Extreme nonlinear optics in a Kerr medium: Exact soliton solutions for a few cycles

A. V. Kim,[1] S. A. Skobelev,[1] D. Anderson,[2] T. Hansson,[2] and M. Lisak[2]
[1]*Institute of Applied Physics, Russian Academy of Sciences, 603950 Nizhny Novgorod, Russia*
[2]*Department of Radio and Space Science, Chalmers University of Technology, SE-412 96 Göteborg, Sweden*
(Received 10 October 2007; published 17 April 2008)

Exact soliton solutions containing only a few cycles are found within the framework of a nonlinear full wave equation in a Kerr medium. It is proven numerically that they are stable and play a fundamental role in the pulse propagation dynamics. These wave solitons cover the range from the fundamental Schrödinger solitons, which occur for long pulses involving many field oscillations, to extremely short pulses, which contain only one optical period.

DOI: 10.1103/PhysRevA.77.043823 PACS number(s): 42.65.Tg

I. INTRODUCTION

The establishment of solitary waves is a key element in the understanding of many widely differing nonlinear wave phenomena (see, e.g., [1]). In particular, optical solitons have played an exceptionally important role in conventional nonlinear optics where the slowly varying envelope approximation (SVEA) is conventionally applied [2]. However, the progress in the last decade in laser science has established a new field of "extreme nonlinear optics," which involves very short laser pulses comprising only a few optical cycles and pulses of high intensities [3]. Such few-cycles laser pulses in high-field science open new possibilities for studying ultrafast phenomena occurring on an attosecond time scale [4,5]. Another important example is pulses of THz frequencies, which also comprise a few cycles of the field [6]. High intensities generated with such electromagnetic pulses are of interest for nonlinear physics with accompanying intensity-dependent corrections to the index of refraction [7,8]. This actually raises the question of whether the concept of solitons can be extended to this new regime. Indeed, some important results have recently been obtained by employing a unidirectional approach, which is based on the slowly evolving wave field approximation (SEWA) and neglects back-reflected waves [9–14]. Within this approach a few-optical-cycle solitons are found and it is shown that they play an important role in the pulse propagation dynamics. However, the establishment of a soliton solution within the framework of an exact full wave equation can shed new light on the fundamental properties of nonlinear waves and can extend the soliton concept to higher intensities and shorter durations where the SEWA is not applicable [1]. There are a few examples in optical physics where soliton solutions of the full wave equation have been considered, but mostly for resonant media. There are also a couple of examples in other areas of physics e.g., relativistic envelope solitons in plasmas [15] and solitary waves in the so-called φ^4 field theory [1]. However, most of these solutions have been found to be unstable.

In this paper, we present results on electromagnetic field dynamics within the framework of a new nonlinear full wave equation. A new class of exact solitary solutions for a few cycles is found, which describes the propagation of extremely short pulses in a nonresonant Kerr medium. An appealing feature of this result is that there is a continuous transition between the new solutions and the classical Schrödinger-type solitons This makes it possible to follow the transition from Schrödinger solitons to few-cycles pulses even down to single-cycle duration.

II. BASIC EQUATION

In general, the vector wave equation for isotropic media can be written in the following form:

$$\frac{\partial^2 \mathbf{E}}{\partial z^2} - \frac{1}{c^2}\frac{\partial^2}{\partial t^2}\int_{-\infty}^{t} \varepsilon(t-t')\mathbf{E}(t')dt' = \frac{4\pi}{c^2}\frac{\partial^2 \mathbf{P}_{nl}}{\partial t^2}, \qquad (1)$$

where ε is the linear permittivity. In media with an ultrabroad spectral region of transparency, $\omega_1 \ll \omega \ll \omega_2$, it follows from the Kramers-Kronig relation that

$$\varepsilon_r(\omega) = 1 + \frac{2}{\pi}\int_0^{\infty} \frac{x\varepsilon_i(x)dx}{x^2-\omega^2}, \qquad (2)$$

where ε_r and ε_i are the real and imaginary parts of ε, respectively. This implies that [16]

$$\varepsilon_r(\omega) = \varepsilon_o - \frac{\omega_p^2}{\omega^2}, \qquad (3)$$

where $\varepsilon_o = 1+(2/\pi)\int_{\omega_2}^{\infty}(\varepsilon_i/x)dx$ is the static permittivity and $\omega_p^2 = (2/\pi)\int_0^{\omega_1} x\varepsilon_i dx$. The latter relation may also be written as $\omega_p^2 = 4\pi e^2 N/m$ in terms of the number of oscillators N responsible for absorption in the frequency range from 0 to ω_1. It should be emphasized that the group velocity dispersion inherent in Eq. (3) has anomalous character, whereas in optics typically the contributions giving rise to anomalous and normal dispersion are equally important. This situation occurs when the second term of Eq. (3) is much smaller than the first one, i.e., $\omega_p^2/\omega^2 \ll \varepsilon_o$, and the higher order term of the Taylor expansion in Eq. (2) (in the small parameter $\omega^2/x^2 \ll 1$ for $x > \omega_2$) must be taken into account. With this expansion and applying the SEWA, a few-cycles soliton theory has been developed [12–14]. However, the main purpose of the present work is to develop a soliton theory for the nonlinear full wave equation.

In the case of nonresonant media, the vectorial Kerr effect can be described by the nonlinear polarization given by $\mathbf{P}_{nl} = \chi^{(3)}|\mathbf{E}|^2\mathbf{E}$ (where $\chi^{(3)}$ is the cubic nonlinear susceptibility) and the nonlinear atomic response is assumed to be instantaneous (see, e.g., [2,4]). In this approximation, the following nonlinear wave equation is obtained:

$$\frac{\partial^2 \mathbf{E}}{\partial z^2} - \frac{\varepsilon_o}{c^2}\frac{\partial^2 \mathbf{E}}{\partial t^2} - \frac{\omega_p^2}{c^2}\mathbf{E} - \frac{\varepsilon_o}{c^2}\frac{\partial^2}{\partial t^2}(|\mathbf{E}|^2\mathbf{E}) = 0, \tag{4}$$

where we have introduced the normalized field, $\mathbf{E} \to (4\pi\chi^{(3)}/\varepsilon_o)^{1/2}\mathbf{E}$.

III. EXACT SOLITON SOLUTIONS FOR A FEW CYCLES

For localized field distributions, the following integral relation can be derived from Eq. (4):

$$\int_{-\infty}^{+\infty} \mathbf{E}dt = 0. \tag{5}$$

This indicates the oscillation character of the solutions which implies that the average field must be equal to zero. In the following analysis we will consider circularly polarized light i.e., $\mathbf{E}(z,t)=\mathcal{E}(z,t)\cos\varphi(z,t)\mathbf{e_x}+\mathcal{E}(z,t)\sin\varphi(z,t)\mathbf{e_y}$. For the amplitude $\mathcal{E}(z,t)$ and the phase $\varphi(z,t)$ we then obtain the following set of equations:

$$\mathcal{E}_{zz} - \mathcal{E}\varphi_z^2 - (\varepsilon_o/c^2)(\mathcal{E}_{tt} - \mathcal{E}\varphi_t^2) - (\omega_p^2/c^2)\mathcal{E} - (\varepsilon_o/c^2)(6\mathcal{E}\mathcal{E}_t^2 + 3\mathcal{E}^2\mathcal{E}_{tt} - \mathcal{E}^3\varphi_t^2) = 0, \tag{6}$$

$$2\mathcal{E}_z\varphi_z + \mathcal{E}\varphi_{zz} - (\varepsilon_o/c^2)(2\mathcal{E}_t\varphi_t + \mathcal{E}\varphi_{tt}) - (\varepsilon_o/c^2)(6\mathcal{E}^2\mathcal{E}_t\varphi_t + \mathcal{E}^3\varphi_{tt}) = 0, \tag{7}$$

where low indices denote derivatives.

We now assume that the amplitude propagates with constant velocity and look for solutions in the form

$$\mathcal{E}(z,t) = \mathcal{E}(\xi), \tag{8}$$

$$\varphi(z,t) = \omega t - kz + F(\xi), \tag{9}$$

where $\xi = z - Vt$ and the function $F(\xi)$ obeys the equation

$$(1-\beta^2A^2)AF_{\xi\xi} + 2(1-3\beta^2A^2)A_\xi F_\xi + 2\left[\gamma^2\left(\frac{\varepsilon_o V\omega}{c^2} - k\right) + \frac{3\varepsilon_o V\omega A^2}{c^2}\right]A_\xi = 0, \tag{10}$$

where $\beta = V\varepsilon_o^{1/2}/c$ and $\gamma = (1 - V^2\varepsilon_o/c^2)^{-1/2}$ is the relativistic factor. To simplify the expression in Eq. (10), we have introduced the normalized amplitude $A = \gamma\mathcal{E}$ and imposed the constraint that $F \to 0$ (or const) as $\xi \to \infty$. In the limit when $A \to 0$, we infer that

$$V\frac{\omega}{k} = \frac{c^2}{\varepsilon_o}, \tag{11}$$

otherwise the frequency ω and the wave number k should be redefined. It should be pointed out that the relation (11) between group $(V = \partial\omega/\partial k)$ and phase velocities (ω/k) is true also for linear waves, since in this case ω and k satisfy the dispersion equation $k = \omega/c(\varepsilon_o - \omega_p^2/\omega^2)^{1/2}$, cf. Eq. (4). Integration of Eq. (10) yields the following expression for the nonlinear phase modulation:

$$F(\xi) = -\frac{\beta k_o}{2}\int_{-\infty}^{\xi}\frac{A^2(3-2\beta^2A^2)}{(1-\beta^2A^2)^2}d\xi', \tag{12}$$

where $k_o = \omega\sqrt{\varepsilon_o}/c$. Substituting the local frequency $\varphi_t = \omega - VF_\xi$ and the local wave number $\varphi_z = -k + F_\xi$, where $F_\xi = -(\beta k_o/2)A^2(3-2\beta^2A^2)/(1-\beta^2A^2)^2$, into Eq. (6), we arrive at the second-order differential equation

$$A_{\xi\xi} - \frac{6\beta^2AA_\xi^2}{1-3\beta^2A^2} - \frac{k_o^2A}{1-3\beta^2A^2}\times\left(\alpha^2 - \frac{A^2[4(1-\beta^2A^2)^2 - \beta^2A^2]}{4(1-\beta^2A^2)^3}\right) = 0, \tag{13}$$

where we have introduced $\alpha^2 = \gamma^2\omega_p^2/(\omega^2\varepsilon_o) - 1$. It is interesting to note that Eq. (13) is similar to that obtained by using the SEWA approach [12,13]. However, since Eq. (13) is obtained within the framework of the full wave equation, it is valid for arbitrary laser pulse velocities and from a physical point of view it describes a new type of field structure.

Equation (13) simplifies significantly in the case of zero group velocity, i.e., for $\beta = 0$. In this case there is no phase modulation, i.e., $F = 0$, and the envelope of the pulse has sech-shaped form, $A(\xi) = \sqrt{2|\varepsilon|/\varepsilon_o}\,\mathrm{sech}(\sqrt{|\varepsilon|/\varepsilon_o}k_o\xi)$, where $\varepsilon = \varepsilon_o - \omega_p^2/\omega^2 < 0$. This solution describes standing solitons in a medium with negative dielectric permittivity, a situation that can occur for carrier frequencies less than some critical value, $\omega < \omega_p/\varepsilon_o^{1/2}$. This field distribution is well known in plasma physics where such nonlinear electromagnetic structures correspond to cavities with locally positive permittivity in a region of high field intensity surrounded by an opaque medium where $\varepsilon < 0$ (see, e.g., [15]).

In the case of arbitrary group velocity and in terms of a new normalized amplitude $a = \beta A = \beta\gamma\mathcal{E}$, Eq. (13) can be integrated once for localized solutions (i.e., when $a, a_\xi \to 0$ for $\xi \to \pm\infty$) to read as

$$a_\xi = \pm\frac{(\omega/V)(1+\delta^2)^{1/2}a}{(1-3a^2)(1-a^2)}\sqrt{\left(1-\frac{3}{2}a^2\right)\left(\frac{1+4\delta^2+\sqrt{1-8\delta^2}}{4(1+\delta^2)} - a^2\right)\left(\frac{1+4\delta^2-\sqrt{1-8\delta^2}}{4(1+\delta^2)} - a^2\right)}. \tag{14}$$

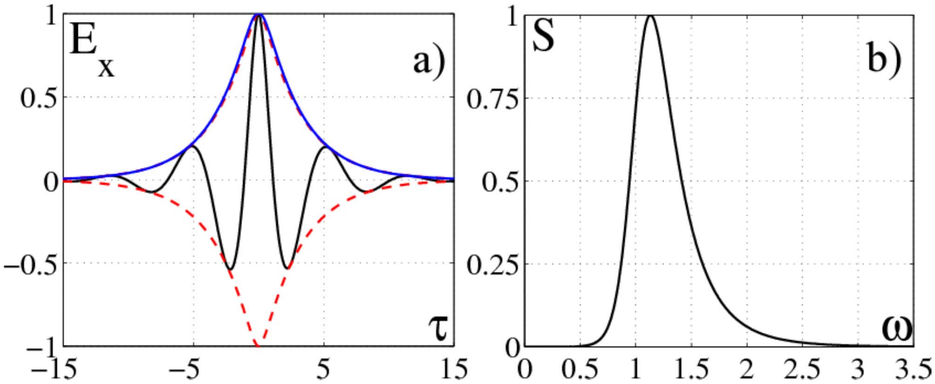

FIG. 1. (Color online) Temporal profile of a few-cycles soliton for δ=0.33: (a) Field distribution (solid line), exact envelope (dashed line), approximate envelope given by Eq. (15) (upper solid line) and (b) spectrum of the soliton.

As is easily seen, Eq. (14) has localized solutions with maximum amplitude $a_{\max}^2=(1+4\delta^2-\sqrt{1-8\delta^2})/[4(1+\delta^2)]$ provided $\delta^2\equiv(\alpha\beta)^2<1/8$, which defines the range of velocities where the wave solitons exist for given carrier frequency ω. Together with the nonlinear self-phase modulation given by Eq. (12), this completely characterizes the wave solitons. An example of such a few-cycle soliton is shown in Fig. 1(a). For convenience we choose to present the result in terms of the retarded time $\tau=\omega t-k_o z/\sqrt{\varepsilon}$. The solitons are localized in space, propagate with constant group velocity, and their duration can be comparable with the optical period. However, compared with the classical solitary waves [1], they are chirped with an ultrabroad spectrum extending over a whole octave as shown in Fig. 1(b). These solitons have a minimum duration of $\tau_{\min}\cong 2.3$ [full width at half-maximum (FWHM) of the intensity] and the corresponding maximum amplitude is equal to $a_{\max}=\sqrt{2/3}$. In this limit case, Eq. (14) can be integrated exactly and its solution is given by the implicit expression $(2-3u^2)^{1/2}/3-(1/2)\text{Arch}(\sqrt{2/3}u^{-1})=\pm\omega\xi/(4V)$ where Arch x denotes the inverse function of cosh x.

In order to present direct evidence of the reduction of the new soliton to the fundamental Schrödinger soliton, we consider solutions of Eq. (14) for small amplitudes, i.e., $a_{\max}^2 \ll 1$, which corresponds to long pulses containing many optical cycles where the SVEA is valid. In this case the solution can be written as

$$\frac{9}{2}\delta\sqrt{\delta^2-\frac{a^2}{2}}-\text{Arch}\left(\frac{\sqrt{2}\delta}{a}\right)=\pm\frac{\omega}{V}\delta\xi. \tag{15}$$

It is interesting to note that this solution, with good accuracy, describes solitons even down to very short durations, except for the limiting case with the shortest duration $\tau=\tau_{\min}$. In Fig. 1(a) the blue line shows the approximate solution given by Eq. (15), which indeed agrees very well with the exact one. This approximate solution could be useful for analysis of the soliton dynamics. For example, by taking the cosh-function of both side of Eq. (15) and then making an expansion of the left-hand side in the small parameter $\delta^2\ll 1/8$, we obtain solutions of modified Schrödinger equations of any orders. To first order we obtain the fundamental Schrödinger soliton $a(\xi)=a_{\text{NLS}}=\sqrt{2}\delta/\cosh(\omega\delta\xi/V)$ [2]. To next order, we obtain a solution of the higher order nonlinear Schrödinger equation (HONSE) which accounts for nonlinear group velocity dispersion where the nonlinear phase modulation becomes important [17–19],

$$a=\frac{2\delta\sqrt{1-9\delta^2}}{\sqrt{1-18\delta^2+\cosh(2\omega\delta\xi/V)}},\quad F=-\frac{3\omega}{2V}\int_{-\infty}^{\xi}a^2d\xi'. \tag{16}$$

Thus, solitary wave structures defined by the nonlinear wave equation, Eq. (4), cover the whole spectrum of wave solitons from the fundamental Schrödinger solitons for long pulses containing many optical oscillations to solitons of extremely short durations, containing just a few cycles. However, even the shortest solitons must contain at least a couple of cycles in order to satisfy the integral relation, Eq. (5), which expresses that the average of the field must be equal to zero.

IV. NUMERICAL ANALYSIS

To study the stability problem of the wave solitons and their dynamical properties we have carried out simulations of Eq. (5). This equation has two constants of motion, the total energy $W=\int_{-\infty}^{+\infty}|\mathbf{E}|^2dt$ and the Hamiltonian $H=\int_{-\infty}^{+\infty}(\omega_p^2|\int_{-\infty}^{t}\mathbf{E}dt'|^2-\varepsilon_o|\mathbf{E}|^2-\varepsilon_o|\mathbf{E}|^4/2)dt$, a fact which has been used for high precision control of the calculations.

A. Single soliton excitation

In order to make long distance modeling and to distinguish between different solitons we have chosen the comoving frame which propagates with the linear group velocity, i.e., we use the retarded time τ. Figure 2 shows the excitation and subsequent stable propagation of a single soliton from a localized initial distribution taken as sech-shaped, $\mathbf{E}=\delta\sqrt{2n/(\omega^2-n)}\text{sech}(\delta\omega\tau)[\cos(\omega\tau)\mathbf{e_x}+\sin(\omega\tau)\mathbf{e_y}]$, where the following notations and input parameters have been used: $\omega/\omega_o\rightarrow\omega$, $\omega_p^2/\omega_o^2=n$ and $\omega=\omega_{in}=1.2$, $n=0.02$, $\delta=\delta_{in}=0.32$. A stable soliton pulse has formed at $z\approx 40$ [measured in linear dispersion lengths for the initial pulse ($L_{\text{dis}}=600$)] and has the parameters $\omega_s\simeq 1.02$ and $\delta_s\simeq 0.27$. In order to identify the physical parameters of the generated few-cycles solitons and to prove their soliton nature, we have used the approximate solution (15) from which the amplitude $\mathcal{E}_{\max}$ and the energy W_s can be obtained as $\mathcal{E}_{\max}^2=2\delta^2 n/(\omega^2-n)$ and

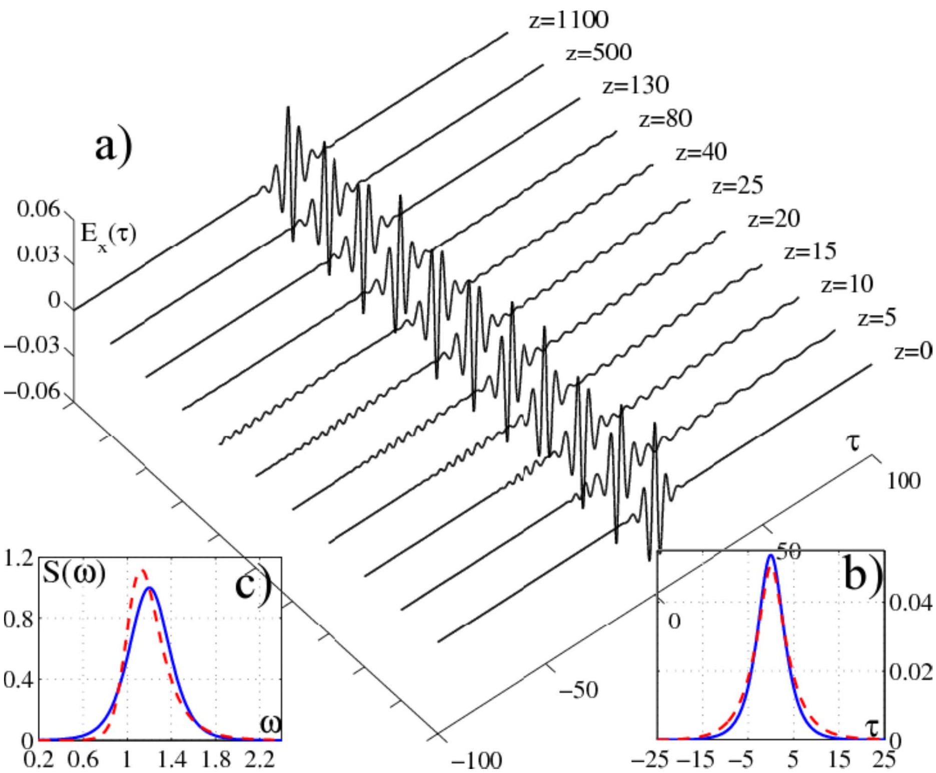

FIG. 2. (Color online) Snapshots of field distributions (a) along the propagation distance; (b), (c) input (solid line) and output (dashed line) pulse envelopes and the corresponding spectra of the pulse, respectively.

$W_s = 4\delta(1-3\delta^2)n/[\omega(\omega^2-n)]$ where the inequalities $n/\omega^2 \ll 1$, $\delta^2 \ll 1$ have been used. From the latter relation we can define the parameters of the few-cycles soliton as follows: Carrier frequency

$$\omega = \{[W_s^2 n - 24\mathcal{E}_{\max}^4 + \sqrt{(W_s^2 n - 24\mathcal{E}_{\max}^4)^2 + 32W_s^2\mathcal{E}_{\max}^2 n(1+3\mathcal{E}_{\max}^2)}]/(2W_s^2)\}^{1/2}$$

and $\delta = \mathcal{E}_{\max}\sqrt{(\omega^2-n)/(2n)}$. The sech-shaped distribution gives a good approximation of the soliton envelope in Fig. 2(b), but in the few-cycles regime, the initial carrier frequency and the spectrum of the soliton are essentially modified.

B. Dynamics of higher order solitons

Another point of fundamental interest is the dynamics of higher order solitons, which plays an exceptionally important role in the theory of the nonlinear Schrödinger equation as well as for different applications, e.g., for the optical pulse compression technique [2]. In analogy with the formalism for the NSE we introduce $K_S = \int_{-\infty}^{+\infty}\mathcal{E}_S(\tau)d\tau$ and the soliton number $N = \int_{-\infty}^{+\infty}\mathcal{E}(z=0,\tau)d\tau/K_S$. However, since for few-cycles solitons, K_S is not a universal value as in the NSE case (where it depends on the medium parameters only), we define an initial distribution in the form of higher order solitons as $\mathcal{E}(z=0,\tau) = N\mathcal{E}_S$, where $\mathcal{E}_S$ is the exact solution of Eq. (14). For very small values of δ ($\delta^2 \ll 1/8$), when the NSE approximation is a valid modeling of Eq. (4), the simulation results are in good agreement with the classical predictions of the higher order soliton dynamics of the NSE. However, when approaching the few-cycles regime, new characteristic features become important. First, for intermediate values of δ ($\delta \lesssim 0.095$), initial higher order soliton distributions split into a number of solitons determined by N, but the carrier frequency of each soliton may differ from the initial one. An example of such a solution is shown in Fig. 3 where the initial pulse data were taken as $\mathcal{E} = 2.05\mathcal{E}_S$ with $\omega = 1$ and $\delta = 0.08$. Eventually this pulse generated two solitons with the following parameters: $\omega_1 = 0.9$, $\delta_1 = 0.24$ and $\omega_2 = 0.95$, $\delta_2 = 0.08$. For comparatively short higher order few-cycles solitons with $\delta \gtrsim 0.095$, the value of N cannot be used for pre-

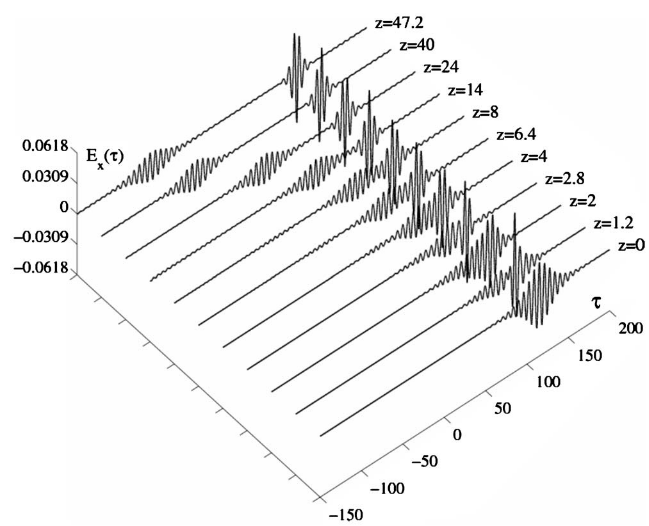

FIG. 3. Evolution of the field (x component) of an electromagnetic pulse during propagation for the case of an input pulse in the form of a higher order soliton with $N=2.05$, $\delta=0.08$, $\omega=1$. The two output solitons have the parameters $\delta_1 \simeq 0.24$, $\omega_1 \simeq 0.9$; $\delta_2 \simeq 0.08$, $\omega_2 \simeq 0.95$. The distance of propagation z is normalized to the dispersion length for the input pulse L_{dis}, which is $L_{\mathrm{dis}} \simeq 7200$.

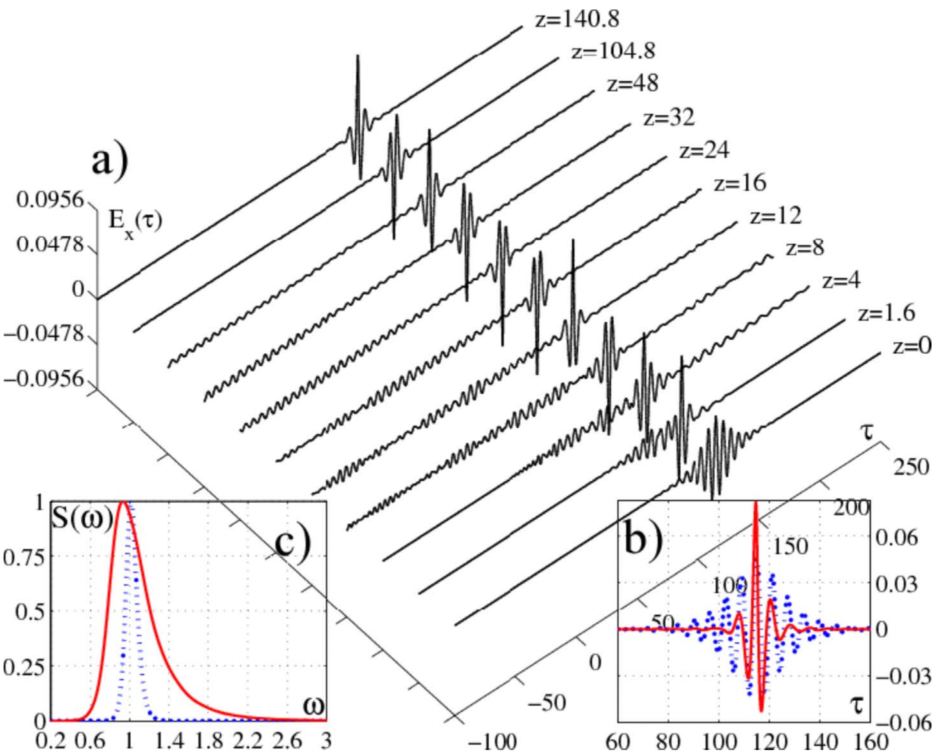

FIG. 4. (Color online) (a) Evolution of the pulse field (x component) during propagation for the case of input pulse as a higher order soliton with the parameters $N=2.05$, $\delta=0.11$, $\omega=1$; (b), (c) input (dashed line) and output (solid line) field distributions and their spectra, respectively. The output single soliton (at $z=140.8$) has $\delta_1 \simeq 0.34$ and $\omega_1=0.78$. The dispersion length is $L_{\mathrm{dis}} \simeq 3000$.

dictions of the higher order soliton dynamics, in particular for the asymptotically emerging number of solitons, as in the case of the NSE. However, it is still true that higher order solitons will transform into shorter duration solitons which is one of the ways to generate extremely short few-cycles solitons, even down to a single optical period. Figure 4 shows the results of simulations similar to those in Fig. 3, but with the input parameter $\delta=0.11$. The dynamics is now completely different, the duration of the generated soliton is significantly shortened (by a factor of 5.9 times compared to the initial one), it also has a broader spectrum [see Figs. 4(b) and 4(c)], and its carrier frequency is down-shifted to $\omega_1=0.78$.

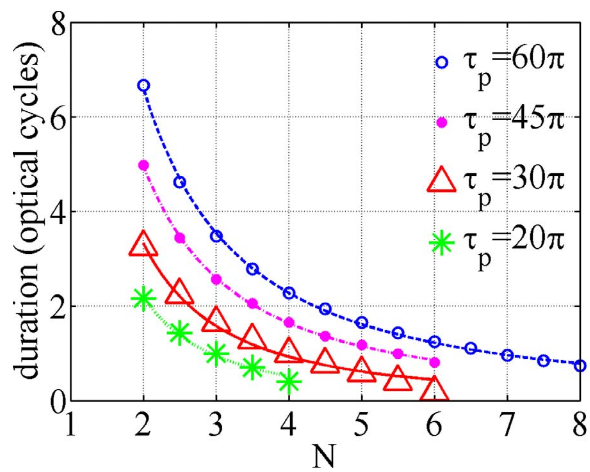

FIG. 5. (Color online) Dependence of the duration of compressed spikes on the soliton number N for different initial pulse durations.

C. Pulse self-compression

It should be noted that the efficiency of soliton excitation and pulse self-compression depends on initial pulse amplitude since back-scattered waves become important at higher amplitudes causing a difference between the full wave equation model and SEWA, which neglects back-reflected waves [12]. However, as follows from our simulations and as shown earlier in Ref. [20], this difference becomes significant only at very high input amplitudes when the Kerr contribution to the refraction index is comparable with the linear one. For the problem of few-cycle pulse generation using the atomic Kerr nonlinearity, the corresponding value is small as compared with unit. For this case we present practically interesting results on pulse self-compression down to single-cycle duration. Here we investigate the self-compression regime as an intermediate state of the pulse dynamics. After self-compression, the pulse starts to broaden, and then (depending on the parameter N) it splits into several few-cycle solitons, which are well separated in space as in Fig. 3. Since in the few-optical-cycle regime, the compression factor depends not only on N, as for Schrödinger solitons, but also on the initial pulse duration, τ_p. Figure 5 presents the dependence of the duration of compressed pulses on the soliton number N for different pulse durations. It is well fitted by the power law $\tau_p/\tau_{\min} \propto N^{\alpha}$ where α actually depends on the initial pulse duration and increases with decreasing τ_p: $\alpha=1.54;1.6;1.85;2.1$ for $\tau_p=60\pi;45\pi;30\pi;20\pi$ (the carrier period is 2π), respectively.

For possible experimental realization we present estimates corresponding to a $\lambda=0.8$ μm, 40-fs laser pulse with energy 1 mJ propagating through a 250 μm diameter capillary filled with helium at 4 bar pressure and pre-ionized argon at a few tens of Torr. For these parameters, our simulations demonstrate pulse compression down to 2 fs, occurring at a distance of $z=80$ cm.

V. CONCLUSION

In conclusion, we have developed a theoretical treatment of the soliton concept for the full nonlinear wave equation describing the propagation of arbitrary duration electromagnetic pulses in a Kerr medium, which predicts the existence of solitary waves with only a few-cycles duration. We have proven that these solitons are stable and can easily be excited by employing the concept of higher order solitons, which can also provide a way for generation of single-cycle solitons.

ACKNOWLEDGMENT

This work was partially supported by RFBR Grant No. N 07-02-00596. S.A.S. gratefully acknowledges the support by Dynasty Foundation and Russian Science Support Foundation.

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