Photons as quasicharged particles

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The Schrödinger motion of a charged quantum particle in an electromagnetic potential can be simulated by the paraxial dynamics of photons propagating through a spatially inhomogeneous medium. The inhomogeneity induces geometric effects that generate an artificial vector potential to which signal photons are coupled. This phenomenon can be implemented with slow light propagating through a gas of double-Λ atoms in an electromagnetically induced transparency setting with spatially varied control fields. It can lead to a reduced dispersion of signal photons and a topological phase shift of Aharonov-Bohm type.

I. INTRODUCTION

It is known since the ground-breaking work of Berry on geometric phases [1] that artificial gauge potentials can be induced if the spatial dynamics of a system that obeys a wave equation is confined in a certain way. An example is the gauge field dynamics of neutral atoms if their internal Hamiltonian contains an energy barrier but the spin eigenstates are spatially varying [2]. In the limit of ray optics, moving atomic ensembles could simulate the propagation of light around a black hole or generate topological phase factors of the Aharonov-Bohm type [3], and inhomogeneous dielectric media could generally exhibit geometric effects such as an optical spin-Hall effect and the optical Magnus force [4].

In this paper, we propose to use electromagnetically induced transparency (EIT) to generate an artificial vector potential for the paraxial dynamics of signal photons that simulates *quantum dynamics* of charged particles in a static electromagnetic field. Not only the ray of light but also its mode structure is affected, resulting in a paraxial wave equation that is equivalent to the Schrödinger equation for charged particles. Furthermore, the form of the artificial vector potential can be easily controlled through spatial variations in the control fields. We suggest configurations that generate homogeneous quasielectric and magnetic fields as well as a vector potential of Aharonov-Bohm type.

Although the treatment in this paper is based on EIT, the effect presented here is more general: it will occur in any medium that supports a set of discrete eigenmodes for propagating signal fields with different indices of refraction. If the parameters governing these eigenmodes vary in space, the signal modes will adiabatically follow, acquiring geometric phases that affect their paraxial dynamics.

II. REVIEW OF EIT WITH MULTI-Λ ATOMS

The effect takes place in an atomic multi-Λ system, in which two ground states are coupled to Q excited states by Q pairs of control (Ω_q) and signal ($\hat{a}_q$) fields [Fig. 1(a)]. An experimentally relevant example of such a system is the fundamental $D1$ transition in atomic rubidium, where both the ground and excited levels are split into two hyperfine sublevels [5]. We assume that the detunings are small so each

signal field $\hat{a}_q$ interacts only with the respective transition $|B\rangle \leftrightarrow |A_q\rangle$ with the associated atomic field operator $\hat{\sigma}_{B,A_q}(\mathbf{x},t)$ that describes an atomic coherence $|B\rangle\langle A_q|$ at position $\mathbf{x}$. The respective coupling constant is given by the vacuum Rabi frequency $g_q = \mathcal{D}_q \sqrt{\omega/(2\varepsilon_0 \hbar V)}$, with $\mathcal{D}_q$ the dipole moment for this transition and quantization volume V, which equals the interaction volume of the mode. The paraxial wave equation for each signal mode can be cast into the form
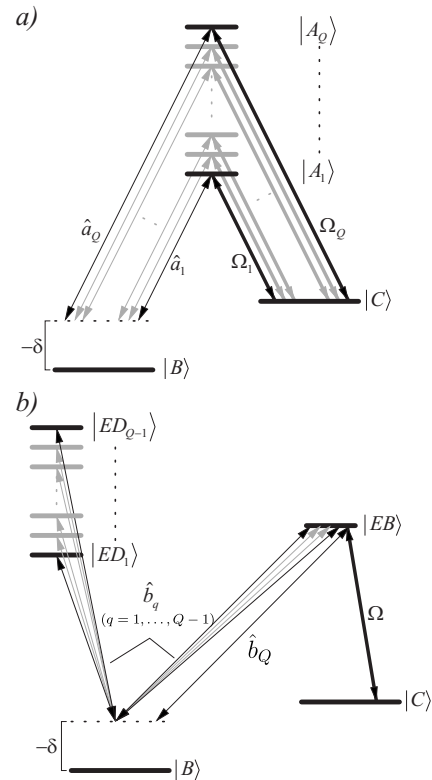

FIG. 1. EIT in a multi Λ-system. (a) In the original basis, Q excited states $|A_q\rangle$ are each coupled by a classical control field Ω_q to the ground state $|C\rangle$ and by a quantized field $\hat{a}_q$ with detuning δ to state $|B\rangle$. (b) In the transformed atomic and optical bases, the atomic states $\{|B\rangle, |EB\rangle, |C\rangle\}$ form a Λ system in which the signal field $\hat{b}_Q$ experiences EIT.

$$\left(\frac{\partial}{\partial t} + c\frac{\partial}{\partial z} - \frac{ic}{2k}\Delta_\perp\right)\hat{a}_q = iNg_q^*\hat{\sigma}_{B,A_q}, \qquad (1)$$

where the wave propagates along the z axis, N is the number of atoms in the interaction volume, and k is the wave vector which we assume approximately independent of q. In this section, where we review the results of our earlier work [6], we neglect transverse variations of all fields and hence the last term in the left-hand side of Eq. (1). We have constructed a unitary mapping $\hat{U}$ of atomic excited states such that one and only one of the new states, the "excited bright state"

$$|EB\rangle = \hat{U}|A_Q\rangle = \sum_{q=1}^{Q}\frac{\Omega_q}{\Omega}|A_q\rangle, \qquad (2)$$

where

$$\Omega \equiv \sqrt{\sum_{q=1}^{Q}|\Omega_q|^2}, \qquad (3)$$

is coupled to the ground state $|C\rangle$ by the control fields. In addition, we defined a unitary transformation of the optical signal modes

$$\hat{a}_q = \sum_{s=1}^{Q}W_{qs}\hat{b}_s \qquad (4)$$

that maps the original field modes a_q to a new set of modes $\hat{b}_q$, such that one and only one of the new modes, $\hat{b}_Q$, couples to an atomic dark state and experiences EIT [6–8].

The transformation W is given explicitly by

$$W_{qq'} = \gamma w_q w_{q'}^* - \delta_{qq'} \qquad (5)$$

with

$$\gamma = R_Q + 1 \quad \text{and} \quad w_q = \frac{\delta_{Qq} + R_q}{\gamma}. \qquad (6)$$

The EIT mode is expressed as

$$\hat{b}_Q = \sum_{q=1}^{Q}R_q^*\hat{a}_q, \qquad (7)$$

where

$$R_q \equiv \frac{\Omega_q}{g_q R} \quad \text{with} \quad R \equiv \sqrt{\sum_{q=1}^{Q}|\Omega_q/g_q|^2} \qquad (8)$$

depend on the control fields.

The EIT mode $\hat{b}_Q$ interacts with the multi-Λ atoms in the same fashion as does the signal field in a regular three-level system: it gives rise to a dark-state polariton associated with zero interaction energy [9] [Fig. 1(b)]. The paraxial propagation equation takes the form

$$\left(\frac{\partial}{\partial t} + c\frac{\partial}{\partial z}\right)\hat{b}_Q = iNg\hat{\sigma}_{B,EB} \qquad (9)$$

with a real coupling constant $g = \Omega/R$ and

$$g\hat{\sigma}_{B,EB} = \sum_{q=1}^{Q}W_{qQ}^*g_q^*\hat{\sigma}_{B,A_q}. \qquad (10)$$

Because the response of an EIT medium is linear with respect to the signal field, we can write

$$Ng\hat{\sigma}_{B,EB} = \frac{ck}{2}\chi_Q\hat{b}_Q, \qquad (11)$$

where the susceptibility is [15]

$$\chi_Q = \frac{2Ng^2}{ck}\frac{\delta}{-\delta\left(\delta + i\frac{\gamma}{2}\right) + \Omega^2} \approx \frac{2Ng^2}{ck}\frac{\delta}{\Omega^2}, \qquad (12)$$

the approximation being valid for $\delta \ll \Omega$.

All other optical modes $\hat{b}_q (q \neq Q)$ couple to absorbing atomic states $|ED_q\rangle = \hat{U}|A_q\rangle$ $(q \neq Q)$ and do not experience EIT. The associated susceptibilities have large imaginary components, and are thus significantly different from the susceptibility of the EIT mode $\hat{b}_Q$. This difference guarantees that, if the amplitudes and phases of the control fields are slowly changed, the composition of the dark-state polariton, and hence the EIT mode, will adiabatically follow. It has been proposed [6] and experimentally demonstrated [5] that, by varying the control fields in time while the signal pulse is inside the medium, one can adiabatically transfer optical states between different signal modes $\hat{a}_q$. In this paper, we focus on *spatial* propagation of the EIT mode under control fields that are constant in time, but varied in space.

III. DERIVATION OF THE GAUGE POTENTIAL

We proceed by expressing Eq. (1) (now taking the spatial term into account) in terms of the new signal modes $\hat{b}_q$. Employing the vector notation $\vec{a} = \{\hat{a}_1, \ldots, \hat{a}_Q\}$ and $\vec{\sigma}_{B,A} = \{g_1^*\hat{\sigma}_{B,A_1}, \ldots, g_Q^*\hat{\sigma}_{B,A_Q}\}$ we get

$$\left(\frac{\partial}{\partial t} + c\frac{\partial}{\partial z} - \frac{ic}{2k}\Delta_\perp\right)\overset{\leftrightarrow}{W}\vec{\hat{b}} = iN\vec{\sigma}_{B,A}. \qquad (13)$$

Throughout the paper, the double arrow denotes a $Q \times Q$ matrix. Because $\overset{\leftrightarrow}{W}$ depends on space and time, the differential operators have to be applied to both $\overset{\leftrightarrow}{W}$ and $\vec{\hat{b}}$. As a result, transformation (4) brings about additional terms into the equation of motion that can be written in the form of a minimal coupling scheme by introducing the Hermitian gauge field

$$\overset{\leftrightarrow}{A}_l \equiv i\overset{\leftrightarrow}{W}{}^\dagger\partial_l\overset{\leftrightarrow}{W}, \qquad (14)$$

where $l = t, x, y, z$. We multiply both sides of Eq. (13) by $\overset{\leftrightarrow}{W}{}^\dagger$ and exploit the unitarity of $\overset{\leftrightarrow}{W}$ to show that $\partial_l\overset{\leftrightarrow}{W}{}^\dagger = -\overset{\leftrightarrow}{W}{}^\dagger(\partial_l\overset{\leftrightarrow}{W})\overset{\leftrightarrow}{W}{}^\dagger$ from which it follows that $-\overset{\leftrightarrow}{W}{}^\dagger\partial_l^2\overset{\leftrightarrow}{W} = \overset{\leftrightarrow}{A}_l^2 + i\partial_l\overset{\leftrightarrow}{A}_l$. The dynamic equation for the $\hat{b}$ modes can then be written as

$$(i\partial_t + \overleftrightarrow{A}_t)\vec{b} = -(ic\partial_z + c\overleftrightarrow{A}_z)\vec{b} + \frac{c}{2k}(-i\overleftrightarrow{\nabla}_\perp - \overleftrightarrow{A}_\perp)^2\vec{b} - \overleftrightarrow{W}^\dagger N\vec{\sigma}_{BA}$$

$$(15)$$

with $(\overleftrightarrow{\nabla}_\perp)_{qq'} \equiv \delta_{qq'}\nabla_\perp$. This equation has the structure of a 2+2 dimensional field theory with minimal coupling.

Under the assumption that the control fields do not depend on t and z we can make a temporal Fourier transformation of the slowly varying amplitudes, which results in the paraxial wave equation

$$i\partial_z\vec{b}(\delta) = \left(-\frac{\delta}{c} + \frac{1}{2k}(-i\nabla_\perp - \overleftrightarrow{A}_\perp)^2\right)\vec{b}(\delta) - \overleftrightarrow{W}^\dagger\frac{N}{c}\vec{\sigma}_{BA}(\delta).$$

$$(16)$$

The gauge potential is given explicitly (see the Appendix) by

$$\overleftrightarrow{A}_\perp = i\sum_{q=1}^{Q} R_q^*(\nabla_\perp R_q)\vec{w}\vec{w}^\dagger - i\gamma(\nabla_\perp\vec{w})\vec{w}^\dagger + i\gamma^*\vec{w}\nabla_\perp\vec{w}^\dagger.$$

$$(17)$$

The full matrix $\overleftrightarrow{A}_\perp$ is a pure gauge: it has emerged solely as a consequence of the unitary transformation (4), which reflects our choice to describe the system in terms of the new modes $\hat{b}_q$ rather than the original modes $\hat{a}_q$. However, this choice is motivated by the fact that the EIT mode $\hat{b}_Q$ is the only mode that is not absorbed. Absorption of other modes $\hat{b}_q$ (with $q \neq Q$) means that the index of refraction for these modes has a significant imaginary part. This separates the EIT mode $\hat{b}_Q$ from other $\hat{b}$ modes and ensures that it will adiabatically follow variations of the control fields. Therefore when analyzing the evolution of $\hat{b}_Q$, we can neglect the off-diagonal terms in the matrix $(-i\nabla_\perp - \overleftrightarrow{A}_\perp)^2$ in Eq. (16) and write

$$i\partial_z\hat{b}_Q(\delta) = -\left(\overleftrightarrow{W}^\dagger\frac{N}{c}\vec{\sigma}_{BA}\right)_Q(\delta) - \frac{\delta}{c}\hat{b}_Q(\delta)$$
$$+ \frac{1}{2k}\sum_{q=1}^{Q}(-i\nabla_\perp - A_\perp)_{Qq}(-i\nabla_\perp - A_\perp)_{qQ}\hat{b}_Q(\delta).$$

$$(18)$$

This equation does not include the whole matrix $\overleftrightarrow{A}_\perp$. Consequently, this potential no longer acts like a pure gauge but attains physical significance in determining the spatial dynamics of the EIT mode.

The first term on the right-hand side of Eq. (18) is responsible for the usual interaction of the signal field with the EIT medium and can be rewritten using Eqs. (10) and (11) as

$$\left(\overleftrightarrow{W}^\dagger\frac{N}{c}\vec{\sigma}_{BA}\right)_Q(\delta) = \frac{k\chi_Q}{2}\hat{b}_Q.$$

$$(19)$$

In linear approximation in the detuning δ, this transforms Eq. (18) to

$$i\partial_z\hat{b}_Q = \left(\frac{1}{2k}[-i\nabla_\perp - (A_\perp)_{QQ}]^2 - \frac{\delta}{v_{\text{EIT}}} + \frac{\Phi}{2k}\right)\hat{b}_Q \quad (20)$$

with the EIT group velocity

$$v_{\text{EIT}} = \frac{c}{\frac{ck}{2}\frac{\partial\chi_Q}{\partial\delta} + 1} \approx \frac{c\Omega^2}{Ng^2} \quad (21)$$

and (see the Appendix for a detailed derivation)

$$(A_\perp)_{QQ} = i\sum_{q=1}^{Q} R_q^*\nabla_\perp R_q = -\sum_{q=1}^{Q}|R_q|^2\nabla_\perp \arg(R_q),$$

$$\Phi \equiv \sum_{q=1}^{Q-1}|(A_\perp)_{Qq}|^2 = -(A_\perp)_{QQ}^2 + \sum_{q=1}^{Q}|\nabla_\perp R_q|^2 \quad (22)$$

being, respectively, the "quasivector" and "quasiscalar" potentials. Note that v_{EIT} depends on the spatial position because Ω does.

We see that the paraxial spatial evolution of the EIT signal mode is governed by the equation that is identical (up to coefficients) to the Schrödinger equation of a charged particle in an electromagnetic field. This is the main result of this work. By arranging the control field in a certain configuration, one can control the spatial propagation of the signal mode through the EIT medium.

IV. CASE OF TWO CONTROL FIELDS: HOMOGENEOUS ELECTRIC AND MAGNETIC QUASIFIELDS

Some steering of the EIT mode is possible even in a single-Λ system by affecting the term δ/v_{EIT} in Eq. (20), which results in nonuniform refraction for this mode. For example, in a "Stern-Gerlach experiment for slow light" [11], a magnetic field gradient inside the interaction volume translates into a gradient of the two-photon detuning, which plays the role of gauge-independent potential energy in Eq. (20). This results in a quasiforce deflecting the EIT beam. Another relevant phenomenon is "waveguiding" [10] of the probe field. Here, again, the potential δ/v_{EIT} is affected: the group velocity is higher in the center of the Gaussian pump beam than at the periphery, resulting in a quasiforce pointing toward the beam center.

Both of these phenomena rely upon spatial variation of the refractive index. The origin of quasigauge potentials (22) is, however, fundamentally different: they can take place only in a multi-Λ system and are a consequence of a unitary gauge transformation [16]. A somewhat paradoxical feature of the resulting quasiforces is that they can occur even at the two-photon resonance, where the refraction index is a constant unity. Below, we present a few important examples of this case.

Of particular practical importance is the simplest nontrivial case with $Q = 2$. We parametrize the control fields by writing

$$R_{1,2} = \sqrt{\frac{1}{2} \pm S} e^{i(\phi \pm \theta)}. \tag{23}$$

The corresponding Rabi frequencies are then $\Omega_i = h(x,y)g_i R_i$, with $h(x,y)$ a certain common prefactor. This parametrization yields the gauge potentials

$$(A_\perp)_{QQ} = -\nabla_\perp \phi - 2S\nabla_\perp \theta,$$

$$\Phi = \frac{(\nabla_\perp S)^2}{1 - 4S^2} + (\nabla_\perp \theta)^2 (1 - 4S^2). \tag{24}$$

Similarly to usual electrodynamics we can use a gauge transformation [12], $A'_{QQ} = A_{QQ} + \nabla_\perp f$, to eliminate the term $\nabla_\perp \phi$ from Eq. (24). The common phase ϕ of the control fields therefore does not contribute and can be set to zero.

A. Electric quasifields

A simple way to generate a term that corresponds to a one-dimensional scalar potential $V(x)$ for a Schrödinger particle is to choose $S=0$ and

$$\theta = \int_0^x dx' \sqrt{2kV(x')}. \tag{25}$$

This choice of control fields leads to $A_{QQ}=0$ and $\Phi = 2kV(x)$.

For the special case of a constant electric quasifield along the x axis, $V(x) = V_0 - Fx$ and subsequently

$$\theta = -\frac{\sqrt{k}}{3F}[2(V_0 - Fx)]^{3/2}. \tag{26}$$

The constant V_0 is physically insignificant and only included to ensure that $V(x)$ is positive in the region of interest. A resonant ($\delta=0$) Gaussian solution to Eq. (20) that has width w when the signal field enters the medium at $z=0$ is then characterized by the following spatial mode function:

$$b_Q = \frac{\mathcal{N}e^{-iV_0 z}}{\sqrt{1+i\zeta}} \exp\left[-\frac{(\xi - \xi_{\text{ctr}})^2}{1 + i\zeta} + iFz_R w\zeta\left(\xi - \frac{1}{3}\xi_{\text{ctr}}\right)\right], \tag{27}$$

where $\mathcal{N}$ is the normalization constant. Here we have ignored the evolution in the y direction for brevity, introduced the Rayleigh length $z_R = kw^2/2$, as well as the scaled variables $\zeta \equiv z/z_R$, $\xi \equiv x/w$, and $\xi_{\text{ctr}} \equiv Fz^2/(2kw)$. The corresponding field intensity $I \sim |b_Q|^2$ has the form of a Gaussian whose center is shifted by an amount $w\xi_{\text{ctr}}$. This is equivalent to the motion of a charged particle in a constant electric field, see Fig. 2.

The control field phase profile (26) can be implemented using, for example, a phase plate. The assumption that the control fields do not depend on z implies that the Fresnel number for these fields must be above 1, i.e., that the characteristic transverse distance over which these fields significantly change must be larger than $\sim \sqrt{\lambda L}$, where L is the EIT cell length. This imposes a limitation on the magnitude of the electric quasifield: from Eq. (26) we find $F \lesssim \lambda^{-1/2}L^{-3/2}$ and
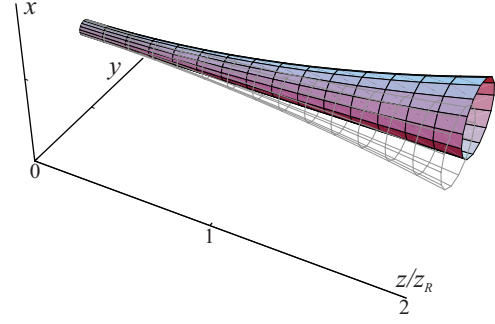

FIG. 2. (Color online) Paraxial propagation of a signal beam over twice the Rayleigh length in the presence (solid line) and absence (gray line) of a constant electric quasifield along the x axis. The dashed line represents the center of the gray beam.

thus $x_{\text{ctr}} \lesssim \sqrt{L/\lambda}$. Assuming that the signal field also has a Fresnel number of at least 1, and thus satisfies $2z_R \gtrsim L$, we find that in a realistic experiment, the maximum possible signal beam displacement due to the quasielectric field is on the order of the signal beam width w. An essential condition for obtaining observable displacements is focusing the beams into the medium so a small Fresnel number geometry is maintained.

B. Magnetic quasifields

To generate a homogeneous magnetic quasifield along the z axis the quantity $\mathbf{B} = \nabla \times A_{QQ} = 2\nabla_\perp \theta \times \nabla_\perp S$ should be constant. However, it seems difficult to simultaneously achieve a vanishing electric quasifield $\mathbf{E} = -\nabla_\perp \Phi$. A choice that minimizes the electric quasifield around the origin is given by $\theta = \sqrt{B/2}x$ and $S = \sqrt{B/2}y$. The quasipotentials then become $\mathbf{A}_{QQ} = -By\mathbf{e}_x$, which corresponds to the Landau gauge in standard electrodynamics, and $\Phi = B + 2B^3 y^4 + O(y^6)$. If Φ is neglected, a Gaussian solution to the paraxial wave equation is given by

$$b_Q = \mathcal{N}\csc u(z) \exp\left\{i\left[\frac{B}{4}\cot u(z)\Delta\mathbf{x}^2 + \Delta\mathbf{x}\cdot\mathbf{p}_c - \frac{B}{2}\Delta x\Delta y\right.\right.$$
$$\left.\left. -\frac{1}{2B}p_{c,x}p_{c,y} + \frac{1}{2}y_c p_{c,y}\right]\right\}, \tag{28}$$

where we have set $\Delta\mathbf{x} \equiv (x - x_c, y - y_c)$, $u(z) \equiv Bz/(2k) - i\tanh^{-1}(2\eta)$, and $\eta \equiv Bw^2/4$. Here $\mathbf{x}_c = \mathbf{x}_0 + (k/B)\{\mathbf{x}_0'\sin(Bz/k) + \tilde{\mathbf{x}}_0'[1-\cos(Bz/k)]\}$ denotes the classical spiral trajectory of a charged particle in a magnetic field, with $\mathbf{x}_c' = d\mathbf{x}_c/dz$, initial position $\mathbf{x}_0$, and initial velocity $\mathbf{x}_0'$. For convenience we also have defined $\tilde{\mathbf{x}}_0' = (y_0', -x_0')$ and the classical canonical momentum $\mathbf{p}_c = k\mathbf{x}' + \mathbf{A}_{QQ}$. Its interpretation in the case of a light beam is that it describes a small initial misalignment between the signal field and the control beams. We remark that $p_{c,x} = kx_0' - By_0$ is a constant of motion. The evolution of the signal mode is displayed in Fig. 3.

A surprising feature of solution (28) is that the diffractive divergence of the signal beam is reduced: the width squared of the Gaussian,
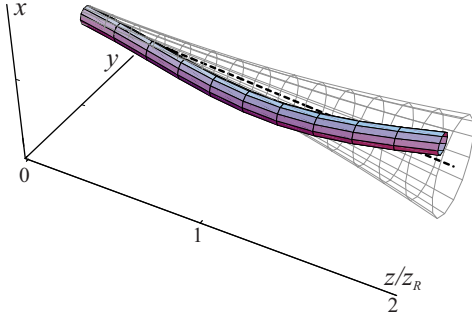
FIG. 3. (Color online) Paraxial propagation of a signal beam over twice the Rayleigh length in the presence (solid line) and absence (gray line) of a constant magnetic quasifield along z. The dashed line represents the center of the gray beam. The effect of the fields is somewhat exaggerated.

$$\frac{2}{\mathrm{Re}(iB\cot u)} = \frac{w^2}{8\eta^2}\left[1 + 4\eta^2 - (1 - 4\eta^2)\cos\left(\frac{Bz}{k}\right)\right],$$ (29)

varies periodically with z instead of monotonically increasing. This effect is known for electron wave packets [13] and can be understood as a consequence of the circular motion of particles in a magnetic field: instead of dispersing, two-dimensional particles in a magnetic field will simply move in circles of different size (depending on their velocity), but with the same angular velocity. The particle cloud will therefore not spread but "breathe."

It remains to show that nonadiabatic coupling to other modes can be suppressed for realistic experimental parameters. For systems described by the Schrödinger equation, nonadiabatic coupling between two states is suppressed if the energy difference between the states is much larger than those terms in the Hamiltonian that induce a transition between both states. In our case, the light modes are described by the paraxial wave equation and the linear susceptibility plays the role of the energy difference. Thus nonadiabatic coupling is suppressed if the gauge field terms coupling b_Q to other modes in Eq. (16) are much smaller than the difference in the respective linear susceptibilities χ_1. Using Eqs. (17) and (23) one can derive the coupling term,

$$(A_\perp)_{12} = \frac{\gamma^*}{\gamma\sqrt{1 - 4S^2}}[-i\nabla_\perp S + (1 - 4S^2)\nabla_\perp \theta],$$ (30)

that describes the transfer of photons from the EIT mode $\hat{b}_Q$ to the other mode. Inserting the specific parameters for the magnetic quasifield one can easily see that at the center of the Gaussian solution (28) the coupling terms between the two modes in

Eq.(16) are given by $(A^2)_{12}/(2k) \sim \nabla_\perp A_{12}/k \sim B/(2k)$, and $A_{12}\nabla_\perp b_Q \sim \sqrt{B}p_c/(2k)$.

For the EIT mode b_Q, χ_2 is defined by Eq. (12); for the other modes it can be approximated by the susceptibility of a two-level medium,

$$\chi_1 = -\frac{Ng^2}{c}\frac{\delta - i\frac{\gamma}{2}}{\delta^2 + \frac{\gamma^2}{4}} \approx -4\frac{Ng^2}{c\gamma^2}\left(\delta - i\frac{\gamma}{2}\right).$$ (31)

Thus at resonance the difference between the susceptibilities of the EIT mode and the second mode is given by $\Delta\chi_1 = 2ig^2N/\gamma$. Requiring that this difference is much larger than the coupling terms yields, for sufficiently small canonical momentum p_c, the condition $B/(2k) \ll \Delta\chi_1$ on the magnetic quasifield strength which can be expressed as $\eta \ll (kw)^2 n3\pi/2$, with $n \equiv N/(Vk^3)$ being the number of atoms in the volume k^{-3}. This condition can easily be fulfilled in an experiment.

V. AHARONOV-BOHM POTENTIAL FOR PHOTONS

One of the most intriguing phenomena of charged quantum particles in electromagnetic fields is the Aharonov-Bohm (AB) effect [14]. Its two astonishing features are (i) a phase shift induced by the vector potential in a region in which electric and magnetic fields are absent, and (ii) its topological nature: the phase shift does not depend on the particle trajectory as long as it encloses a magnetic flux. Because (unlike genuine electromagnetism) the potential (14) is a differential function of the control fields, it is impossible to simulate feature (i) with quasicharged photons. However, we will show here that a mathematically equivalent topological phase shift does exist for the optical case.

To generate an AB potential for photons we propose to use two counter-rotating Laguerre-Gaussian control fields, i.e., fields that possess an orbital angular momentum. If these control fields are spatially wider than the signal fields, the corresponding Rabi frequencies can be approximated in cylindrical coordinates (r, φ) by $\Omega_1 = g_1 s_1 r e^{i\varphi}$ and $\Omega_2 = g_2 s_2 r e^{-i\varphi}$. The gauge potentials (24) then become $A_{QQ} = -2S/r\vec{e}_\varphi$ and $\Phi = (1 - 4S^2)/r^2$, with

$$S = \frac{1}{2}\frac{|s_1|^2 - |s_2|^2}{|s_1|^2 + |s_2|^2} = \frac{1}{2}\frac{|\Omega_1|^2 - |\Omega_2|^2}{|\Omega_1|^2 + |\Omega_2|^2},$$ (32)

where the last equality only holds for $|g_1| = |g_2|$. The potential A_{QQ} corresponds exactly to an Aharonov-Bohm potential for charged particles as it is created by a solenoid.

Because of $\Omega \sim r$, the EIT group velocity can be written as $v_{\mathrm{EIT}} = \tilde{v}r^2$ with $\tilde{v} \equiv c\sqrt{|s_1|^2 + |s_2|^2}/N$. Therefore signal light closer to the propagation axis travels slower. We remark that this also implies that the EIT model breaks down close to the propagation axis because the pump field goes to zero. To find a solution of the paraxial wave Eq. (20) we introduce the scaled variable $u = z/(2k)$ and cylindrical coordinates r, ϕ to rewrite the dynamical equation as

$$i\partial_u b_Q = \left(\frac{1 - \frac{2k\delta}{\tilde{v}}}{r^2} - i\frac{2S}{r^2}\partial_\phi - \frac{1}{r}\partial_r r\partial_r - \frac{1}{r^2}\partial_\phi^2\right)b_Q.$$ (33)

For monochromatic signal fields, the Aharonov-Bohm term $\propto S$ generates a rotation of the transverse mode structure. A quick way to understand this is to ignore all terms but the Aharonov-Bohm term, which results in the simplified equation
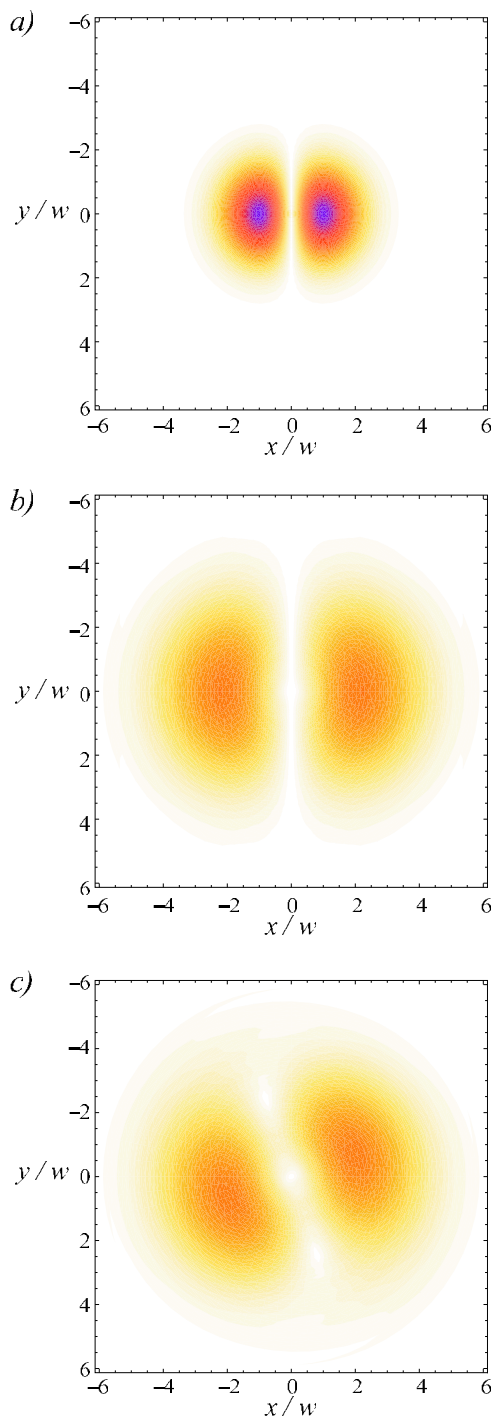
FIG. 4. (Color online) Photonic Aharonov-Bohm effect for a signal field in a TEM-01 mode. (a) Initial field, (b) field after a propagation length of 2.8 Rayleigh lengths without Aharonov-Bohm effect ($S=0$), and (c) with Aharonov-Bohm effect ($S=0.4$).

$$\left(i\partial_u + 2i\frac{S}{r^2} \right) b_Q = 0. \tag{34}$$

The general solution of this equation is given by $b_Q(\phi,u) = b_Q(\phi - 2uS/r^2, 0)$, which indicates that after propagating over a distance z the mode structure is rotated by an angle

$$\Delta\phi_{AB} = -\frac{z}{k}\frac{S}{r^2}. \tag{35}$$

For $S = \pm 1/2$ the Aharonov-Bohm potential transfers a unit amount of angular momentum to the signal light, but generally the amount can vary continuously between $-\hbar$ and $\hbar$. Signal photons in the EIT mode therefore form a two-dimensional bosonic quantum system in an Aharonov-Bohm potential. A realistic estimate for $\Delta\phi_{AB}$ can be given by considering the rotation of a mode structure whose radial extent r is comparable to its minimum width w. After propagation over two Rayleigh lengths, $z = kw^2$, the rotation angle is then given by $\Delta\phi_{AB} = -S$ which could easily be observed.

When the signal field mode is expanded as $b_Q = \Sigma_{m \in \mathbb{Z}} B_m(r,u)\exp(im\varphi)$, solutions to the paraxial wave equation are given by Bessel functions,

$$B_m = e^{-i\kappa^2 u}[\alpha_m J_\nu(\kappa r) + \beta_m Y_\nu(\kappa r)], \tag{36}$$

with $\nu = \sqrt{1 + m^2 + 2mS - 2k\delta/\tilde{v}}$. This can be exploited to write down the general solution of Eq. (33) in terms of a Hankel transformation,

$$B_m(r,u) = \int_0^\infty k\,dk\,J_\nu(kr)e^{-i\kappa^2 u}B_m(k), \tag{37}$$

$$B_m(k) = \int_0^\infty r\,dr\,J_\nu(kr)B_m(r,0). \tag{38}$$

In Fig. 4 we have plotted the effect of the Aharonov-Bohm potential for an incident signal field of the form $b_Q(x,y,0) \propto x\exp[-(x^2+y^2)/(2w^2)]$, which corresponds to a TEM 01 mode with $B_{\pm 1}(r,0) \propto r\exp[-r^2/(2w^2)]$. The integrals appearing in Eqs. (37) and (38) have been evaluated numerically. The rotation angle of the mode structure is in agreement with the simple estimate based on neglecting the radial dependence.

VI. CONCLUSION

We have shown that EIT in a multi-Λ system can be used to generate a variety of geometric effects on propagating signal pulses that mimic the behavior of a charged particle in an electromagnetic field. We found specific arrangements of two spatially inhomogeneous pump fields in a double-Λ system which generate quasigauge potentials which correspond to constant electric and magnetic fields. Furthermore, topological effects like the Aharonov-Bohm phase shift can be induced. The latter is significantly different from the proposal of Ref. [3] in that it is based on spatially inhomogeneous pump fields rather than the Doppler effect in moving media.

This paper investigated EIT in systems with two ground levels. In such a system, there is only one EIT mode, which results in an Abelian U(1) gauge theory, making the physics analogous to electromagnetism. By extending to multiple ground levels, it may be possible to obtain multiple EIT modes and model non-Abelian gauge potentials. This will be explored in a future publication.

APPENDIX: DERIVATION OF EQS. (17) and (22)

To derive Eq. (17) one uses the definitions of the quantities $W_{qq'}$, γ, and w_q in terms of R_q. With

$$\sum_{q=1}^{Q} |R_q|^2 = 1 \tag{A1}$$

one finds

$$\sum_{r=1}^{Q} w_r w_r^* = \frac{\gamma + \gamma^*}{|\gamma|^2}. \tag{A2}$$

Inserting this in the matrix element of the gauge potential,

$$(A_\perp)_{qq'} = \sum_{r=1}^{Q} i W_{rq}^* \nabla_\perp W_{rq'}, \tag{A3}$$

and sorting all terms into those multiplying $w_q w_{q'}^*$, $w_q \nabla_\perp w_{q'}^*$, or $(\nabla_\perp w_q) w_{q'}^*$ yields

$$(A_\perp)_{qq'} = i w_q w_{q'}^* \left(\frac{\gamma}{\gamma^*} \nabla_\perp \gamma + |\gamma|^2 \sum_{r=1}^{Q} w_r^* \nabla_\perp w_r \right) - i\gamma (\nabla_\perp w_q) w_{q'}^*$$
$$+ i\gamma^* w_q \nabla_\perp w_{q'}^*. \tag{A4}$$

Using that $\nabla_\perp \gamma = \nabla_\perp R_Q$ one then can show that

$$\sum_{r=1}^{Q} w_r^* \nabla_\perp w_r = \frac{1}{|\gamma|^2} \left(-\frac{\gamma^*}{\gamma} \nabla_\perp R_Q + \sum_{r=1}^{Q} R_r^* \nabla_\perp R_r \right), \tag{A5}$$

from which Eq. (17) follows.

The explicit form of the quasivector potential in Eq. (22) follows from Eq. (17) if one notices that $w_Q = 1$. Furthermore, from $R_q = |R_q| e^{i \arg R_q}$ we find

$$R_q^* \nabla_\perp R_q = |R_q| \nabla_\perp |R_q| + i |R_q|^2 \nabla_\perp \arg R_q. \tag{A6}$$

When we sum over all q's, the first term in the above equation vanishes due to Eq. (A1):

$$\sum_{q=1}^{Q} |R_q| \nabla_\perp |R_q| = \frac{1}{2} \nabla_\perp \left(\sum_{q=1}^{Q} |R_q|^2 \right) = 0, \tag{A7}$$

yielding the expression for $(A_\perp)_{QQ}$.

In order to find the quasiscalar potential, one can write with the help of Eq. (17)

$$(A_\perp)_{Qq} = (A_\perp)_{QQ} w_q^* + i\gamma^* \nabla_\perp w_q^*. \tag{A8}$$

Now employing the results (A2) and (A5), as well as $\nabla_\perp R_Q = \nabla_\perp \gamma$ and

$$|\gamma|^2 \sum_{q=1}^{Q-1} |\nabla_\perp w_q|^2 = \sum_{q=1}^{Q} |\nabla_\perp R_q|^2 + i(A_\perp)_{QQ} \left(\frac{\nabla_\perp \gamma^*}{\gamma^*} - \frac{\nabla_\perp \gamma}{\gamma} \right), \tag{A9}$$

yields expression (22) for Φ.

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