rangle = \kappa\delta(t-t'), \quad (6)$$

$$\langle\hat{F}_1(t)\hat{F}_1(t')\rangle = [\gamma_1\langle N_1\rangle + R(1-p)]\delta(t-t'), \quad (7)$$

$$\langle\hat{F}_2(t)\hat{F}_2(t')\rangle = \gamma_2\langle N_2\rangle\delta(t-t'), \quad (8)$$

$$\langle\hat{F}_p^{\dagger}(t)\hat{F}_p(t')\rangle = [(2\gamma_{\perp} - \gamma_1)\langle N_1\rangle + R]\delta(t-t'), \quad (9)$$

$$\langle\hat{F}_p(t)\hat{F}_p^{\dagger}(t')\rangle = (2\gamma_{\perp} - \gamma_2)\langle N_2\rangle\delta(t-t'), \quad (10)$$

$$\langle\hat{F}_p(t)\hat{F}_1(t')\rangle = \gamma_1\langle P\rangle\delta(t-t'), \quad (11)$$

$$\langle\hat{F}_2(t)\hat{F}_p(t')\rangle = \gamma_2\langle P\rangle\delta(t-t'). \quad (12)$$

As is demonstrated in Refs. [8,9,11], relying on the model as sketched above, one can introduce the pumping statistics. It can be modeled via distributions of the time instants when excited atoms enter the cavity. This process is described by the single parameter $p \leq 1$, where $p=1$ corresponds to the regular pumping resulting in the sub-Poissonian photon statistics, $p=0$ to the Poissonian pumping, and $p<0$ to the super-Poissonian one.

In order to deal with the photodetection, one needs not only the above-mentioned correlation functions but also their normally ordered counterparts. These nonzero correlations are obtained in Ref. [8] and read

$$\langle:\hat{F}_1(t)\hat{F}_1(t'):\rangle = [\gamma_1\langle\hat{N}_1\rangle - g\langle\hat{a}^\dagger\hat{P} + \hat{a}\hat{P}^\dagger\rangle + R(1-p)]\delta(t-t'), \quad (13)$$

$$\langle:\hat{F}_2(t)\hat{F}_2(t'):\rangle = [\gamma_2\langle N_2\rangle - g\langle\hat{a}^\dagger\hat{P} + \hat{a}\hat{P}^\dagger\rangle]\delta(t-t'), \quad (14)$$

$$\langle:\hat{F}_1(t)\hat{F}_2(t'):\rangle = g\langle\hat{a}^\dagger\hat{P} + \hat{a}\hat{P}^\dagger\rangle\delta(t-t'), \quad (15)$$

$$\langle:\hat{F}_p^\dagger(t)\hat{F}_p(t'):\rangle = [(2\gamma_\perp - \gamma_1)\langle N_1\rangle + R]\delta(t-t'), \quad (16)$$

$$\langle:\hat{F}_p(t)\hat{F}_p(t'):\rangle = 2g\langle\hat{a}\hat{P}\rangle\delta(t-t'), \quad (17)$$

$$\langle:\hat{F}_p(t)\hat{F}_2(t'):\rangle = \gamma_2\langle P\rangle\delta(t-t'). \quad (18)$$

Having specified the model and its theoretical description, we proceed with finding the solution applying reasonable approximations.

In order to solve the system of Eqs. (1)–(4), we employ the following commonly used approximations. First, we assume that the polarization $\hat{P}$ and the population $\hat{N}_2$ of the lower laser level can be adiabatically eliminated. This is justified given that the requirement $\gamma_2, \gamma_\perp \gg \gamma_1, \kappa$ is fulfilled. These relations between the spectral parameters are typical for semiconductor lasers. After this operation, the system is essentially simplified in that only two equations for the population of the upper atomic state $\hat{N}_1$ and for the field amplitude $\hat{a}$ survive. The remaining equations are nonlinear differential equations that also cannot be solved analytically. The next approximation is based on the assumption that in the stationary regime, the system variables deviate only slightly from the corresponding semiclassical solutions. This approach allows us to linearize the equations of motion.

First, let us apply the adiabatic approximation relative to the fast variables $\hat{P}$ and $\hat{N}_2$. After the adiabatic illumination of these variables, the simplified system only for the field amplitude and the population of the upper laser level is obtained. This system reads

$$\dot{\hat{N}}_1 = -\gamma_1\hat{N}_1 - c\hat{a}^\dagger\hat{a}\hat{N}_1 + R + \hat{\xi}_N(t), \quad c = 2g^2/\gamma_\perp, \quad (19)$$

$$\dot{\hat{a}} = -\frac{\kappa}{2}(\hat{a} - a_{\text{in}}) + \frac{c}{2}\hat{N}_1\hat{a} + \hat{\xi}_a(t), \quad (20)$$

where the new noise sources are given by

$$\hat{\xi}_a = \hat{F}_a + g/\gamma_\perp\hat{F}_p - g^2/\gamma_2^2\hat{F}_2\hat{a},$$

$$\hat{\xi}_N = \hat{F}_1 + c/\gamma_2\hat{a}^\dagger\hat{a}\hat{F}_2 - g/\gamma_\perp(\hat{a}^\dagger\hat{F}_p + \hat{a}\hat{F}_p^\dagger). \quad (21)$$

We shall find the solutions of the system of Eqs. (19) and (20) assuming that these solutions can be considered as slow deviations near the semiclassical stationary solutions. Formally, this means that the quantities

$$\hat{a}(t) = [\sqrt{n} + \delta\hat{a}(t)]e^{i\varphi_{\text{in}}}, \quad \hat{N}_1(t) = N_1 + \delta\hat{N}_1(t) \quad (22)$$

should satisfy the requirement

$$\sqrt{n} \gg \delta\hat{a}(t), \quad N_1 \gg \delta\hat{N}_1(t). \quad (23)$$

It should be stressed here that the first of the inequalities is valid only if the phase diffusion in the laser is suppressed.

As for the semiclassical stationary solutions N_1, $\bar{a}$, it is not difficult to see that in the only interesting saturation regime $\gamma_1 \ll cn$,

$$N_1 = \frac{R}{\gamma_1}, \quad \bar{a} = \sqrt{n}e^{i\varphi_{\text{in}}}. \quad (24)$$

The stationary real amplitude $\sqrt{n}$ satisfies the following quadratic equation:

$$\sqrt{n}(\sqrt{n} - \sqrt{n_{\text{in}}}) = \frac{R}{\kappa}. \quad (25)$$

It is convenient for us to introduce the parameter

$$\mu = \sqrt{\frac{n_{\text{in}}}{n}}, \quad (26)$$

which reflects the role of the external synchronizing factor in a production of the full power of the operation. Furthermore, we will choose that this role is very small, which means that

$$\mu \ll 1. \quad (27)$$

Then in particular, Eq. (25) provides us with the solution $n \approx R/\kappa$.

B. Linearization of the equations relative to fluctuations

Under requirement (23), one can linearize the adiabatic equations (19) and (20) and obtain them in the following form:

$$\delta\dot{\hat{x}} = -\kappa\mu/2\,\delta\hat{x} + c\sqrt{n}/2\,\delta\hat{N}_1 + \hat{\xi}_x(t), \quad (28)$$

$$\delta\dot{\hat{y}} = -\kappa\mu/2\,\delta\hat{y} + \hat{\xi}_y(t), \quad (29)$$

$$\delta\dot{\hat{N}}_1 = -\Gamma_1\delta\hat{N}_1 - 2\kappa(1-\mu)\sqrt{n}\,\delta\hat{x} + \hat{\xi}_N(t), \quad \Gamma_1 = \gamma_1 + cn. \quad (30)$$

Here we have introduced for further convenience instead of the field amplitude fluctuations $\delta\hat{a}$ the Hermitian quadrature operators

$$\delta\hat{x} = \frac{1}{2}(\delta\hat{a} + \delta\hat{a}^\dagger), \quad \delta\hat{y} = -\frac{i}{2}(\delta\hat{a} - \delta\hat{a}^\dagger). \quad (31)$$

Here it must be stressed that these functions are not quite the quadrature field components inside the cavity. Indeed, ac-

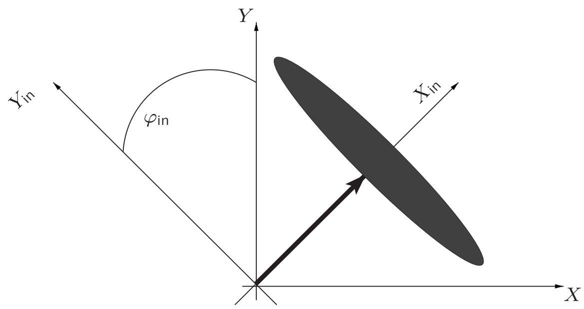

FIG. 2. Amplitude and phase quadratures in rotated frame.

cording to Eq. (22), we have to take into account an additional exponential factor coupled with the external field phase. Thus the quadrature components are expressed by

$$\delta\hat{x} = \frac{1}{2}(e^{i\varphi_{\rm in}}\delta\hat{a} + e^{-i\varphi_{\rm in}}\delta\hat{a}^\dagger), \quad \delta\hat{y} = -\frac{i}{2}(e^{i\varphi_{\rm in}}\delta\hat{a} - e^{-i\varphi_{\rm in}}\delta\hat{a}^\dagger). \tag{32}$$

It is worth noting that these operators acquire different physical meaning depending on the phase $\varphi_{\rm in}$ of the control field. In fact, they are the usual amplitude and phase quadratures but in the coordinate frame rotated by the angle $\varphi_{\rm in}$, see Fig. 2. One can see that as $\varphi_{\rm in}=0$, the definitions (31) and (32) are perfectly equivalent.

The new noise sources in Eqs. (28)–(30) read

$$\hat{\xi}_x = \frac{1}{2}(e^{-i\varphi_{\rm in}}\hat{F}_a + e^{i\varphi_{\rm in}}\hat{F}_a^\dagger) + \frac{g}{2\gamma_\perp}(e^{-i\varphi_{\rm in}}\hat{F}_p + e^{i\varphi_{\rm in}}\hat{F}_p^\dagger) - \frac{c\sqrt{n}}{2\gamma_2}\hat{F}_2, \tag{33}$$

$$\hat{\xi}_y = -\frac{i}{2}(e^{-i\varphi_{\rm in}}\hat{F}_a - e^{i\varphi_{\rm in}}\hat{F}_a^\dagger) - \frac{ig}{2\gamma_\perp}(e^{-i\varphi_{\rm in}}\hat{F}_p - e^{i\varphi_{\rm in}}\hat{F}_p^\dagger), \tag{34}$$

$$\hat{\xi}_N = \hat{F}_1 - \frac{g\sqrt{n}}{\gamma_\perp}(e^{-i\varphi_{\rm in}}\hat{F}_p + e^{i\varphi_{\rm in}}\hat{F}_p^\dagger) + \frac{cn}{\gamma_2}\hat{F}_2. \tag{35}$$

To find the partial solution of inhomogeneous linear equations (28)–(30), it is convenient to pass to the Fourier domain. Defining the Fourier transformation for a function $G(t)$ as

$$G_\omega = \frac{1}{\sqrt{2\pi}}\int_{-\infty}^{+\infty} G(t)e^{i\omega t}dt, \quad G(t) = \frac{1}{\sqrt{2\pi}}\int_{-\infty}^{+\infty} G_\omega e^{-i\omega t}d\omega, \tag{36}$$

one comes to the following equations:

$$-i\omega\delta\hat{N}_{1\omega} = -\Gamma_1\delta\hat{N}_{1\omega} - 2\kappa(1-\mu)\sqrt{n}\,\delta\hat{x}_\omega + \hat{\xi}_{N\omega}, \tag{37}$$

$$-i\omega\delta\hat{x}_\omega = -\kappa\mu/2\,\delta\hat{x}_\omega + c/2\sqrt{n}\,\delta\hat{N}_{1\omega} + \hat{\xi}_{x\omega}, \tag{38}$$

$$-i\omega\delta\hat{y}_\omega = -\kappa\mu/2\,\delta\hat{y}_\omega + \hat{\xi}_{y\omega}. \tag{39}$$

After some simple algebra, the solutions for the field-related fluctuations are found to be

$$\delta\hat{x}_\omega = \frac{c\sqrt{n}/2\hat{\xi}_{N\omega} + (\Gamma_1 - i\omega)\hat{\xi}_{x\omega}}{(\kappa\mu/2 - i\omega)(\Gamma_1 - i\omega) + cn\kappa(1-\mu)},$$

$$\delta\hat{y}_\omega = \frac{\hat{\xi}_{y\omega}}{\kappa\mu/2 - i\omega}, \tag{40}$$

and the population fluctuation reads

$$\delta\hat{N}_{1\omega} = \frac{-2\kappa(1-\mu)\sqrt{n}\hat{\xi}_{x\omega} + (\kappa\mu/2 - i\omega)\hat{\xi}_{N\omega}}{(\kappa\mu/2 - i\omega)(\Gamma_1 - i\omega) + cn\kappa(1-\mu)}. \tag{41}$$

Let us define the spectral densities $(\xi_x^2)_\omega$, $(\xi_y^2)_\omega$, and $(\xi_N^2)_\omega$ of the stochastic sources in Eqs. (28)–(30) as factors in front of the δ functions in the pair-correlation functions,

$$\langle\hat{\xi}_{x\omega}\hat{\xi}_{x\omega'}\rangle = (\xi_x^2)_\omega\delta(\omega+\omega'), \tag{42}$$

$$\langle\hat{\xi}_{y\omega}\hat{\xi}_{y\omega'}\rangle = (\xi_y^2)_\omega\delta(\omega+\omega'), \tag{43}$$

$$\langle\hat{\xi}_{N\omega}\hat{\xi}_{N\omega'}\rangle = (\xi_N^2)_\omega\delta(\omega+\omega'). \tag{44}$$

Similarly one can define the spectral densities $(\xi_y\xi_x)_\omega$, $(\xi_x\xi_N)_\omega$, and $(\xi_y\xi_N)_\omega$ for the cross-correlation.

After straightforward algebra, one can obtain the following results for the field variances:

$$(\xi_x^2)_\omega = (\xi_y^2)_\omega = 2i(\xi_y\xi_x)_\omega = -2i(\xi_x\xi_y)_\omega = \kappa/2(1-\mu/2), \tag{45}$$

$$(\xi_N^2)_\omega = \kappa(1-\mu)/c\Gamma_1(2-p), \tag{46}$$

$$(\xi_x\xi_N)_\omega = (\xi_N\xi_x)_\omega = -i(\xi_y\xi_N)_\omega = i(\xi_N\xi_y)_\omega = -\kappa/2(1-\mu)\sqrt{n}. \tag{47}$$

Now it is possible to derive the spectral intracavity variances in the explicit form

$$(\delta x^2)_\omega = \frac{\kappa(1-\mu/2)}{2} \times \frac{\omega^2 + \Gamma_1^2 - cn\Gamma_1 p/2(1-\mu)/(1-\mu/2)}{[\omega^2 - cn\kappa(1-\mu) - \kappa\mu/2\Gamma_1]^2 + \omega^2(\Gamma_1 + \kappa\mu/2)^2}, \tag{48}$$

$$(\delta y^2)_\omega = \frac{\kappa(1-\mu/2)}{2}\frac{1}{\omega^2 + \kappa^2\mu^2/4}. \tag{49}$$

In the saturation regime $\gamma_1 \ll cn \rightarrow \Gamma_1 \approx cn$,

$$(\delta x^2)_\omega = \frac{\kappa}{2}\frac{1-\mu/2-(1-\mu)p/2}{\kappa^2(1-\mu/2)^2+\omega^2}. \tag{50}$$

We remind the reader that in our theory the synchronizing parameter μ is much less than 1. Nevertheless, we have to survive it in our formulas because there is compensation in the main order.

C. Spectral field variances outside the cavity

The single-mode light leaving the cavity for a photodetector is described by the normalized amplitude $\hat{a}(t)$ inside and the other normalized amplitude $\hat{A}(t)$ outside the cavity. Normalization is fulfilled such that a value $\langle \hat{a}^\dagger \hat{a} \rangle$ becomes the mean photon number inside the cavity and a value $\langle \hat{A}^\dagger \hat{A} \rangle$ becomes the mean photon number per second through the cross section of the outgoing beam. The boundary condition at the output mirror can be derived in the form [10]

$$\hat{A}(t) = \sqrt{\kappa}\hat{a}(t) - \hat{A}_{\text{vac}}(t). \tag{51}$$

This condition takes into account not only the field leaving the cavity but also the vacuum field reflected by mirror. This ensures that the field amplitude $\hat{A}(t)$ obeys the canonical commutation relations

$$[\hat{A}(t),\hat{A}^\dagger(t')] = \delta(t-t'), \quad [\hat{A}(t),\hat{A}(t')] = 0 \tag{52}$$

and it is independent of the processes inside the cavity.

Introducing again the quadrature components for the external field as real and imaging parts of the amplitude,

$$\hat{X} = \frac{1}{2}(\hat{A} + \hat{A}^\dagger), \quad \hat{Y} = \frac{1}{2i}(\hat{A} - \hat{A}^\dagger), \tag{53}$$

one can couple the spectral variances inside and outside the cavity,

$$(\delta X^2)_\omega = \frac{1}{4} + \kappa(:\delta x^2:)_\omega, \quad (\delta Y^2)_\omega = \frac{1}{4} + \kappa(:\delta y^2:)_\omega. \tag{54}$$

One can see that the external variances are expressed via the normally ordered variances inside the cavity. The latter can be obtain on the basis of the normally ordered variances for the noise sources,

$$(:\xi_x^2:)_\omega = (:\xi_y^2:)_\omega = \kappa/2(1-\mu), \quad (:\xi_x \xi_N:)_\omega = -\kappa(1-\mu)\sqrt{n},$$
$$(:\xi_N^2:)_\omega = \kappa(1-\mu)/c\Gamma_1(2-p), \tag{55}$$

and then it is not difficult to get

$$(:\delta x^2:)_\omega = \frac{\kappa(1-\mu)}{2} \times \frac{\omega^2 + \Gamma_1^2 - cn\Gamma_1(1+p/2)}{[\omega^2 - cn\kappa(1-\mu) - \kappa\mu/2\Gamma_1]^2 + \omega^2(\Gamma_1 + \kappa\mu/2)^2}, \tag{56}$$

$$(:\delta y^2:)_\omega = \frac{\kappa(1-\mu)}{2}\frac{1}{\omega^2 + \kappa^2\mu^2/4}. \tag{57}$$

In the saturation regime, the first variance is simplified to

$$(:\delta x^2:)_\omega = -\frac{\kappa(1-\mu)p}{4}\frac{1}{\kappa^2(1-\mu/2)^2 + \omega^2}. \tag{58}$$

D. Locking the laser phase

One of our main goals is to understand how the external synchronization affects the laser radiation properties. First, we note that the power of the synchronizing field must not be high, otherwise it will impose the Poissonian photon statistics on the laser radiation, i.e., it will destroy the laser squeezing. This is a physical reason for the assumption $\mu \ll 1$ made above that limits the power of the synchronizing field.

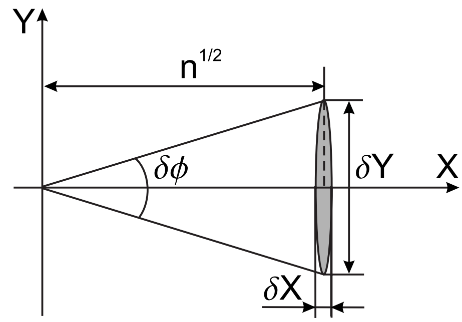

FIG. 3. Amplitude and phase variances.

However, under this restriction we risk locking the phase improperly. For example, although the phase is locked, its fluctuations $\delta\varphi$ could remain high (on the level of 1 or even more), which would render the laser completely useless for our aims. Below, we will prove that the inequalities $\mu \ll 1$ and $\delta\varphi \ll 1$ are perfectly compatible.

Figure 3 will help us to evaluate $\delta\varphi$. In the case $\varphi_{\text{in}}=0$, it is possible to write

$$\langle \delta\varphi^2 \rangle = \langle \delta\hat{y}^2 \rangle / n. \tag{59}$$

The integral y variance is found from the spectral one by integrating over frequencies

$$\langle \delta\hat{y}^2 \rangle = \frac{1}{2\pi}\int (\delta y^2)_\omega d\omega = \sqrt{\frac{n}{4n_{\text{in}}}} \gg 1, \tag{60}$$

where the spectral variance $(\delta y^2)_\omega$ is given in Eq. (49). Combining this result with Eq. (59) yields

$$\langle \delta\varphi^2 \rangle = \frac{1}{\sqrt{4n_{\text{in}}n}}. \tag{61}$$

This means that the condition $\langle \delta\varphi^2 \rangle \ll 1$ is always fulfilled as $n_{\text{in}} \gg 1/n$. Since we require that $n \gg 1$, the appropriate phase locking is easily realizable without contradiction of any other requirements. Thus the synchronization by the external field is perfectly effective even for very weak synchronizing fields, thus the presented analysis is self-consistent.

III. QUANTUM DENSE CODING PROTOCOL

The quantum dense coding protocol was first proposed and experimentally realized for discrete variables, namely qubits [13,14], and later generalized and experimentally realized for continuous variables in [15,16]. This protocol is well known as a traditional approach for transfer information. Its feature in comparison with the other protocols is as follows. For many-channel information schemes, the signal is inserted to one channel only, however all of the channels work equally for the transfer of information. In this section, we shall investigate how the capacity of a two-channel scheme could be improved on the basis of the use of phase-locked sub-Poissonian lasers.

A. The Duan criterium for continuous variable entanglement

The Duan criterium [12] allows us to predict whether the application of the laser radiation can in principle be effective for quantum information purposes. In the case of multimode entanglement, this criterium can be formulated in the following way: two beams are entangled if there is a frequency region where the collective canonic coordinates $\delta\hat{Q}_{1\omega}$ and $\delta\hat{Q}_{2\omega}$ and the canonic momenta $\delta\hat{P}_{1\omega}$ and $\delta\hat{P}_{2\omega}$ satisfy

$$2[(\delta Q_1+\delta Q_2)^2]_\omega,\quad 2[(\delta P_1-\delta P_2)^2]_\omega<1. \tag{62}$$

Let us mix two light beams from two independent phase-locked lasers on a symmetrical beamsplitter; then two initial amplitudes $\hat{S}_1$ and $\hat{S}_2$ are modified according to the equalities

$$\hat{E}_1=\frac{1}{\sqrt{2}}(\hat{S}_1+\hat{S}_2),\quad \hat{E}_2=\frac{1}{\sqrt{2}}(\hat{S}_1-\hat{S}_2). \tag{63}$$

Introducing the quadrature components after and before the beamsplitter according to

$$\hat{E}_i=\hat{Q}_i+i\hat{P}_i,\quad \hat{S}_i=\hat{X}_i+i\hat{Y}_i,\quad i=1,2, \tag{64}$$

one can obtain for the coordinates

$$\delta\hat{Q}_{1,\omega}=\frac{1}{\sqrt{2}}(\delta\hat{X}_{1,\omega}+\delta\hat{X}_{2,\omega}),\quad \delta\hat{Q}_{2,\omega}=\frac{1}{\sqrt{2}}(\delta\hat{X}_{1,\omega}-\delta\hat{X}_{2,\omega}) \tag{65}$$

and for the momenta

$$\delta\hat{P}_{1,\omega}=\frac{1}{\sqrt{2}}(\delta\hat{Y}_{1,\omega}+\delta\hat{Y}_{2,\omega}),\quad \delta\hat{P}_{2,\omega}=\frac{1}{\sqrt{2}}(\delta\hat{Y}_{1,\omega}-\delta\hat{Y}_{2,\omega}). \tag{66}$$

Due to the assumption that the lasers are statistically independent, we can derive

$$[(\delta Q_1+\delta Q_2)^2]_\omega=2(\delta X_1^2)_\omega,\quad [(\delta P_1-\delta P_2)^2]_\omega=2(\delta Y_2^2)_\omega. \tag{67}$$

Now we require that both lasers are identical except the value of the phase φ_{in}: let us take $\varphi_{in}=0$ for the first laser and $\varphi_{in}=\pi/2$ for the second one. Then taking into account Eqs. (54)–(58), we obtain the following result:

$$2[(\delta Q_1+\delta Q_2)^2]_\omega=2[(\delta P_1-\delta P_2)^2]_\omega =\frac{\omega^2+\kappa^2[\mu^2/4+(1-p)(1-\mu)]}{\omega^2+\kappa^2(1-\mu/2)^2}. \tag{68}$$

In the case of Poissonian lasers ($p=0$), $2[(\delta Q_1+\delta Q_2)^2]_\omega=2[(\delta P_1-\delta P_2)^2]_\omega=1$ and we have no possibility to think about the entanglement.

However as $p=1$ (sub-Poissonian lasing),

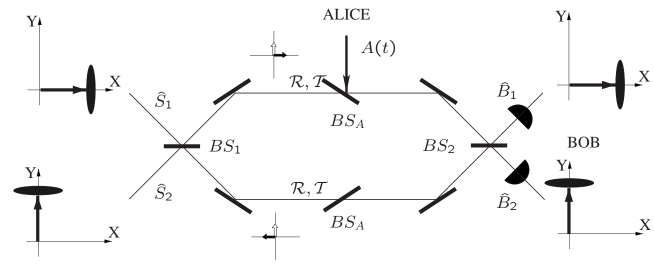

FIG. 4. Quantum dense coding protocol.

$$2[(\delta Q_1+\delta Q_2)^2]_\omega=2[(\delta P_1-\delta P_2)^2]_\omega=\frac{\omega^2+\kappa^2\mu^2/4}{\omega^2+\kappa^2}. \tag{69}$$

In this regime, the Duan criterium is well fulfilled for the frequency region $\omega\ll\kappa$.

Thus we come to the conclusion that the phase-locked sub-Poissonian lasers can serve as an effective source of entanglement for different protocols of quantum information. Below, we are going to prove this by analyzing quantum dense coding and teleportation protocols.

B. The dense coding setup

We will discuss the same setup as in Ref. [3]. The scheme of the protocol realization is shown in Fig. 4. The setup is based on the Mach-Zehnder interferometer. All space is divided by optical elements into several domains, therefore it is convenient to use different letters to denote the field amplitudes in different domains. Two sources of squeezed light emit the beams with amplitudes $\hat{S}_1(t)$ and $\hat{S}_2(t)$. We require that these sources be perfectly identical, but their fields are squeezed in mutually orthogonal quadratures. Being mixed at the input symmetrical beamsplitter BS_1, these fields produce two beams in an entangled state. We denote the amplitudes of the entangled beams by $\hat{E}_1(t)$ and $\hat{E}_2(t)$ [Eq. (63)]. We reserve the notation $\hat{A}(t)$ for Alice's signal, which is supposed to be transmitted through the dense coding setup.

The beam $\hat{E}_1$ in the upper arm of the interferometer interferes at the beamsplitter BS_A with Alice's signal $\hat{A}(t)$, which is intended to be delivered to Bob. We assume that Alice's signal comprises the information-carrying c-number amplitude $A(t)$ (classical signal) and the vacuum noise $\hat{A}_{vac.1}$,

$$\hat{A}(t)=A(t)+\hat{A}_{vac.1}(t). \tag{70}$$

Thus at BS_A, the field amplitude in the upper arm of the interferometer is transformed according to

$$\hat{E}_1(t)\to\sqrt{\mathcal{T}}\hat{E}_1(t)+\sqrt{\mathcal{R}}[A(t)+\hat{A}_{vac.1}(t)], \tag{71}$$

where $\mathcal{T}$ and $\mathcal{R}$ are the transmission and reflection coefficients of BS_A, respectively ($\mathcal{T}+\mathcal{R}=1$).

The important property of the Mach-Zehnder interferometer is the ability to restore the quantum state of the input field at the output. Two independent squeezed beams at BS_1 would be the same independent squeezed beams after BS_2

without BS_A in the upper arm. In other words, the property to restore quantum states refers to a symmetrical interferometer. Thus an ancillary beamsplitter identical to BS_A has to be introduced in the lower arm of the interferometer to preserve the symmetry. Then the beam $\hat{E}_2$ will also be transformed as

$$\hat{E}_2(t) \to \sqrt{\mathcal{T}}\hat{E}_2(t) + \sqrt{\mathcal{R}}\hat{A}_{\text{vac.2}}(t). \tag{72}$$

Finally, at Bob's site the beams leaving the interferometer are

$$\hat{\hat{B}}_1(t) = \sqrt{\mathcal{R}/2}[A(t) + \hat{A}_{\text{vac.1}}(t) + \hat{A}_{\text{vac.2}}(t)] + \sqrt{\mathcal{T}}\hat{S}_1(t), \tag{73}$$

$$\hat{\hat{B}}_2(t) = \sqrt{\mathcal{R}/2}[A(t) + \hat{A}_{\text{vac.1}}(t) - \hat{A}_{\text{vac.2}}(t)] + \sqrt{\mathcal{T}}\hat{S}_2(t). \tag{74}$$

One sees that the signal sent by Alice appears in both output beams, although it was introduced only in the upper arm. The next step of the protocol is detecting by Bob the squeezed quadratures of the output beams in order to extract the information sent by Alice.

C. Spectral signal-to-noise ratio

In order to calculate some specific signals, in particular the signal-to-noise ratio or the capacity of the dense coding channel, we need to model the Alice signal. Often it is assumed that the random Alice amplitude A_ω obeys the Gaussian statistics, so the corresponding Wigner function reads

$$W(A_\omega) = \frac{1}{\pi\sigma_\omega^A}\exp\left(-\frac{|A_\omega|^2}{\sigma_\omega^A}\right). \tag{75}$$

Further, under the calculation of the Shannon information, we will choose the explicit form for the variance σ_ω^A as

$$\sigma_\omega^A = \frac{P}{\sqrt{\pi\Delta\omega_A^2/2}}\exp\left(-\frac{\omega^2}{\Delta\omega_A^2/2}\right), \tag{76}$$

where P is the integral Alice stream dense (the mean photon numbers per second) and $\Delta\omega_A$ is the spectral width of Alice's signal.

Bob can measure the output fields $\hat{B}_1(t)$ and $\hat{B}_2(t)$ with two independent photodetectors. Then the corresponding photocurrent operators read

$$\hat{i}_m(t) = \hat{B}_m^\dagger(t)\hat{B}_m(t), \quad m = 1,2. \tag{77}$$

In the dense coding scheme one needs, however, to measure the quadratures, thus a balanced homodyne detection has to be used. The fluctuation current operators $\delta\hat{i}_m(t) = \hat{i}_m(t) - \langle\hat{i}_m\rangle$ in this case can be written as

$$\delta\hat{i}_m(t) = \beta_m^*\delta\hat{B}_m(t) + \beta_m\delta\hat{B}_m^\dagger(t), \tag{78}$$

where β_m is the amplitude of the local oscillator. We choose the phase of the local oscillator in such a way that $\beta_1 = \beta_1^* \equiv \beta$ and $\beta_2 = -\beta_2^* \equiv i\beta$. Then the photocurrent fluctuations read

$$\delta\hat{i}_1(t) = \beta[\delta\hat{B}_1(t) + \delta\hat{B}_1^\dagger(t)], \quad \delta\hat{i}_2(t) = i\beta[\delta\hat{B}_2(t) - \delta\hat{B}_2^\dagger(t)]. \tag{79}$$

This choice ensures that two mutually orthogonal squeezed field quadratures from the different sources are selected on the different detectors.

By applying the well-known procedure of calculation, one gets the photocurrent spectrum of each of the photodetectors in the form

$$(\delta i_m^2)_\omega = \beta^2[\mathcal{R} + \mathcal{T}4(\delta X_1^2)_\omega + \mathcal{R}\sigma_\omega^A]. \tag{80}$$

Here we assume again that both lasers forming the Einstein-Podolsky-Rosan (EPR) light are completely identical but they are synchronized to have different phases zero and $\pi/2$, respectively. This means that $(\delta X_1^2)_\omega = (\delta Y_2^2)_\omega$ and $(\delta Y_1^2)_\omega = (\delta X_2^2)_\omega$. Defining the signal-to-noise ratio as

$$\text{SNR}_\omega = \frac{\mathcal{R}\sigma_\omega^A}{\mathcal{R} + \mathcal{T}4(\delta X_1^2)_\omega}, \tag{81}$$

it is not difficult to derive for this quantity the following result in the saturation laser regime:

$$\text{SNR}_\omega = \frac{\mathcal{R}\sigma_\omega^A[\omega^2 + (1-\mu/2)^2\kappa^2]}{\omega^2 + (1-\mu/2)^2\kappa^2 - \mathcal{T}p(1-\mu)\kappa^2}. \tag{82}$$

Performing this calculation, we assumed that the Fourier component A_ω of the Alice amplitude obeys the Gaussian statistics.

If the laser operates in the Poissonian regime ($p=0$), then the signal is observed on the level of the shot noise. We regard this value of the signal-to-noise ratio as the minimal one. It reads

$$\text{SNR}_\omega^{\min} = \mathcal{R}\sigma_\omega^A. \tag{83}$$

In the sub-Poissonian regime as $p=1$, the spectral variance becomes frequency-dependent,

$$\text{SNR}_\omega = \frac{\omega^2 + \kappa^2}{\omega^2 + (\mathcal{R} + \mu^2/4)\kappa^2}\mathcal{R}\sigma_\omega^A, \tag{84}$$

and achieves its maximum as $\Delta\omega_A \ll \kappa$.

The factor in front of the minimal signal-to-noise ratio contains three parameters: $\mathcal{R}$ and $\mathcal{T}$ ($\mathcal{R}+\mathcal{T}=1$) determine the input possibilities of Alice and $\mu \ll 1$ restricts the power of the synchronizing field in the laser. It is readily seen that this factor can be made very large as $\mathcal{R} \ll 1$ and the laser operates in the appropriate regime (see Fig. 5).

D. Shannon mutual information

To calculate the information capacity due to the dense coding protocol, we apply the same theoretical consideration as in Ref. [3]. As discussed in these references, the appropriate quantitative characteristic in this case is the so-called Shannon mutual information (SMI) stream density. Assuming a Gaussian information channel, one can express the SMI stream density for this channel as

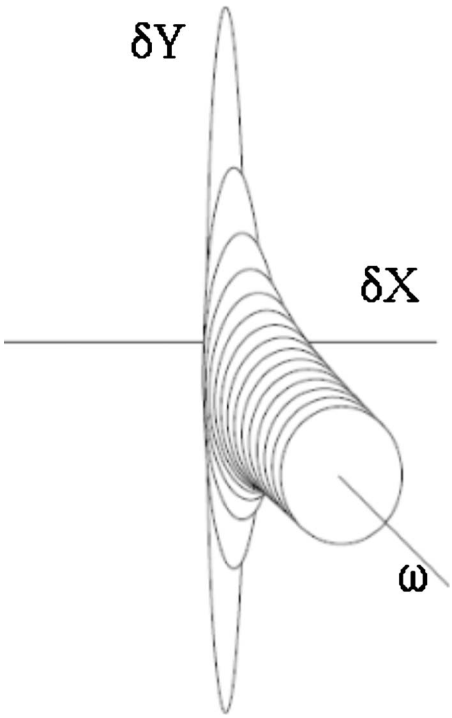

FIG. 5. The uncertainty ellipse versus the frequency of the mode.

$$I^{\mathrm{Sh}} = \int_{-\infty}^{+\infty} \ln(1 + \mathrm{SNR}_{\omega}) d\omega. \tag{85}$$

Substituting Eq. (84) here and choosing the spectral variance for Alice's signal as the Gaussian distribution (76), we can calculate the SMI.

In the case of the Poissonian laser ($p=0$), SMI stream density reads

$$I^{\mathrm{Sh}} = \int_{-\infty}^{+\infty} \ln\left(1 + \frac{\mathcal{R}P}{\sqrt{\pi\Delta\omega_A^2/2}} e^{-(\omega^2/\Delta\omega_A^2/2)}\right) d\omega. \tag{86}$$

For a weak Alice signal as $\mathcal{R}P \ll \Delta\omega_A$, this integral can be explicitly evaluated to give

$$I^{\mathrm{Sh}} = \mathcal{R}P, \tag{87}$$

i.e., the SMI stream density is exactly equal to the Alice signal stream density in the information channel.

Being interested in quantum regimes, we therefore choose the sub-Poissonian laser regime $p=1$. Then a combined expression for the SMI can be written in the following form:

$$I^{\mathrm{Sh}} = \int_{-\infty}^{+\infty} \ln\left(1 + \frac{\omega^2 + \kappa^2}{\omega^2 + \kappa^2(\mathcal{R} + \mu^2/4)} \times \frac{\mathcal{R}P}{\sqrt{\pi\Delta\omega_A^2/2}} e^{-(\omega^2/\Delta\omega_A^2/2)}\right) d\omega. \tag{88}$$

To perform numerical integration in Eq. (88), it is convenient to introduce the dimensionless parameters

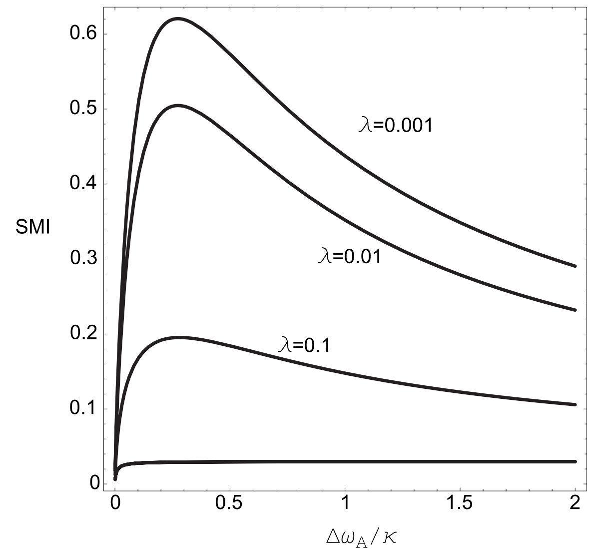

FIG. 6. Frequency dependence of the mutual information stream under the parameters $\sqrt{\mathcal{R}}=0.1, \mathcal{P}=2\pi P/\kappa=3$.

$$2\pi I^S/\kappa = \mathcal{I}^S, \quad \omega/\kappa \to \omega, \quad 2\pi\Delta\omega_A/\kappa = d_A, \quad 2\pi P/\kappa = \mathcal{P}. \tag{89}$$

The results of the integration are shown in Fig. 6. All the curves represent the dependence of the SMI on the scaled bandwidth d_A. The lower curve corresponds to the Poissonian regime of the laser ($p=0$) while all other curves to the sub-Poissonian regime ($p=1$) with different $\lambda=\mu^2/4$ belonging to the interval from 0.1 to 0.001. The uppermost curve corresponds to $\lambda=0.001$. Note that further decreasing of this parameter does not shift the curve. The reason for this is the fact that the λ parameter appears in the equations only in the combination $\lambda+\mathcal{R}$, where we have chosen $\mathcal{R}=0.01$. Thus the information transfer can be improved further by decreasing the reflection of the BS_A.

From the figure, one can see the essential advantage of using the sub-Poissonian laser regime. In this case, the SMI can achieve the value of 0.6, while for information transfer with classical light one has only 0.04. This difference depends on the value P/κ that has been chosen to be equal to 3. If P/κ is increased, then the difference becomes less pronounced and practically disappears at $P/\kappa=3000$.

IV. QUANTUM TELEPORTATION PROTOCOL

A. General scheme

We shall discuss the optical scheme for the teleportation similar to that proposed and realized in [2]. This scheme is shown in Fig. 7. As for the dense coding, the amplitudes $\hat{S}_1(t)$ and $\hat{S}_2(t)$ presenting the independent laser beams are converted on the symmetrical beam splitter to $\hat{E}_1(t)$ and $\hat{E}_2(t)$. The last amplitudes describe the entangled two-beam light. One of the beams is mixed again on the symmetrical beamsplitter with the Alice signal $\hat{A}_{\mathrm{in}}(t)$,

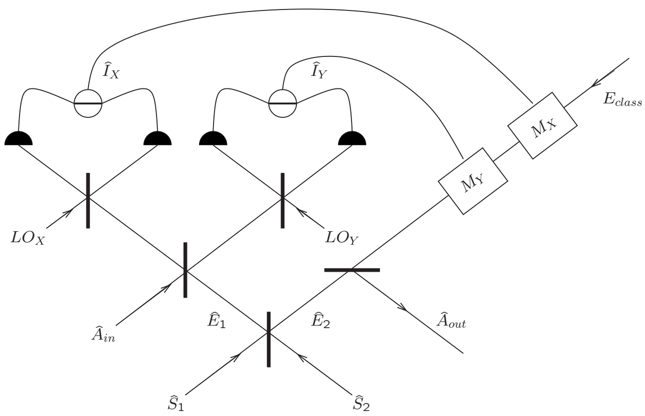

FIG. 7. Quantum teleportation protocol.

$$\hat{B}_x(t) = \frac{1}{\sqrt{2}}[\hat{A}_{\text{in}}(t) + \hat{E}_1(t)], \quad \hat{B}_y(t) = \frac{1}{\sqrt{2}}[-\hat{A}_{\text{in}}(t) + \hat{E}_1(t)]. \tag{90}$$

Then with the help of appropriately chosen local oscillators LO_X and LO_Y, the amplitude quadrature of the field $\hat{B}_x(t)$ and the phase quadrature of $\hat{B}_y(t)$ are detected. This in particular means that the photocurrents

$$\hat{i}_x(t) = \beta[\hat{B}_x^\dagger(t) + \hat{B}_x(t)], \quad \hat{i}_y(t) = i\beta[\hat{B}_y^\dagger(t) - \hat{B}_y(t)] \tag{91}$$

are registered. These photocurrents are sent from Alice to Bob via two classical communication lines. Bob uses these photocurrents for reconstruction of the field $\hat{A}_{\text{out}}(t)$ via two modulators M_x and M_y which modulate the corresponding quadrature components of an incoming plane coherent light wave with the mean amplitude E_0. At the output of the modulators, the amplitude reads

$$\hat{E}(t) = E_0 + \xi[\hat{i}_x(t) - i\hat{i}_y(t)], \tag{92}$$

where ξ describes the efficiency of the modulation. Mixing this field with the other part of the EPR pair on the well reflecting beamsplitter ($\mathcal{R}\approx 1, \mathcal{T}\ll 1, \mathcal{R}+\mathcal{T}=1$), Bob obtains his copy of Alice's signal,

$$\hat{A}_{\text{out}}(t) = \hat{A}_{\text{in}}(t) + \hat{F}(t), \quad \hat{F}(t) = \sqrt{2}[\delta\hat{X}_1(t) - i\delta\hat{Y}_2(t)] \tag{93}$$

given that the following equalities hold:

$$\xi\beta\sqrt{2\mathcal{T}} = 1, \quad \sqrt{\mathcal{T}/2}E_0 + \langle X_1\rangle - i\langle Y_2\rangle = 0. \tag{94}$$

Here we introduce the quadrature component $\hat{X}$ and $\hat{Y}$ according to

$$\hat{S}_m = \hat{X}_m + i\hat{Y}_m, \quad m = 1,2 \tag{95}$$

and their fluctuations

$$\hat{X}_m = \langle\hat{X}_m\rangle + \delta\hat{X}_m, \quad \hat{Y}_m = \langle\hat{Y}_m\rangle + \delta\hat{Y}_m. \tag{96}$$

Thus the output field completely reproduces the input one if the noise contribution $\hat{F}(t)$ is negligibly small. Using the quadrature squeezed states of the laser radiation, one reduces $\delta\hat{X}_1$ and $\delta\hat{Y}_2$, thereby increasing the fidelity of the teleportation.

In the Fourier domain, the operator at the output of the teleportation scheme reads

$$\hat{A}_{\text{out},\omega} = \hat{A}_{\text{in},\omega} + \hat{F}_\omega, \quad F_\omega = \sqrt{2}(\delta\hat{X}_{1,\omega} - i\delta\hat{Y}_{2,\omega}). \tag{97}$$

B. Spectral fidelity of the teleportation protocol

Addressing the fidelity of a teleportation scheme, we deal with optical fields propagating in free space, which can be presented as a set of oscillators parametrized by their frequencies ω. We denote by $\hat{\rho}_\omega^{\text{in}}$ and $\hat{\rho}_\omega^{\text{out}}$ the density operators of the ω oscillator at the input and at the output of our teleportation scheme, respectively. Let the spectral fidelity of this scheme be defined as

$$\mathcal{F}_\omega = \text{Tr}(\hat{\rho}_\omega^{\text{in}}\hat{\rho}_\omega^{\text{out}})/\text{Tr}(\hat{\rho}_\omega^{\text{in}})^2. \tag{98}$$

For the pure state, this expression is converted into well-known $|\langle\psi_{\text{in}}|\psi_{out}\rangle|^2$.

Using the formalism of the Wigner function, this definition can be cast into the form containing ordinary functions,

$$\text{Tr}(\hat{\rho}_\omega^{\text{in}}\hat{\rho}_\omega^{\text{out}}) = \pi\int d^2\alpha_\omega W_\omega^{\text{in}}(\alpha_{\text{in},\omega})W_\omega^{\text{out}}(\alpha_{\text{out},\omega}),$$

$$\text{Tr}(\hat{\rho}_\omega^{\text{in}})^2 = \pi\int d^2\alpha_\omega[W_\omega^{\text{in}}(\alpha_{\text{in},\omega})]^2. \tag{99}$$

Let the Wigner distributions be factorized in the form

$$W_\omega^{\text{in,out}}(\alpha_{\text{in,out}\omega}) = W_\omega^{\text{in,out}}(X_{\text{in,out}\omega})W_\omega^{\text{in,out}}(Y_{\text{in,out}\omega}), \tag{100}$$

where

$$\alpha_\omega = X_\omega + iY_\omega. \tag{101}$$

Let the quadrature component fluctuations obey the Gaussian distribution; then the integrals (99) can be calculated and the spectral fidelity is expressed via the in, out variances in the explicit form

$$\mathcal{F}_\omega = \left(\frac{1}{1 + (\delta X_1^2)_\omega/(\delta X_{\text{in}}^2)_\omega}\right)^{1/2}\left(\frac{1}{1 + (\delta Y_2^2)_\omega/(\delta Y_{\text{in}}^2)_\omega}\right)^{1/2}. \tag{102}$$

If one teleports light in a coherent state, then $(\delta X_{\text{in}}^2)_\omega = (\delta Y_{\text{in}}^2)_\omega = \frac{1}{4}$ and the fidelity reads

$$\mathcal{F}_\omega = \left(\frac{1}{1 + 4(\delta X_1^2)_\omega}\right)^{1/2}\left(\frac{1}{1 + 4(\delta Y_2^2)_\omega}\right)^{1/2}. \tag{103}$$

Setting then $(\delta X_1^2)_\omega = (\delta Y_2^2)_\omega$, and using the result for the quadrature spectral variance of the sub-Poissonian laser, one ends up with the following result for the spectral fidelity:

$$\mathcal{F}_\omega = \frac{1}{2}\frac{\omega^2 + \kappa^2}{\omega^2 + \kappa^2(1 - p/2)}, \quad F_{\omega=0} = \frac{1}{2 - p}. \tag{104}$$

For the laser in a Poissonian regime ($p=0$), the expected result $F_\omega = \frac{1}{2}$ is revealed. This is just the classical limit for the teleportation fidelity, however for the sub-Poissonian lasing ($p=1$), the fidelity appears to be

$$F_{\omega}=\frac{1}{2}\frac{\omega^2+\kappa^2}{\omega^2+\kappa^2/2}. \quad (105)$$

One can see that at zero frequency $F_{\omega=0}=1$, which indicates quantitatively the improved quality of the teleportation with nonclassical light and proves the feasibility of the proposed sources.

V. SUMMARY

Aiming to develop an effective source of entangled light, we consider in this paper the quantum properties of radiation emitted by the sub-Poissonian laser with external synchronization. It has been shown that these systems can operate in essentially nonclassical regimes and hence can be used in temporally multimode quantum information schemes. In particular, we have shown that weak external synchronization can greatly reduce laser phase diffusion without disturbing nonclassical photon statistics. This allows for generation of quadrature-squeezed light. Moreover, varying the phase of the synchronizing field, one can squeeze different quadratures. Thus we believe that the laser source discussed in this paper can serve as an effective and flexible tool for quantum information.

To prove the feasibility of the considered systems, the operation of two well-known quantum information protocols based on these systems has been studied. First, quantum dense coding has been addressed, where the Shannon mutual information of the channel based on the phase-locked sub-Poissonian laser has been calculated. It has been shown that the regimes for the laser exist, where the Shannon information exceeds considerably that for the channel based on a coherent beam. This regime is realized for regular pumping (sub-Poissonian generation) and a weak synchronizing external field, i.e., in the case of best squeezing.

The proposed systems have also been tested in a quantum teleportation scheme. To quantify the efficiency of the systems in the teleportation protocol, the spectral fidelity has been considered. It has been shown that operating in the essentially quantum regimes discussed above, both systems demonstrate a zero-frequency fidelity of $\frac{2}{3}$, whereas teleportation with a classical field yields $\frac{1}{2}$. Furthermore, the results for the information protocols have been confirmed on a more general basis of applying the Duan entanglement criterium.

ACKNOWLEDGMENTS

This work was performed within the French-Russian cooperation program "Lasers and Advanced Optical Information Technologies" with financial support from the following organizations: INTAS (Grant No. 7904), YS-INTAS (Grant No. 6078), RFBR (Grant No. 05-02-19646), and Ministry of Education and Science of RF (Grant No. RNP 2.1.1.362). We would like to thank M. Kolobov and C. Fabre for productive discussions.

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