Relativistic time dilation and the spectrum of electrons emitted by 33-TeV lead ions penetrating thin foils

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We study the energy distribution of ultrarelativistic electrons produced when a beam of 33-TeV $Pb^{81+}(1s)$ ions penetrates a thin Al foil. We show that, because of a prominent role of the excitations of the ions inside the foil, which becomes possible solely due to the relativistic time dilation, the width of this distribution can be much smaller than in the case when the ions interact with rarefied gaseous targets. We also show that a similar reduction in the width of the energy distribution of ultrarelativistic electrons arises when 33-TeV Pb^{82+} ions penetrate a thin Au foil. These results shed some light on the origin of the very narrow electron energy distributions observed experimentally about a decade ago.

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I. INTRODUCTION

Atomic physics normally does not deal with objects exposed to extreme conditions. One of the interesting and important exceptions to this rule is represented by the studies of various phenomena accompanying the penetration of targets by highly charged projectile ions moving with velocities very close to the speed of light. During the interaction between the ion and a target atom both of these particles are exposed to extremely intense and extraordinarily short pulses of the electromagnetic fields.

For instance, in collisions of 33-TeV hydrogenlike $Pb^{81+}(1s)$ ions with Al (which will be considered below) the typical durations of the electromagnetic pulses acting on the electron bound in the ion are $\lesssim 10^{-21}$ s (in the rest frame of the ion). The peak pulse intensities in this frame can reach $\sim 10^{28}-10^{29}$ W/cm^2 which enables, despite the very short interaction time, transitions of the very tightly bound electron of the ion to be induced with a noticeable probability [1].

The first experimental results on the total cross section for the electron loss from 33 TeV $Pb^{81+}(1s)$ were reported in [2] together with data for electron capture by 33-TeV bare Pb^{82+} ions [3].

Compared to the study of the total cross sections, much more information can be obtained when differential cross sections are explored. The first experimental results on the differential cross sections for such collisions were reported in [4]. In that experiment the incident beams of 33-TeV $Pb^{81+}(1s)$ and 33-TeV Pb^{82+} were penetrating Al and Au foils, respectively. In both cases it was found that the penetration is accompanied by the emission of ultrarelativistic electrons whose energy distributions have the form of a cusp with a maximum at an energy corresponding to the electrons moving in the laboratory frame with velocities equal to that of the ions.

One of the unexpected results reported in [4] was that the measured distribution of the high-energy electrons produced under the bombardment of a thin Al foil was found to be much narrower than one could expect based on the consideration of the width of the Compton profile of the electron state in the incident $Pb^{81+}(1s)$ ions (see also [5]). Moreover, in a more rigorous calculation performed in [6] for the energy spectrum of electrons emitted from a 33-TeV $Pb^{81+}(1s)$ ion colliding with an Al atom, it was confirmed that this spectrum is indeed much broader than that observed in the experiment [4].

Another intriguing finding of [4] was that, for 33-TeV Pb^{82+} ions incident on a thin Au foil, the shape of the measured energy distribution of high-energy electrons emerging from the foil was very similar to that obtained for the beam of 33-TeV $Pb^{81+}(1s)$ ions incident on the Al foil.

It is known that the total and differential loss cross sections depend on the bound state from which the electron leaves the ion (see, e.g., [7–11] and references therein). Therefore, it was speculated in [4] that in the case of the incident 33-TeV Pb^{82+} ions the very narrow shape of the electron cusp might be a signature of electron capture into excited states. However, for $Pb^{81+}(1s)$ ions incident on the Al foil, the possible influence of excited states of these ions on the electron cusp was not considered seriously because of the common experience that excitations of very heavy hydrogenlike ions inside thin foils of relatively light elements do not have a noticeable impact on the electron loss process.

For instance, in the recent experimental-theoretical study [12] on 200 MeV/u $Ni^{27+}(1s)$ ions incident on gaseous and solid targets, it was found that the fraction of the ions excited inside the solids does not exceed 5–6 %, leading to corrections of about 20% to the (effective) electron loss cross section. Effects of similar order, caused by the population of excited states of the ions, have been observed in the energy distributions of the emitted electrons: in [13] it was found that the width of the energy distribution of electrons emitted by 390 MeV/u Ar and Fe ions can be reduced by 15–35 %.

Moreover, even such rather modest effects seem to be hardly reachable for very heavy hydrogenlike ions since, compared to the case of relatively light ions, the penetration of matter by the very heavy ions possesses the following two important differences. First, because of a very tight binding of the electron in such ions, cross sections for collision-induced electron transitions are much smaller. Therefore, for highly charged ions like Pb^{81+} moving inside solids the mean free path with respect to the collision-induced transitions will be much larger. Second, the lifetimes of the excited states with respect to the spontaneous radiative decay in such ions are much shorter.

The above two points mean that there will be much more time between successive collisions for the excited ion to relax into the ground state via the spontaneous radiative decay. As a result, there might seem to be sound grounds for the skeptical attitude to the possible role played by the excitations of the incident 33-TeV $Pb^{81}(1s)$ ions in the formation of the electron cusp. However, it will be demonstrated below that the expectations based on the experience accumulated when exploring collisions at moderate relativistic impact energies have to be substantially corrected in the case of the extreme relativistic energies studied in [4].

II. THEORY

Our consideration of the energy spectrum of the cusp electrons assumes that the foil materials are amorphous (not crystals) and includes three main ingredients.

First, the basis of our consideration is represented by calculations of cross sections for the projectile-electron excitation (deexcitation) and loss occurring in the ion-atom collisions. We also calculate cross sections for the bound-free pair production. In our calculations we use the Dirac-Coulomb wave functions to describe bound and continuum states of the electron (and the positron) in the field of the bare lead nucleus. The ion-atom interaction is described in the first-order relativistic theory [14] including the screening of the atomic electrons; in addition, their antiscreening contribution is evaluated as well. We have also checked our first-order results by applying the so-called light cone approximation [15] and found no noticeable deviations from the first-order results [16].

We also consider the channels for radiative electron capture and kinematic capture. At an impact energy of 33 TeV, as our calculations [18] for the radiative capture show, this capture channel is much weaker than the bound-free production. The kinematic capture channel is even weaker than the radiative capture and can simply be ignored.

Within our basic atomic physics analysis we also calculate rates for the spontaneous radiative decay of excited hydrogenlike lead ions to all possible internal states with lower energies [18].

In the second step we solve the kinetic equations which describe the population $N_j(t)$ of the internal states of the ion inside the foil and obtain these populations as a function of time t [or of the ionic coordinate $z=vt$ inside the foil; z and t are measured in the laboratory frame, and $\mathbf{v}=(0,0,v)$ is the projectile velocity].

The system of kinetic equations reads

$$\frac{dN_0}{dt} = -\frac{N_0}{\tau^{\text{capt}}} + \sum_{j=1}^{N_{\max}} \frac{N_j}{\tau_j^{\text{loss}}},$$

$$\frac{dN_j}{dt} = \frac{N_0}{\tau_j^{\text{capt}}} - \frac{N_j}{\tau_j^{\text{loss}}} - N_j \sum_{i=1}^{i<j} \frac{1}{\tau_{j\to i}^{\text{sp}}} - N_j \sum_{i=1\ (i\neq j)}^{N_{\max}} \frac{1}{\tau_{j\to i}} + \sum_{i=1\ (i\neq j)}^{N_{\max}} \frac{N_i}{\tau_{i\to j}}, \quad (1)$$

where N_0 is the number of bare ions, N_j is the number of ions with one electron in the jth internal state $(j=1,2,\ldots,N_{\max})$, and $N_{\max}$ is the total number of involved bound states. Moreover, τ_j^{capt} is the mean time for electron (vacuum and radiative) capture into the jth state and τ^{capt} is the mean time for electron capture to any state $(1/\tau^{\text{capt}}=1/\tau_1^{\text{capt}}+1/\tau_2^{\text{capt}}+\cdots)$. As for the capture mean times, we can introduce (i) the mean time for the electron loss from a state j to the continuum, τ_j^{loss}, (ii) the mean time for the collision-induced transition from the internal state i to the internal state j, $\tau_{j\to i}$, and (iii) the lifetime of the state j with respect to the spontaneous radiative transition to any possible state i, $\tau_{j\to i}^{\text{sp}}$.

The mean decay, loss, and capture times in Eq. (1) can be obtained in the usual way from the elementary cross sections and spontaneous decay rates obtained during the first step of the consideration. In particular, $\tau_{j\to i}^{\text{sp}}=\gamma/\Gamma_{j\to i}^{\text{sp}}$ where $\Gamma_{j\to i}^{\text{sp}}$ is the spontaneous decay rate for the transition $j\to i$ calculated in the rest frame of the ion and $\gamma=1/\sqrt{1-v^2/c^2}$ is the collision Lorentz factor. In contrast to the spontaneous decay, the collision-induced transition times depend also on the atomic density of the target n_a as, for example, $\tau_j^{\text{loss}}=1/(n_a\sigma_j^{\text{loss}}v)$, where σ_j^{loss} is the cross section for the electron loss from the jth internal state of the ion and n_a is the atomic density of the target.

After the calculation of the elementary cross sections and, hence, the mean decay, loss, and capture times, we can integrate the system of kinetic equations (1) and to find the population of the internal states N_j as a function of time or ionic coordinate. By making use of these populations, we can obtain the energy spectrum of the electrons emitted from the ion traversing a solid foil of a thickness L which is given by

$$\frac{dn_e}{d\varepsilon_p} = n_a \sum_{j=1}^{N_{\max}} \frac{d\sigma_j^{\text{loss}}}{d\varepsilon_p} \int_0^L dz\, N_j(z), \quad (2)$$

where ε_p is the total electron energy in the laboratory frame and $\frac{d\sigma_j^{\text{loss}}}{d\varepsilon_p}$ is the energy distribution of the electrons emitted from the internal state j.

The third step of our consideration deals with the transport of the emitted electrons through the foil. The detailed analysis of this step represents in general quite a delicate task, but in our case is substantially simplified by the fact that the electrons leaving the ions have in the laboratory frame extremely high values of energy. There are two main effects that can influence the shape of the electron energy distribution when the electrons penetrate the foil.

The first concerns energy losses of the ultrarelativistic electrons traversing the foil. These losses are caused by (i) the excitation of the electrons of the foil and (ii) the emission of the radiation by the ultrarelativistic electrons because of their acceleration during the interactions with the atomic nuclei in the foil. However, for the foil parameters used in the experiment [4] the energy losses can simply be ignored because they are very small ($\lesssim 0.5\%$) compared to the initial energies of these electrons.

The second effect that may possibly influence the shape of the measured energy distributions is that collisions in the foil broaden the distribution of the ultrarelativistic electrons over the transverse components (p_x,p_y) of their momenta. For the foil parameters used in [4], the multiple collisions suffered

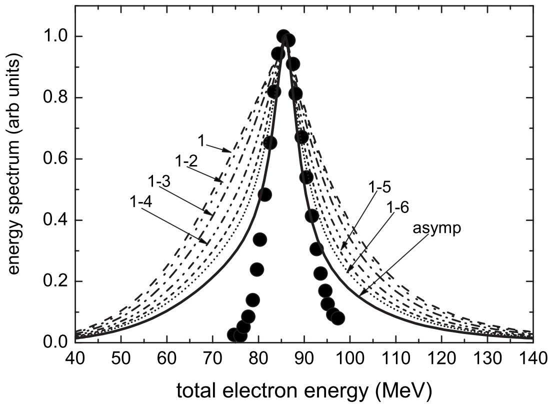

FIG. 1. Energy distribution of the electron cusp produced in collisions between an incident beam of 33-TeV $Pb^{81+}(1s)$ with Al foil with a thickness of 2.85×10^{-2} cm (for more explanation see the text). Circles show the electron energy distribution measured in [4] for 33-TeV Pb^{81+} colliding with an Al foil of the same thickness. All the distributions are given in the laboratory frame and are normalized to 1 at the maximum.

by the ultrarelativistic electrons inside the foil substantially increase the width of their (p_x,p_y) distribution compared to the one these electrons have when leaving the 33-TeV nuclei.

Nevertheless, even after this increase the transverse components ($\sim10^2$ a.u.) remain very small compared to the total electron momenta ($\simeq2\times10^4$ a.u.), which means that the broadening of the (p_x,p_y) distribution may have an impact on the measured electron momentum distribution only if special geometric conditions are employed in an experiment [19]. Since we do not possess all necessary information about the real conditions of the experiment [4], in our calculations for the energy spectra, discussed below, we simply take all electrons (whichever angle they have after leaving the foil) into account.

Under such conditions the changes in the electron momenta during the electron transport through the foil do not have an impact on the final electron energy distribution. Therefore, the main essential difference between the previous estimates [4,6] for the shape of the electron cusp in the case of 33-TeV $Pb^{81+}(1s)$ incident on an Al foil (where it was assumed that the electron loss occurs in the single-collision regime) and our present model is that the latter takes into account electron transitions to the continuum not only directly from the ground state of the ions but also via the intermediate excitations to higher bound states occurring when the ions penetrate the foil.

III. RESULTS AND DISCUSSION

The initial expectation, that in the case of very heavy ions their excitations are of minor importance for the loss process, seems to be just confirmed if we compare in Fig. 1 the curves labeled with 1 and 1–2. In this figure, where we present results for the electron energy spectrum in the case of 33-TeV $Pb^{81+}(1s)$ ions incident on Al foil, the curve 1 was obtained by ignoring all excited bound states, while in the calculation resulted in the curve 1–2 the states with the principal quantum number $n=2$ were also taken into account. Yet there is just a tiny difference in the widths of these two curves.

However, when we add the states with $n=3$ into our analysis (the curve in Fig. 1 labeled by 1–3), the width-reducing effect becomes quite visible. Adding into the analysis the states with $n=4$ leads to a further reduction in the calculated width and this reduction is even larger than that observed when the states with $n=3$ are added. The reduction of the width continues further when we add states with $n=5$ and 6 (see Fig. 1); however, it proceeds at a smaller pace compared to that when the states with $n=3$ and 4 were added.

Note that the inclusion of the states with $n=1-6$ into the analysis means that we calculated the collision-induced and spontaneous radiative transitions in a system of levels involving 182 quantum bound states of the Pb^{81+} ion as well as the electron and positron continua in the field of the nucleus Pb^{82+}. This is quite demanding and a computationally expensive task. For obvious reasons, in our calculations we cannot increase indefinitely the number of bound states. Therefore, we have attempted to extrapolate our results in order to get the asymptotic limit for the electron cusp shape corresponding to taking into account all bound states ($n\to\infty$).

Using standard software tools [20] we analyzed changes in the shape of the electron cusp occurring when we start with the $n=1$ case and subsequently add states with $n=2$, 3, 4, and 5. In this way we obtained the first asymptotic limit of the cusp shape. Then we repeated the same procedure but now including into the set of the calculated cusps one more spectrum which is obtained when the levels with $n=6$ are also taken into account. The second asymptotic limit practically coincided with the first one.

In addition, within each of these two simple schemes we also used several different options in order to analyze the changes in the cusp shape appearing when the bound states with $n=2-6$ are sequentially added in the analysis and to extrapolate them to get the asymptotic limit of the cusp shape. All these options yielded practically the same results. Thus, we have very good grounds to believe that the asymptotic limit that we found is physically meaningful.

Comparing the energy distributions in Fig. 1 we see that their asymptotic width is about a factor of 3 smaller than the width obtained by assuming that the cusp is produced under the single-collision conditions. This strong effect is caused by the excitation of the ions inside the foil, which involves rather high-lying bound states: when the ions move in the foil the electron cloud surrounding the ionic nuclei has enough time to expand tremendously in size before it almost completely disappears due to transitions to the continuum. The key factor making this possible is the relativistic time dilation, which effectively decreases the spontaneous decay rates of the excited states of the ions by a factor of ≈170 [21].

Compared to the ground state, the excited states have larger loss cross sections (and, thus, shorter free paths with respect to the loss) and narrower Compton profiles which, along with the relative decrease in the population of the ground state due to the excitations, lead to the narrowing of the electron energy distribution.

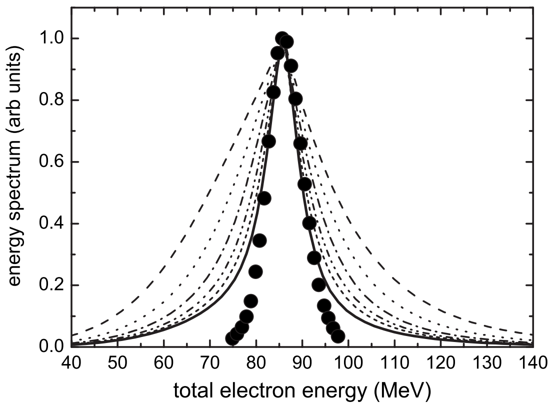

FIG. 2. Same as in Fig. 1 but for an incident beam of 33-TeV Pb^{82+} penetrating Au foil with a thickness of 8.81×10^{-4} cm corresponding to the conditions of the capture experiment [4]. Circles show the electron energy distribution measured in [4].

One more point that should be mentioned is that cross sections for electron capture are relatively very small. As a result, in the formation of the electron cusp in the case of hydrogenlike ions incident on the Al foil the capture channels do not play any noticeable role.

Let us now turn to the consideration of the electron cusp formed when 33-TeV Pb^{82+} ions are incident on a Au foil. Our results for this case are shown in Fig. 2.

Of course, in the case of incident bare projectiles the electron capture from vacuum becomes of paramount importance for the very existence of the electron cusp. One should note, however, that the capture cross sections decrease very rapidly when n and j_e increase (j_e is the total angular momentum of the electron in a bound state). Therefore, most of the excited bound states having a very important impact on the energy spectrum are populated not by capturing the electron directly from the vacuum but via excitations from a few states with the lowest values of n and j_e, for which the capture is efficient. This indirect path becomes especially effective because in collisions with Au atoms the excitation cross sections are much larger than in the case with Al.

Comparing the spectra shown in Fig. 2 with those displayed in Fig. 1, we see that the changes in the form of the calculated spectrum in Fig. 2 (occurring when we allow for more bound states in our analysis) are accumulating at a different pace. In addition, the asymptotic cusp shape in Fig. 2 has less pronounced wings. These differences are related to two basic reasons: (i) the excitation and loss cross sections in a Au foil are much larger while the spontaneous decay rates remain exactly the same as in the case of an Al foil, and (ii) the initial step in the cusp formation is now represented by the capture process, which also somewhat increases the relative population of the excited states compared to the case when a beam of $Pb^{81+}(1s)$ ions was incident on the Al foil.

Curiously, however, the asymptotic width in Fig. 2 is again about three times smaller than the initial width and the shape of the asymptotic spectra in both cases looks similar (which is also in agreement with the experimental observations of [4]). In general, such a similarity will not hold when the foil parameters (for instance, their thicknesses) are changed and, in this sense, it is accidental. Yet in both cases the strong reductions in the widths of the energy distributions are caused by the excitations of the electrons to rather high-lying bound ionic states occurring when the ions penetrate the foils.

IV. SUMMARY

In summary, briefly, we have considered the energy spectra of the ultrarelativistic electrons emitted when incident 33-TeV $Pb^{81+}(1s)$ and Pb^{82+} ions penetrate Al and Au foils, respectively. The foil thicknesses were taken to be the same as used in the experiment [4]. We have found that these spectra are much narrower than those that would be produced under single-collision conditions and have similar shapes. The similarity in the shapes in general will not hold if foils with other parameters are used and thus is fortuitous. However, in both cases the strong width reduction is caused by the excitations of the ions when they penetrate the target foils and suffer multiple collisions with the target atoms. Such a profound role of the excitations in the case of very heavy ions is in contrast to the previous experience gained when exploring collisions in the low and intermediate relativistic domains of impact energies. In the case under consideration, the excitations become so effective because of the relativistic time dilation, which decreases very strongly the spontaneous decay rates of excited states in ions moving with velocities very closely approaching the speed of light.

Our present consideration sheds some light on the origin of the unexpectedly narrow shape of the electron cusp produced by the ultrarelativistic heavy ions. However, a more careful analysis taking into account all real conditions of the experiment [4] would be necessary in order to make a detailed comparison between the experimental and theoretical results on the electron cusp.

[1] These field parameters may be compared with the parameters of state of the art laser systems whose shortest pulse lengths are about $\sim 10^{-16}-10^{-15}$ and whose peak intensities do not exceed $\sim 10^{21}-10^{22}$ W/cm^2.

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