Sensitivity of terahertz photonic receivers

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We theoretically discuss sensitivity limitations of a THz receiver which is based on up-conversion of the THz radiation into optical domain using high quality factor crystalline whispering gallery mode resonators. We show that the sensitivity of the receiver operating in the nonlinear regime approaches the sensitivity of an ideal THz photon counter. Thermal noise of the counter can be substantially reduced because of transparency of the nonlinear material in the THz frequency range. We also show that the density of power fluctuations of the receiver operating in the linear regime is given by the THz shot noise.

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I. INTRODUCTION

The nonlinear frequency mixing is a quite common approach for detection and reception at long wavelength [1–6] where no efficient detectors are available. The signal wave that belongs to infrared, THz, or microwave range of the electromagnetic spectrum is converted to the visible radiation range, where highly sensitive and low-noise detectors are readily available. This conversion is achieved by generating the sum frequency of the infrared radiation and one or more laser fields through use of second- or third-order nonlinear optical interactions. However, the general usefulness of this technique has been limited by the low conversion efficiencies.

In this paper we describe a concept of the photonic THz receiver based on a crystalline whispering gallery mode (WGM) resonator. WGM resonators have been playing a significant role in the nonlinear optics in general and in microwave photonics reception particularly [7–10]. We show that the WGM based devices can possess ideal THz reception properties and that the resonant technique can produce a unity conversion efficiency from the THz to the visible light in a continuous regime. The basic advantage of the WGM resonators is that they can be resonant with the THz radiation as well as with both the driving and signal laser fields. This triple resonance along with small mode volume of the WGMs insures high efficiency of the nonlinear frequency conversion, even if the resonator is fabricated from a material with low nonlinearity, as is usually the case for optically transparent materials. The discussed concept is also valid for any triple-resonant scheme allowing up-conversion of the THz radiation into optical domain.

It was shown recently that it is possible to achieve phase matching between light and THz radiation by manipulating morphology of the resonator [11]. As the result, light scattering on the polariton modes of the resonator was observed. Similar process can be achieved if a THz signal is sent from outside into an optically pumped resonator. The THz signal will be converted into optical domain due to the nonlinearity of the resonator. Because of the unitary character of this conversion, the emitted light carries information about quantum properties of the THz signal, which is, however, masked by optical shot noise and thermal noise in the THz mode. When the receiver is in equilibrium with the environment, the latter noise amounts to the value of $k_B T$ per mode. If the detector is hotter than the environment, its excess noise can be considerably suppressed by virtue of the fluctuation-dissipation theorem, if the receiver material has low absorption in the THz range (see Fig. 1 for explanation). The temperature of the THz mode of the detector is therefore effectively lower than the receiver temperature. This may be important for applications of the room-temperature photonic receiver in the cold (low noise) THz environment. Complete quantum state of the THz signal can be transferred to the light at zero receiver temperature if the probability of the up-conversion of a single THz photon into optical photon approaches unity. Hence, the photonic receiver can be used as a THz photon counter. Ordinarily, the sensitivity of the reception of the coherent THz signals is limited by the energy of a single THz photon, similarly to the sensitivity of linear heterodyne receivers.

The nearly ideal THz reception is possible if spontaneous emission processes are suppressed in the receiver. Spontaneous oscillations observed in [11] can deteriorate sensitivity of the receiver if they occur at the reception frequency. Even subthreshold oscillations introduce extra noise to the con-

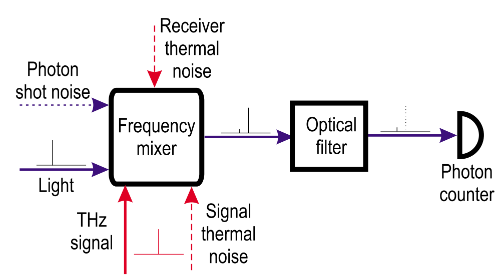

FIG. 1. (Color online) Schematic of the THz photonic receiver. Pump light and THz radiation is sent into a nonlinear mixer containing a WGM resonator. The light exiting the mixer has an anti-Stokes sideband carrying information about the THz signal. The optical carrier is removed from the exiting light using an optical stop-band filter and the anti-Stokes sideband is detected using a photodetector. Coherent detection of the sideband using an optical local oscillator is also possible.

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verted optical signal. Spontaneous noise is absent in case of the anti-Stokes conversion, when the generated optical photon has frequency higher than the carrier frequency of the optical pump. We consider such case and study interaction of two optical and a single THz resonator modes via $\chi^{(2)}$ nonlinearity, focusing on an interaction of externally pumped optical and THz modes having finite Q factors. We study the generation of an anti-Stokes sideband in another optical mode occurring as the result of the interaction and show that the anti-Stokes light carries information about THz radiation. The frequencies of the anti-Stokes sideband, $\omega+\omega_M$, is given by frequencies of optical (ω) and THz (ω_M) pumping, respectively (a conceptual diagram of the receiver is shown in Fig. 1). We assume that the neighboring optical mode is nearly resonant with the sideband and that the process is phase matched in the sense of the orbital numbers.

II. PHASE MATCHING BETWEEN LIGHT AND THZ WGMs

The parametric interaction takes place if the phase-matching condition is fulfilled,

$$\eta_{pm} = \frac{1}{\mathcal{V}}\int_{\mathcal{V}} d\mathcal{V}\Psi_a\Psi_b^*\Psi_c \neq 0, \tag{1}$$

where $\mathcal{V}$ is the volume of the optical mode, Ψ_a, Ψ_b, and Ψ_c are the normalized dimensionless spatial distributions of the modes. We assume that the optical WGMs are nearly identical, $\mathcal{V}_a \simeq \mathcal{V}_b = \mathcal{V}$ and that the volume of the THz mode is much larger than the volume of the optical modes, so the integration can be taken only in the volume occupied by the light. Notice that Eq. (1) enforces a specific ratio for the quantum numbers of the optical as well as THz modes.

The phase matching can be achieved by selecting the proper size as well as morphology of the resonator. We assume that $\omega \approx 10^{15}$ s^{-1} and $\omega_M \approx 2\pi \times 1$ THz. The indexes of refraction of the material are 2.2 in optical and 5 in THz domains. We assume that the resonator is a cylinder with radius R and thickness L and select all the WGMs to belong to the basic TE mode family. The frequencies of the modes obey to equation

$$\omega_l \simeq \frac{c}{Rn}\left\{\left[l + \alpha_1\left(\frac{l}{2}\right)^{1/3} - \frac{n}{\sqrt{n^2-1}}\right]^2 + \left(\frac{\pi R}{L}\right)^2\right\}^{1/2}, \tag{2}$$

where l is the mode number, $\alpha_1 \simeq 2.338$, and n is the refractive index of the mode. Equation (2) describes optical as well as the microwave modes dispersion. It is strongly dominated by the leading-order l term for the optical modes. Therefore, the dispersion is practically flat and the free spectral range (FSR) can be introduced, $\Omega \equiv \omega_{l+1} - \omega_l \approx c/Rn = \text{const}|_{l,L}$.

Equation (1) together with the energy conservation condition $\omega_b - \omega_a = \omega_c$ leads to the following phase-matching condition: $\omega_M(L, l_c) = \Omega l_c$, whose graphic solutions are given in Fig. 2 for $R = 0.1$ cm. We see that those solutions are quite numerous and can be found for any THz orbital number l_c. More accurately the phase matching can be found taking into account the effect of the evanescent field, which effectively modifies the bulk refraction index for the microwaves. The effect becomes especially important when the thickness of the resonator is comparable or less than the wavelength of THz radiation in the resonator host material. This requires numerical simulation and is outside the scope of this paper.

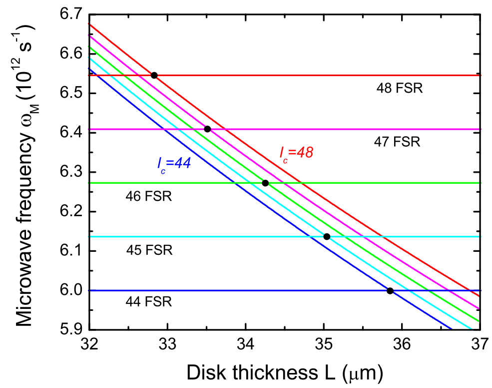

FIG. 2. (Color online) Microwave frequency dispersion for l_c taking on values from 44 to 48 as a function of the disk thickness L. Phase matching (represented by thick black dots) is achieved where each dispersion curve crosses the level of l_c times the optical FSR.

The phase matching becomes possible because unlike the optical modes, the THz modes have strong dispersion as a function of the disk thickness L, as the latter begins to approach the microwave wavelength. Therefore, one can tune to the desired phase matching by adjusting the resonator thickness. This technique is similar to the phase-matching technique of nonlinear processes via geometrical confinement of light [12].

III. BASIC EQUATIONS

Interaction of the optical and THz fields is described by the following equations:

$$\dot{\hat{A}} = -\gamma\hat{A} - ig^*\hat{C}^\dagger\hat{B}_+ + \hat{F}_A, \tag{3}$$

$$\dot{\hat{B}}_+ = -\gamma\hat{B}_+ - ig\hat{C}\hat{A} + \hat{F}_{B+}, \tag{4}$$

$$\dot{\hat{C}} = -\gamma_M\hat{C} - ig^*\hat{A}^\dagger\hat{B}_+ + \hat{F}_C, \tag{5}$$

where $\hat{A}$, $\hat{B}_+$, and $\hat{C}$ are the slowly varying amplitudes pump (optical), idler (optical), and signal (THz) fields; γ and γ_M are the optical and THz decay rates, respectively (we assume that the optical modes are overloaded so the intrinsic losses can be neglected, and γ is the decay rate of the loaded modes); g is the coupling constant that will be determined later in the paper [Eq. (26)], $\hat{F}_A$, are $\hat{F}_{B+}$ are the Langevin forces whose properties are given by

$$\langle\hat{F}_A\rangle = \sqrt{\frac{2P\gamma}{\hbar\omega}}, \tag{6}$$

$$\langle\hat{F}_A(t)\hat{F}_A^\dagger(t')\rangle = \langle\hat{F}_{B+}(t)\hat{F}_{B+}^\dagger(t')\rangle = 2\gamma\delta(t-t'), \tag{7}$$

where $\langle\cdots\rangle$ stands for the ensemble averaging, P is the power of the external optical pump of the mode A. We assume that the sources of thermal noise (see Fig. 1) are uncorrelated. The Langevin force describing the thermal fluctuations consists of two uncorrelated parts,

$$\hat{F}_C = \hat{F}_{Cs} + \hat{F}_{Cr}, \quad \langle \hat{F}_{Cs} \rangle = \sqrt{\frac{2P_{mw}\gamma_{Ms}}{\hbar\omega_c}}, \tag{8}$$

$$\langle \hat{F}_{Cs}(t)\hat{F}^{\dagger}_{Cs}(t') \rangle = 2\gamma_{Ms}(\bar{n}_{ths}+1)\delta(t-t'), \tag{9}$$

$$\langle \hat{F}_{Cr}(t)\hat{F}^{\dagger}_{Cr}(t') \rangle = 2\gamma_{Mr}(\bar{n}_{thr}+1)\delta(t-t'), \tag{10}$$

where γ_{Ms} and γ_{Mr} stand for coupling of the thermal fluctuations from the signal channel and receiver channel to the THz mode (the spectral width of the THz resonance is given by $\gamma_M = \gamma_{Ms} + \gamma_{Mr}$), $\bar{n}_{ths} = [\exp(\hbar\omega_c / k_B T_s) - 1]^{-1}$ is the averaged number of thermal photons coming from the signal, $\bar{n}_{thr} = [\exp(\hbar\omega_c / k_B T_r) - 1]^{-1}$ is the averaged number of thermal photons coming from the receiver, P_{mw} is the power of the external THz pump of mode C. The other expectation values, quadratic deviations, and correlations are equal to zero.

IV. SOLUTION

We solve a linearized set of differential equations (3)–(5) using standard Fourier technique assuming an undepleted classical pump, $\hat{A} \to A = \langle \hat{F}_A \rangle / \gamma = \text{const}$. This approximation is valid if $|g|^2 \langle \hat{n}_c \rangle / \gamma^2 \ll 1$, $\langle \hat{n}_c \rangle$ is the expectation value of the number of THz photons in the mode. We present the forces and the field operators as a sum of an expectation and fluctuational parts, such as

$$\hat{F}_{B+} = \int_{-\infty}^{\infty} \hat{f}_{B+}(\tilde{\omega}) e^{-i\tilde{\omega}t} \frac{d\tilde{\omega}}{2\pi}, \tag{11}$$

$$\hat{F}_C = \langle \hat{F}_{Cs} \rangle + \int_{-\infty}^{\infty} [\hat{f}_{Cs}(\tilde{\omega}) + \hat{f}_{Cr}(\tilde{\omega})] e^{-i\tilde{\omega}t} \frac{d\tilde{\omega}}{2\pi}, \tag{12}$$

where $\langle \hat{F}_{Cs} \rangle$ stands for the THz signal, $\hat{f}_{B+}(\omega)$ and $\hat{f}_C(\omega)$ are the noise Fourier components, $\langle \hat{f}_{B+}(\omega)^{\dagger} \hat{f}_{B+}(\omega) \rangle = 4\pi\gamma\delta(\omega - \omega')$ and $\langle \hat{f}_{Cs}(\omega)^{\dagger} \hat{f}_{Cs}(\omega) \rangle = 4\pi\gamma_{Ms}(\bar{n}_{ths}+1)\delta(\omega-\omega')$ [similar expression for $\hat{f}_{Cr}(\omega)$]; and the field operators have presentation similar to that one of the Langevin forces

$$\hat{B}_+ = \int_{-\infty}^{\infty} \delta\hat{B}_+(\tilde{\omega}) e^{-i\tilde{\omega}t} \frac{d\tilde{\omega}}{2\pi}, \tag{13}$$

$$\hat{C} = C + \int_{-\infty}^{\infty} \delta\hat{C}(\tilde{\omega}) e^{-i\tilde{\omega}t} \frac{d\tilde{\omega}}{2\pi}. \tag{14}$$

The solution of the linearized set (3)–(5) with respect to the fluctuations is given by the Fourier amplitudes

$$\delta\hat{B}_+(\tilde{\omega}) = \frac{\Gamma_M}{\Gamma\Gamma_M + |g|^2|A|^2} \hat{f}_{B+}(\tilde{\omega}) - \frac{igA}{\Gamma\Gamma_M + |g|^2|A|^2} \hat{f}_C(\tilde{\omega}),$$

$$\delta\hat{C}(\tilde{\omega}) = \frac{\Gamma}{\Gamma\Gamma_M + |g|^2|A|^2} \hat{f}_C(\tilde{\omega}) - \frac{ig^* A^*}{\Gamma\Gamma_M + |g|^2|A|^2} \hat{f}_{B+}(\tilde{\omega}),$$

where we use notations $\hat{f}_C(\tilde{\omega}) = \hat{f}_{Cs}(\tilde{\omega}) + \hat{f}_{Cr}(\tilde{\omega})$, $\Gamma = \gamma - i\tilde{\omega}$, and $\Gamma_M = \gamma_M - i\tilde{\omega}$.

Neglecting the optical saturation of the pump field, we obtain expectation values for the fields from (3)–(5),

$$A \simeq \frac{\langle \hat{F}_A \rangle}{\gamma}, \tag{15}$$

$$C \simeq \frac{\langle \hat{F}_C \rangle}{\gamma_M} \frac{\gamma\gamma_M}{\gamma\gamma_M + |g|^2|A|^2}, \tag{16}$$

$$B_+ \simeq \frac{igA}{\gamma} C. \tag{17}$$

It is important to note at this point that there exists a maximum of the sideband amplitude B_+ with respect to the pump amplitude $|A|$ for a given signal amplitude $(\langle \hat{F}_{Cs} \rangle)$. This is similar to the saturation phenomena in Raman lasers occurring due to the power broadening.

We finally derive expressions for the expectation values

$$E_{\text{out}} \simeq E_{\text{in}}, \quad E_{\text{out}+} = E_{\text{in}} \frac{2ig}{\gamma} C \tag{18}$$

as well as fluctuations of the output optical field of the sideband

$$\hat{e}_{\text{out}+}(\tilde{\omega}) = \frac{\Gamma^*\Gamma_M - |g|^2|A|^2}{\Gamma\Gamma_M + |g|^2|A|^2} \hat{e}_{\text{in}+}(\tilde{\omega}) - \frac{2igA\sqrt{\gamma}}{\Gamma\Gamma_M + |g|^2|A|^2} [\sqrt{\gamma_{Ms}} \hat{e}_{Cs}(\tilde{\omega}) + \sqrt{\gamma_{Mr}} \hat{e}_{Cr}(\tilde{\omega})]. \tag{19}$$

We used expressions $\langle A \rangle \sqrt{2\gamma\tau} = 2E_{\text{in}}$, $F_A = E_{\text{in}} \sqrt{2\gamma/\tau}$, $E_{\text{out}\pm} = B_{\pm}\sqrt{2\gamma\tau}$, τ is the resonator round trip time $\tau = 2\pi R / c$, where R is the radius of the resonator. We assume, for the sake of simplicity, that the resonator is phase matched (or empty), so that $\tau = \tau_M$.

Equation (19) shows that the fluctuations of the anti-Stokes sideband are given by the optical shot noise fluctuations as well as by the thermal fluctuations in the THz mode and the receiver. It is easy to see that the thermal noise contributions come with coefficients describing the coupling of the thermal sources to the resonator mode. The effective mode temperature is (in the limit of large temperature, when $n_{th} \approx k_B T / \hbar\omega_c$)

$$T_{\text{eff}} \approx \frac{\gamma_{Ms}}{\gamma_M} T_s + \frac{\gamma_{Mr}}{\gamma_M} T_r. \tag{20}$$

The contribution from the receiver temperature is significantly suppressed if the resonator is over coupled, $\gamma_{Ms} \gg \gamma_{Mr}$.

The fluctuations of the outcoming light do not depend on the optical pumping. The photon number in the anti-Stokes mode is given by the thermal fluctuations and is independent on the pumping power if there is no THz signal. Hence, the excess power fluctuations of the power of the pump laser will not impact the sensitivity in the case of the undepleted pump.

A. Detection of weak THz signals

Let us consider an important for THz astronomy case and assume that the noise background $\hat{e}_{Cs}$ is our signal ($P_{mw}=0$). The power noise of the anti-Stokes light is given by the following expression:

$$\langle \Delta P_{mw}^2 \rangle \simeq 2\frac{\gamma_{Mr}^2}{\gamma_{Mc}^2} n_{thr}^2 \left(\frac{\hbar\omega_c}{2} \frac{\gamma\gamma_{Mc} + |g|^2|A|^2}{\gamma + \gamma_{Mc}} \right)^2 .$$

In other words, the detector temperature is $T_{\text{eff}} \simeq T_r \gamma_{Mr}/\gamma_{Mc}$ now.

B. Detection of strong THz signals

Let us consider another example and assume that one measures photon number $\hat{E}^\dagger_{\text{out}+}\hat{E}_{\text{out}+}$ in the anti-Stokes field to detect a monochromatic THz signal with averaged power $P_{\text{in } mw}$ when $T_r = T_s = T$. One will observe the averaged signal

$$P_{mw} = P_{\text{in } mw} + \frac{\gamma\gamma_M + |g|^2|A|^2}{\gamma + \gamma_M} \frac{k_B T}{2} \tag{21}$$

masked by noise

$$\langle \Delta P_{mw}^2 \rangle = 2n_{th}(n_{th}+1) \left(\frac{\hbar\omega_c}{2} \frac{\gamma\gamma_M + |g|^2|A|^2}{\gamma + \gamma_M} \right)^2 + P_{\text{in } mw} \int_{-\infty}^{\infty} \left[\frac{\hbar\omega_c}{4} \left(\frac{\sqrt{\gamma\gamma_M}}{|g||A|} + \frac{|g||A|}{\sqrt{\gamma\gamma_M}} \right)^2 + 2k_B T \left| \frac{\gamma\gamma_M + |g|^2|A|^2}{\Gamma\Gamma_M + |g|^2|A|^2} \right|^2 \right] \frac{d\tilde{\omega}}{2\pi}. \tag{22}$$

In the case that the signal has significant average power $P_{\text{in } mw} \neq 0$ and $|\tilde{\omega}| < \gamma, \gamma_M$ we have

$$\frac{\langle \Delta P_{mw}^2 \rangle}{P_{\text{in } mw}^2} \simeq \frac{\int \Delta P_{mw}(\tilde{\omega}) d\tilde{\omega}/(2\pi)}{P_{\text{in } mw}}. \tag{23}$$

The density of power fluctuations $\Delta P_{mw}(\tilde{\omega})$ is given by

$$\Delta P_{mw}(\tilde{\omega}) = \frac{\hbar\omega_c}{4} \left(\frac{\sqrt{\gamma\gamma_M}}{|g||A|} + \frac{|g||A|}{\sqrt{\gamma\gamma_M}} \right)^2 + 2k_B T. \tag{24}$$

The receiver has the smallest noise floor

$$\Delta P_{mw}(\tilde{\omega})|_{\min} = \hbar\omega_c + 2k_B T. \tag{25}$$

This is an obvious result for the maximum sensitivity of a linear coherent detector of THz radiation. Now, if one uses the photonic receiver in the THz photon counting mode, so that $P_{\text{in } mw}=0$, the detection noise (22) coincides with the noise of an ideal quadratic THz receiver. Thermal noise in the THz mode determines the maximum measurement sensitivity at room temperature. Shot noise determines the sensitivity in a cooled detector.

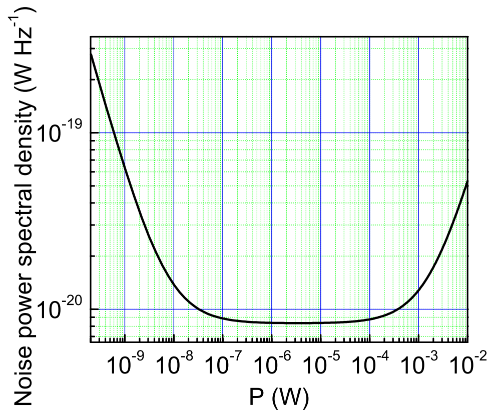

FIG. 3. (Color online) Spectral density of power fluctuations [as defined in Eq. (24)] vs power of the optical pump for the receiver operating in the linear regime. The spectral density has meaning of the noise equivalent power of the receiver. It shows that the sensitivity of the receiver goes to zero at small and large optical powers. It happens because of small conversion efficiency at small optical power and because of the saturation of the conversion efficiency at large optical powers [see Eq. (16)]. The minimum value of the spectral density is given by Eq. (25).

C. Estimation of the sensitivity

Let us estimate the optimum optical power for achieving the maximum sensitivity of the WGM receiver operating in the linear regime. To do this, we recall that [13]

$$g = \frac{4\pi\omega}{\epsilon_a} \chi^{(2)} \sqrt{\frac{2\pi\hbar\omega_c}{\epsilon_c \mathcal{V}_c}} \eta_{pm} \tag{26}$$

is a coupling constant, $\chi^{(2)}$ is the effective second-order electro-optic constant for the material of the dielectric cavity, ϵ_a and ϵ_c are the dielectric susceptibilities for the optical and THz frequencies, $\mathcal{V}_c$ is the volume of the microwave field. Using (26) we derive

$$P_{\text{opt}} = \frac{\omega \mathcal{V}_c}{(\chi^{(2)}\eta_{pm})^2} \frac{\epsilon_a^2 \epsilon_c}{(8\pi)^3 Q^2 Q_M}. \tag{27}$$

Selecting optimistic parameters $\omega = 10^{15}$ rad/s, $\chi^{(2)} = 10^{-6}$ esu, $\eta_{pm}=1$, $\mathcal{V}_c = 10^{-5}$ cm^3, $Q_M = 10^3$, $Q = 10^8$, $\epsilon_a = 4.6$, and $\epsilon_c = 26$ we find $P_{\text{opt}} \approx 3.5$ μW. This is a rather small value. The spectral density of power fluctuations [see Eq. (24)] calculated for the parameters is shown in Fig. 3.

V. CONCLUSION

We have proposed theoretically a THz photonic receiver based on a crystalline WGM resonator. We have shown that the receiver can operate both in the photon counting mode as well as in the linear reception mode and approach the quantum limit of the measurement sensitivity at experimentally achievable parameters.

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