Diffusion-induced decoherence of stored optical vortices

T. Wang,[1] L. Zhao,[1] L. Jiang,[2] and S. F. Yelin[1,3]
[1]*Department of Physics, University of Connecticut, Storrs, Connecticut 06269, USA*
[2]*Department of Physics, Harvard University, Cambridge, Massachusetts 02138, USA*
[3]*ITAMP, Harvard-Smithsonian Center for Astrophysics, Cambridge, Massachusetts 02138, USA*
(Received 2 August 2007; published 10 April 2008)

We study the coherence properties of optical vortices stored in atomic ensembles. In the presence of thermal diffusion, the topological nature of stored optical vortices is found not to guarantee slow decoherence. Instead the stored vortex state's decoherence is surprisingly larger than the stored Gaussian mode. Furthermore, calculation of the coherence factor shows that the center of the stored vortex becomes completely incoherent once diffusion begins and, when a reading laser is applied, the optical intensity at the center of the vortex becomes nonzero. Its implication for quantum information is discussed. A comparison of classical diffusion and quantum diffusion is also presented.

DOI: 10.1103/PhysRevA.77.043815 PACS number(s): 42.50.Gy, 03.67.−a

Photons can carry orbital angular momentum (OAM), which can be created [1], manipulated [2], and detected [3]. The OAM states are associated with vortices of a helical phase $e^{im\phi}$. Each vortex is a topological defect, characterized by a winding number m, obtained from the $2\pi m$ phase twist around the vortex. The light with OAM is exemplified by the Laguerre-Gaussian (LG) modes LG_p^m with p the number of nodes in the radial direction [4,5]. Entanglement through the OAM degree of freedom has been demonstrated for photon pairs from parametric down conversion [6]. Since the winding number can be any integers, the OAM states have been proposed theoretically for multiple-alphabet quantum cryptography with higher information density and a higher margin of security [7,8]. Other proposals, such as quantum coin tossing and violation of Bell inequalities, using the OAM states have been reviewed in Ref. [9].

One of the key challenges for optics based quantum-information processing (QIP) is the difficulty of storing optical fields. It has been demonstrated [10–12] that a superposition of the OAM states can be stored in a nonrotating Bose-Einstein condensates (BEC) in terms of vortex states of the condensate. Meanwhile, the OAM states can also be stored in "hot" atomic ensembles using slow light techniques [13–17]. The information of photonic states, namely, the amplitude and phase, is continuously transformed into Raman coherence, i.e., spin-density waves, of the atomic ensemble, and can be later retrieved. In practice, factors such as inhomogeneous magnetic field [18] and/or thermal diffusion can lead to the decay of the Raman coherence. To what extent can the optical vortex states really be stored coherently? Are they going to be more robust than the Gaussian state? The answers to these questions will determine how the OAM are used in QIP.

Generally speaking, the topological structure of a vortex makes it a good candidate for QIP [11,19,20] because it is stable against continuous deformations which cannot cause it to decay or to "unwind." Actual studies of such robustness against various processes are of course necessary. In particular, one needs to study the robustness in the presence of atom diffusion [21,22] for the stored coherence, which is crucial for applications such as quantum repeaters [23,24] and multiple beam splitters for generating entangled single photons [25]. Toward answering this question, Pugatch *et al.* [17] have shown that after some time of diffusion, the dark center of a stored OAM mode is well preserved, and the dark center of a Gaussian mode generated by blocking its center disappears. They attributed the stable dark center of the stored OAM mode to the robustness of the topological nature of a vortex against diffusion.

In this paper, we provide a careful study for the robustness of the stored vortex states (exemplified by $p=0$ unless otherwise stated) in the presence of diffusion. We find that the stable dark center of the vortex states is not directly associated with the topological robustness. The vortex states are actually more vulnerable to diffusion than the Gaussian state. This is because (1) for vortex states, diffusion induces destructive interference of the coherence, and (2) the diffusion is a global process and can destroy the topological order.

Our basic assumption is that there is no other dynamics besides diffusion. The evolution therefore obeys the diffusion equation $\dot{\rho}=D\nabla^2\rho$, where ρ and D are the density matrix and the diffusion coefficient, respectively. We consider a three-level lambda system as generally used for light storage [13,14]: the weak probe laser is applied between the ground state $|1\rangle$ and the excited state $|3\rangle$, and the pump laser is addressing the transition between $|3\rangle$ and another ground state $|2\rangle$.

Solving the diffusion equation gives the atomic coherence of a LG mode after diffusion time t [17],

$$\rho_{12}(\vec{r},t)=\frac{(-g/\Omega)}{\sqrt{s(t)}^{|m|+1}}A_m(r,\sqrt{s(t)}w_0)e^{-im\theta}, \tag{1}$$

with g the vacuum Rabi frequency for the probe field, Ω the Rabi frequency for the pump laser, $s(t)=(w_0^2+4Dt)/w_0^2$ an evolution factor, w_0 the waist, and $A_m(r,w_0)=(1/w_0)\sqrt{2P/\pi|m|!}(\sqrt{2}r/w_0)^{|m|}\exp(-r^2/w_0^2)$ the radial profile of a LG mode, where P is the total intensity. Note that Eq. (1) applies to the Gaussian mode ($m=0$) as well. For example, the evolution of stored coherence is shown in Figs. 1(a) and 2(a) for $m=1$ and $m=0$, respectively. At large r, the coherence homogeneously approaches zero due to the exponential factor in A_m. Note that Eq. (1) describes both a spread of the coherence, indicated by $\sqrt{s(t)}$ inside A_m, and a decay of the coherence compared to a purely coherent spread.

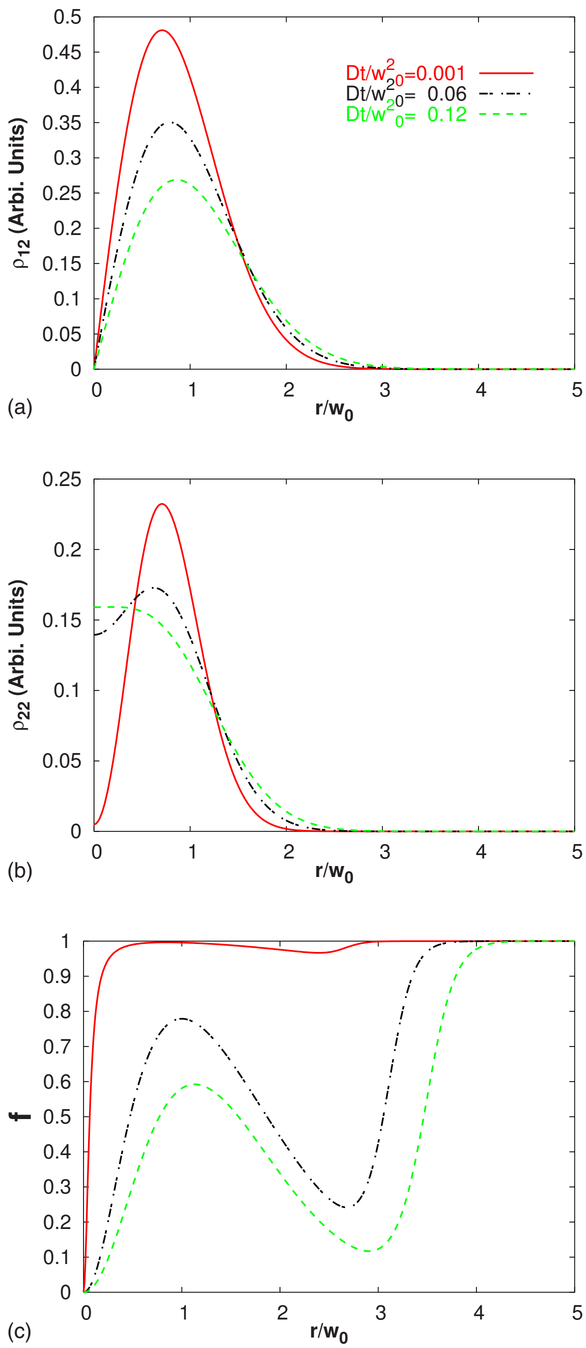

FIG. 1. (Color online) Effects of diffusion on a stored vortex with topological charge $m=1$. Plotted are (a) coherence ρ_{12}, (b) population ρ_{22}, and (c) coherence factor f as functions of radius, for different diffusion times. w_0,D are the waist of the LG mode and the diffusion coefficient, respectively.

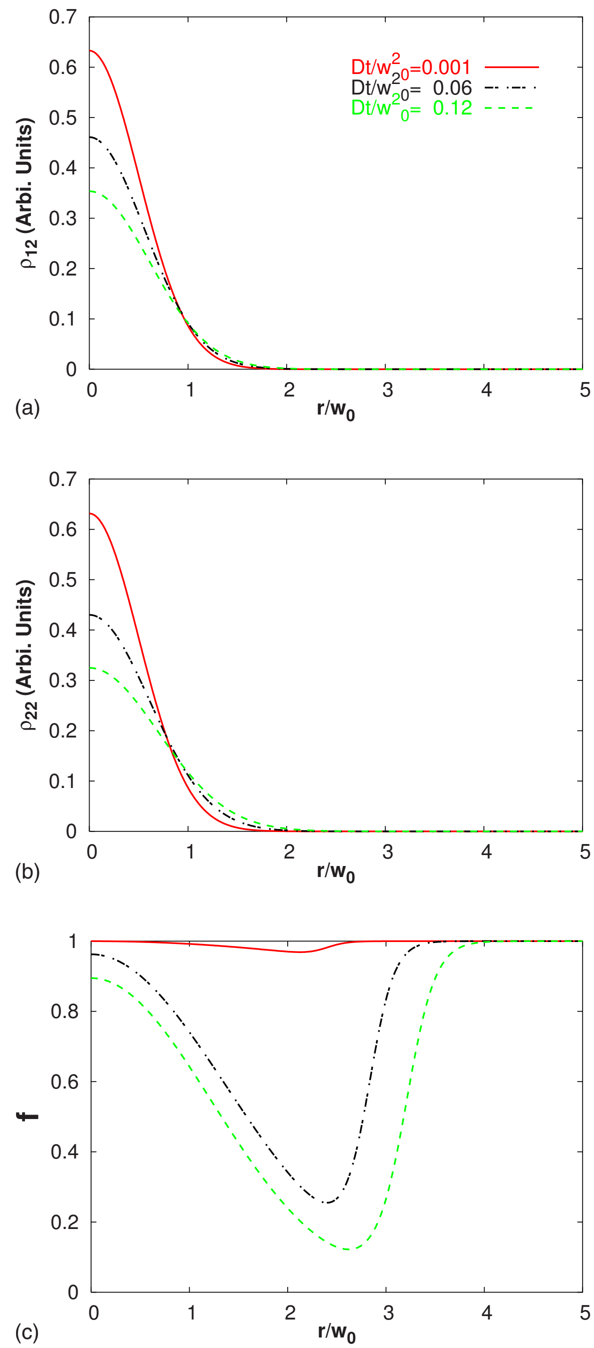

FIG. 2. (Color online) Effects of diffusion on a stored Gaussian mode. Plotted are (a) coherence ρ_{12}, (b) population ρ_{22}, and (c) coherence factor f as functions of radius, for different diffusion times. w_0,D are the waist of the Gaussian mode and the diffusion coefficient, respectively.

The decay is given by $\sqrt{s(t)^{|m|+1}}$ in the denominator. Then $F=1/s(t)^{|m|+1}$ gives the fidelity of the stored coherence. F shows that angular momentum states with different $|m|$ have different coherence decay factors. Although we will come back to the decay of coherence later, we note here that the larger OAM $|m|$, the larger the decay factor. The Gaussian mode, which has no phase singularity, has smaller coherence decay factor than all vortex states. This is because, without a vortex, the coherence is always in phase for different locations and there is no destructive interference to destroy the coherence. One can show that the retrieval efficiency along forward direction is the same as the fidelity F. Therefore, we come to a counterintuitive result—a Gaussian state has a higher storage fidelity (or retrieval efficiency) than the LG modes.

This may make the applications of optical vortex (with storage) for QIP questionable. Even though the excitation loss of a stored Gaussian or optical vortex state can be identified as a detected error and the threshold for detected errors allowed by quantum computation are realistic [26], fast quantum computation with reasonable resource overhead still requires relatively low error rates. This is also true for quantum communication [23,24], where, although the excitation loss can be fully controlled by its intrinsic purification, a fast quantum repeater still needs a fairly high retrieval efficiency.

The scaling factor $\sqrt{s(t)}$ in A_m of Eq. (1) means that the functional forms of coherence are preserved for both the LG mode and the Gaussian mode. This implies that the functional form stability of stored coherence against diffusion does *not* require topological defects. Indeed, the disappearance of the dark center of the center-blocked Gaussian mode after diffusion [17] demonstrates the stability of the Gaussian mode: diffusion tries to restore the nonzero intensity at its center. Of course, the restoration can also be understood by decomposing the center-blocked Gaussian mode to LG_p^0 and noting that $p\neq 0$ modes decay faster than the $p=0$ mode and what is left after some time of diffusion is just the Gaussian mode.

From the above discussion, it is clear that for the LG light $(m\neq 0)$, the coherence at the center $r=0$ stays zero during diffusion. However, as we show now, population at $r=0$ does not stay zero. To simplify the discussion, we assume that, at time $t=0$, the populations of atoms are $\rho_{11}(\vec{r},t=0)=1$ and $\rho_{22}(\vec{r},t=0)=|\rho_{12}(\vec{r},t=0)|^2$, which is a good assumption for strong pump and weak probe lasers as usually used in light-storage experiments. At time t after diffusion, we have $\rho_{11}(\vec{r},t)=1$ and

$$\rho_{22}(r,t)=\frac{4e^{-2r^2/8Dt+w_0^2}P(32D^2t^2+r^2w_0^2+4Dtw_0^2)}{\pi(8Dt+w_0^2)^3} \qquad (2)$$

for the $m=1$ vortex state and

$$\rho_{22}(r,t)=\frac{2e^{-2r^2/8Dt+w_0^2}P}{\pi(8Dt+w_0^2)} \qquad (3)$$

for a Gaussian mode. The evolution of the population is plotted in Figs. 1(b) and 2(b). While it seems that coherence only diffuses outwards in Fig. 1(a), Eq. (2) and Fig. 1(b) clearly show that diffusion goes in all directions as it should be. The outwards moving coherence during diffusion is because interference cancels the inwards diffusing coherence. In contrast, population does not interfere with itself and thus diffusion toward the center is clearly seen. Indeed, the population at the center quickly approaches a global maximum as time increases [Fig. 1(b)]. We also note that the integrated ρ_{22} over the whole space is conserved during the diffusion, i.e., no population is actually transferred.

We have seen that coherence and population diffuse differently [Eqs. (1)–(3)]. This brings in phase decoherence for the stored coherence. To characterize the decoherence, we define a coherence factor $f=\lim_{\eta\to 0}\frac{|\rho_{12}|^2+\eta}{\rho_{11}\rho_{22}+\eta}$. f is a function with $f=1$ for a pure state and $f=0$ for a completely mixed state. Thus, f is a good parameter to describe the (local) coherence property. As a specific example, we plot the coherence factor f in Figs. 1(c) and 2(c). Figure 1(c) shows that right after diffusion begins, the coherence factor $f(r=0)$ of the stored vortex drops from 1 to 0 because at $r=0$, the population becomes nonzero when diffusion starts while the coherence stays zero. We note that such sudden changes within an infinitesimal time are very uncommon in physical processes [32] and $f(r)$ approaches zero at very large times. This latter result holds for a Gaussian mode as well [shown in Fig. 2(c)]. We also note that at those distances where the diffusion of stored inhomogeneous coherence and population has not yet arrived, the coherence factor f stays at 1 because all population is in $|1\rangle$, a pure state. But the weight ρ_{22}, justified by its non-negative and conserved integration of these coherence factors for the retrieved light approaches zero.

Here is how the coherence factor f may be obtained from the experiments. When the reading pulse is applied, the incoherent part will be retrieved as fluorescence in all directions. Collecting both the fluorescent and the coherent emission then allows extracting f. Of course, setting the detector at the forward direction as generally used, e.g., in [17], can only collect a finite fraction of the fluorescence, while the coherent part is collected by the detector [27]. What we want to emphasize is that incoherence makes intensity at the center of a retrieved vortex *nonzero*, very different from a coherent vortex state. As the diffusion time increases, nonzero intensity at the center builds up. Therefore, generation of a mixed state makes an additional loss of retrieved fidelity. Diffusion of the population makes visibility decrease and finally kills the vortex. Of course, this part of the reduction of fidelity can be alleviated by using spatial filtering of the optical mode to prevent spontaneously emitted photons from going to the detector. In this case, the retrieval probability is just the fidelity $F=1/s(t)^{|m|+1}$.

The different collection efficiencies of fluorescent photons and coherent photons by a forward-set detector help to explain why the hole of the center-blocked Gaussian mode disappears very quickly, while the hole of a vortex disappears very slowly [17]. The homogeneous phase of the center-blocked Gaussian mode makes nonzero coherence inside the hole after diffusion and thus the disappearance of the hole once the coherence is read. This is in contrast with a stored vortex, whose coherence at the center remains zero all the time. The nonzero intensity at its center comes only from incoherence of the center. However, if most of the fluorescent photons are collected by the detector, the dark center of a vortex would disappear quicker than that of the Gaussian mode. It is the spatial filtering of the optical mode that helps to overcome the fluorescence from the center.

We now come back to the decay of coherence. We noted that the coherence of stored LG modes decays according to a power law $F=1/s(t)^{|m|+1}$. The larger the order of phase singularity given by m is, the faster the coherence decays. An additional example of the exponential decay rate $2Dk^2$ due to the diffusion of a plane wave e^{-ikx}, which is faster than any power law decay, also corroborates this idea, because a plane wave has an infinite number of phase singularities. We also note that the larger is the k of a plane wave, the larger is the decay rate, which is because a larger k means that the pattern has a higher spatial frequency and the diffusion cancels the coherence faster. Such diffusion of the plane wave happens if the pump and probe lasers have different wave vectors.

As a direct application, the smaller the phase gradient is, the more stable is the stored coherence against diffusion. For example, diffusion of a stored general mode LG_p^m with both winding number and number of radial nodes being nonzero [5] induces faster decay for $p>0$ than for $p=0$. Furthermore, we note that although the number of nodes in the coherence does not change with diffusion time, the positions for the off-center radial nodes change, which is different from the center node. The nonmoving position of the center node comes from geometric symmetry of the vortex. Since the decay rate depends on both $|m|$ and p for the mode LG_p^m, if OAM states are to be used as bases for quantum information, a preferred basis to reduce loss of entanglement for quantum information is actually two modes with the same p but opposite m.

So far, we have only discussed decoherence induced by the *classical* diffusion associated with the inhomogeneous distribution of coherence. We note that decoherence can also happen in a homogeneous system as a result of the quantum diffusion. The quantum diffusion in a light storage system happens when pump and probe lasers couple different momentum states, which introduces decoherence [28]. But this decoherence is reversible, e.g., by photon echo techniques, in contrast with the classical diffusion. This is because the quantum diffusion is described by a complex Schrödinger equation while classical diffusion is not. Finally, we note that when the same momentum states are coupled by choosing pump and probe lasers to have the same wave vectors, the quantum diffusion disappears [28]. Incidentally, inhomogeneous magnetic fields also induce quantum diffusion [18].

Our results indicate that diffusion actually introduces more decoherence in a stored vortex mode than a stored Gaussian mode, which implies that optical vortex states may not be preferable for quantum information when diffusion is involved. Of course, we also do not rule out that in other processes, such as a quantum gate operation, vortex states are possibly much better than the Gaussian state. Nor did we discuss diffusion-free systems such as BEC [10] and bound excitons in semiconductors [29,30]. Additionally, we believe that the formation of vortex solitons [31] due to nonlinear interaction in the presence of diffusion is interesting.

In conclusion, we have found that during diffusion, the coherence of stored vortex states decays faster than that of Gaussian states. This is surprising because vortex states are associated with topological properties, and are presumably more stable than Gaussian states. The underlying reason is that diffusion is a nonlocal process. More generally, the less the phase gradient in stored coherence, the better it is against diffusion. Furthermore, calculations show that the center of a stored vortex becomes completely incoherent once diffusion begins, and when the reading laser is applied, the optical intensity at its center builds up. The implication of our results to quantum information is discussed. Finally, we have compared the classical diffusion and the quantum diffusion.

This work was supported by NSF. T.W. and L.Z. acknowledge Y. Xiao for bringing Ref. [17] to their attention, and thank J. Javanainen for helpful discussions.

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