Broadcasting of continuous-variable entanglement

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We present an example for broadcasting of the entanglement of a two-mode squeezed state of the electromagnetic field shared by two distant parties into two nonlocal bipartite entangled states. Using the technique of covariance matrices we demonstrate the entanglement between the nonlocal output modes and the separability of the local output modes. We find the range of values for the squeezing parameter and the amplifier phase for which broadcasting of continuous-variable entanglement can be implemented for physical states.

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I. INTRODUCTION

Quantum entanglement is now recognized as a powerful resource in communication and computation protocols. The first nontrivial consequence of entanglement on quantum ontology was noticed many years ago within the context of continuous-variable systems [1]. In recent times there has been a rapid development of the theory of entanglement pertaining to infinite-dimensional Hilbert spaces [2]. Several interesting applications of storage and distribution of continuous-variable information are being formulated [3].

Unlike classical correlations, quantum entanglement cannot be freely shared by several systems. An important issue is that of broadcasting of entanglement, viz., whether the entanglement shared by two parties can be transmitted to two less entangled states by local operations. Such a procedure, if implemented practically, could have applications in information processing tasks. For example, as is well known in the case of quantum teleportation, the quantum channel that is an entangled state is lost forever once the state is teleported. However, if one were able to produce two copies of the quantum channel, then it could be possible to perform two different quantum informational tasks with a somewhat reduced amount of entanglement in both the channels. The resources involved in other methods of producing two similarly entangled pairs might be comparitively higher.

The process of broadcasting involves copying of local information. Since exact cloning of an unknown quantum state is impossible [4], it is expected for any scheme of broadcasting to be limited to specific input states that lead to optimal fidelity for the output states. For the case of discrete variables, it is known that imperfect broadcasting of entanglement is possible for restricted input states [5]. No scheme has yet been proposed however, for the broadcasting of continuous-variable entanglement.

The inexact copying of local continuous-variable information has nonetheless, been performed in several works. Various schemes for duplication of coherent states with optimal fidelity and economical means have been suggested using cloning machines comprising of networks of linear amplifiers and beam splitters [6,7]. The idea of telecloning has been extended for continuous variables [8] where clones of an unknown state are generated locally, and then teleported to a distant party by means of previously shared entanglement. Further, by employing a number of input copies it has been shown how superbroadcasting, i.e., the purification of output clones, can be achieved for continuous-variable mixed states too [9].

None of the above studies are about broadcasting of continuous-variable entanglement though, and all of them involve imperfect cloning of unentangled local modes. It is thus all the more relevant to investigate whether such ideas of copying local information can be extended for the purpose of mapping entangled and nonlocal states of continuous variables. To this end we formulate a scheme for transmission of a bipartite entangled two-mode squeezed state into a pair of nonlocal but again bipartite and less entangled states. We employ one cloner [6] at each mode locally, which is comprised of a linear amplifier and a beam splitter available with each party.

II. LOCAL CLONING MACHINE

We begin by describing in terms of the covariance matrix (CM) [10] approach the process of imperfect copying of local continuous-variable information through the cloning machine proposed by Braunstein *et al.* [6]. A single-mode squeezed vacuum state of the electromagnetic field is represented by the squeezing transformation operator acting on the vacuum mode

$$S_i(r) = \begin{pmatrix} e^r & 0 \\ 0 & e^{-r} \end{pmatrix}, \tag{1}$$

where r is the squeezing parameter $(r>0)$. The (CM) corresponding to the single-mode (say, i) squeezed vacuum state is given by

$$\sigma_i(r) = S_i(r) S_i^T(r) = \begin{pmatrix} e^{2r} & 0 \\ 0 & e^{-2r} \end{pmatrix}. \tag{2}$$

The cloning of this state proceeds as follows. First, a linear amplifier [10] mediates the interaction between the mode i and an ancilla (say, a) prepared in the vacuum state, which is represented by the linear transformation

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$$\mathcal{A}_{ia}(r,\phi) = [S_{ia}(r,\phi)] \cdot [S_i \oplus I_a] \tag{3}$$

with ϕ being the phase of the amplifier. After this interaction the squeezed state mode together with the ancilla mode and another blank state mode (say b) are incident on a three-mode 50:50 beam splitter B_{iab} which we define through a symplectic transformation as

$$B_{iab} = \begin{pmatrix} \sqrt{1/2} & 0 & 0 & 0 & \sqrt{1/2} & 0 \\ 0 & \sqrt{1/2} & 0 & 0 & 0 & \sqrt{1/2} \\ 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 \\ \sqrt{1/2} & 0 & 0 & 0 & -\sqrt{1/2} & 0 \\ 0 & \sqrt{1/2} & 0 & 0 & 0 & -\sqrt{1/2} \end{pmatrix}. \tag{4}$$

The above beam splitter is defined in such a way that it does not affect the ancilla mode. The total cloning operation is thus represented by the transformation

$$T_{iab}(r,\phi) = [B_{iab}] \cdot [\mathcal{A}_{ia} \oplus I_b] \tag{5}$$

with the corresponding CM given by

$$\sigma_{iab} = T_{iab}(r,\phi) T^{\dagger}_{iab}(r,\phi). \tag{6}$$

This procedure leads to symmetric cloning resulting in the two cloned output modes i and b.

The fidelity of the two clones can be evaluated through the relation [11]

$$F = \frac{1}{\sqrt{\mathrm{Det}(\sigma_{\mathrm{in}} + \sigma_{\mathrm{out}}) + \delta} - \sqrt{\delta}}, \tag{7}$$

where σ_{in} is given by Eq. (2), and σ_{out} is obtained by tracing out the ancilla mode from the CM in Eq. (6), i.e.,

$$\sigma_{\mathrm{out}} = \begin{pmatrix} P & 0 \\ 0 & M \end{pmatrix} \tag{8}$$

with $P=[e^{2r}(c-hs)^2+k^2s^2+1]/2$, $M=[e^{-2r}(c+hs)^2+k^2s^2+1]/2$, and $\delta=4(\mathrm{Det}[\sigma_{\mathrm{in}}]-1/4)(\mathrm{Det}[\sigma_{\mathrm{out}}]-1/4)$, where

$$c = \cosh(2r), \quad s = \sinh(2r), \quad h = \cos(2\phi), \quad k = \sin(2\phi). \tag{9}$$

The fidelity for the above phase sensitive cloning machine is thus given by

$$F = \frac{1}{\sqrt{(P + e^{2r})(M + e^{-2r}) + 3(PM - 1/4)} - \sqrt{3(PM - 1/4)}}. \tag{10}$$

If the phase of the amplifier is set to $\phi=0$, the fidelity becomes $F=2/(\sqrt{8c^2+12c+5}-\sqrt{3+6c})$. Since the fidelity of the clones depend on the input state, the cloning is said to be state dependent. It follows that as $r\to\infty$, $F\to 0$, and as $r\to 0$, $F\to 1$.

III. PROTOCOL FOR BROADCASTING ENTANGLEMENT

Let us now consider a continuous-variable entangled state (an entangled state of the electromagnetic field) which is shared by two parties located far apart at sites I and J, respectively, and is represented by the generic two-mode (i and j) squeezed state with one mode at each end. The two-mode squeezed vacuum state is obtained by applying the transformation [10]

$$T_{ij}(r) = B_{ij}(1/2) \cdot [S_i(r) \oplus S_j(r)] \tag{11}$$

on two uncorrelated squeezed vacuum modes $[S_i(r)$ and $S_j(r)]$ given by Eq. (1). $B_{ij}(1/2)$ denotes a balanced 50:50 beam splitter with the matrix form

$$B_{ij}(1/2) = \begin{pmatrix} \sqrt{1/2} & 0 & \sqrt{1/2} & 0 \\ 0 & \sqrt{1/2} & 0 & \sqrt{1/2} \\ \sqrt{1/2} & 0 & -\sqrt{1/2} & 0 \\ 0 & \sqrt{1/2} & 0 & -\sqrt{1/2} \end{pmatrix}. \tag{12}$$

The CM corresponding to the two-mode squeezed state is given by

$$\sigma_{ij}(r) = T_{ij}(r) T^{\dagger}_{ij}(r) = \begin{pmatrix} c & 0 & s & 0 \\ 0 & c & 0 & -s \\ s & 0 & c & 0 \\ 0 & -s & 0 & c \end{pmatrix}. \tag{13}$$

The two-mode squeezed vacuum state is the quantum optical representative for bipartite continuous-variable entanglement. In the Heisenberg picture the quadrature operators of the two-mode squeezed state are given by

$$\hat{x}_i = \frac{e^r \hat{x}_i^{(0)} + e^{-r} \hat{x}_j^{(0)}}{\sqrt{2}}, \quad \hat{p}_i = \frac{e^{-r} \hat{p}_i^{(0)} + e^{r} \hat{p}_j^{(0)}}{\sqrt{2}},$$

$$\hat{x}_j = \frac{e^r \hat{x}_i^{(0)} - e^{-r} \hat{x}_j^{(0)}}{\sqrt{2}}, \quad \hat{p}_j = \frac{e^{-r} \hat{p}_i^{(0)} - e^{r} \hat{p}_j^{(0)}}{\sqrt{2}}, \tag{14}$$

where the superscript (0) denotes the initial vacuum modes, the operators $\hat{x}$ and $\hat{p}$ represent the electric quadrature amplitudes (the real and imaginary parts, respectively, of the mode's annihilation operator).

Our purpose here is to broadcast the above two-mode squeezed state to two less entangled two-mode squeezed states. For broadcasting of the above state we apply local cloning machines on the two individual modes of the entangled bipartite state, located at the sites I and J, respectively. Our scheme for broadcasting proceeds as follows (Fig. 1). The local cloner acting on the mode at site I copies the information available locally onto two modes (i and b). Similarly, the cloner acting on the mode at site J copies information onto two modes (j and b'). Note that since the two modes on which the cloners act at sites I and J, respectively, are the constituents of an initially entangled bipartite state, the forms of the output local clones will be different in general, from the outputs of the cloning for a single-mode squeezed state given by Eq. (8). More importantly, the properties of entanglement between the output states are now dependent on the initial entangled state to be broadcasted.

The definition of successful imperfect broadcasting [5] of the entangled two-mode squeezed state (13) can be elaborated as follows. The initial entangled state is broadcasted if

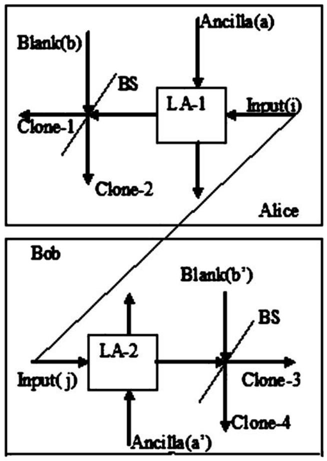

FIG. 1. Schematic diagram for the broadcasting protocol. Alice and Bob have the modes i and j, respectively, which are entangled. Alice inputs her mode i and an ancilla mode a on the linear amplifier LA1. The output mode from LA1 and a blank mode b are made to fall on a beam splitter and the resultant modes 1 and 2 emerge from it. Bob uses a similar setup (cloning machine) on his side to input his mode j and obtain the output modes 3 and 4. It is shown that the entanglement between i and j is broadcasted to the entanglement between the nonlocal modes 1 and 3 (and 2 and 4), but the local pairs 1 and 2 (and 3 and 4) form separable states for a range of values of the squeezing parameter r.

both the nonlocal pairs of output modes (i and b' on the one hand), and (j and b on the other hand) are entangled. Further, since our task is to create output states that are bipartite entangled, the local pairs of output modes (i and b at site I) and (j and b' at site J) should be separable, simultaneously. Our aim is to verify whether the above conditions are satisfied for the output states when the two cloners act at their respective sites. To this end, we formulate this bilocal cloning procedure using the CM approach [10].

The broadcasting operation is implemented through an ancilla mode, a linear amplifier and a beam splitter located at both sites. Thus, after introducing the ancillas a and a' (at the ends I and J, respectively), the CM of the joint two-mode squeezed state with the ancillas takes the form

$$\sigma_{iaja'}(r)=\begin{pmatrix} c & 0 & 0 & 0 & s & 0 & 0 & 0\\ 0 & c & 0 & 0 & 0 & -s & 0 & 0\\ 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0\\ 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0\\ s & 0 & 0 & 0 & c & 0 & 0 & 0\\ 0 & -s & 0 & 0 & 0 & c & 0 & 0\\ 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0\\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \end{pmatrix}. \tag{15}$$

Next, both parties apply linear amplifiers on their respective modes and ancillas. The local amplifiers can be jointly represented as

$$\mathcal{A}(r,\phi)=\begin{pmatrix} \mathcal{A}_1 & 0\\ 0 & \mathcal{A}_2 \end{pmatrix}, \tag{16}$$

where the $\mathcal{A}_i$ are given by [10]

$$\mathcal{A}_i=\begin{pmatrix} c-hs & 0 & ks & 0\\ 0 & c+hs & 0 & -ks\\ ks & 0 & c+hs & 0\\ 0 & -ks & 0 & c-hs \end{pmatrix} \tag{17}$$

for $i=1,2$. After interaction with the local amplifiers, the CM of the two modes with their ancillas is transformed to

$$\sigma'_{iaja'}(r,\phi)=\mathcal{A}^T\sigma_{iaja'}(r,\phi)\mathcal{A}. \tag{18}$$

Thereafter, both parties introduce their respective blank modes (b and b') on which the information of the original modes is to be copied. Both parties now have three local modes each, i.e., original, ancilla, and blank mode, which fall on a 50:50 beam splitter defined through a symplectic transformation in Eq. (4), at each end. The two local beam splitters can be represented jointly by

$$B=\begin{pmatrix} B_{iab} & 0\\ 0 & B_{ja'b'} \end{pmatrix} \tag{19}$$

and the resultant CM at the end of the cloning processes at both the ends is given by

$$\sigma''_{iabja'b'}=B^T\sigma'_{iabja'b'}B. \tag{20}$$

Using the above CM we can now check if the criteria for successful broadcasting are satisfied. In order to verify the entanglement of the nonlocal pairs of modes shared by the two sides, we obtain the reduced CMs corresponding to these modes, which (after tracing out the remaining modes from $\sigma''_{iabja'b'}$) are given by

$$\sigma^{\mathrm{nonlocal}}_{ib'}(r,\phi)=\sigma^{\mathrm{nonlocal}}_{jb}(r,\phi) =\begin{pmatrix} \frac{G+1}{2} & 0 & \frac{E}{2} & 0\\ 0 & \frac{H+1}{2} & 0 & \frac{-E}{2}\\ \frac{E}{2} & 0 & \frac{G+1}{2} & 0\\ 0 & \frac{-E}{2} & 0 & \frac{H+1}{2} \end{pmatrix}, \tag{21}$$

where $E=s(c-hs)^2$, $G=(c-hs)^2c+ks^2$, and $H=(c+hs)^2c+ks^2$.

In order to describe a physical state a CM σ must satisfy the uncertainty principle

$$\sigma+i\mathcal{J}\geq 0 \tag{22}$$

with

$$\mathcal{J}=\begin{pmatrix} 0 & 1\\ -1 & 0 \end{pmatrix}\oplus\begin{pmatrix} 0 & 1\\ -1 & 0 \end{pmatrix}. \tag{23}$$

The uncertainty principle is equivalent to the following conditions [10,13]:

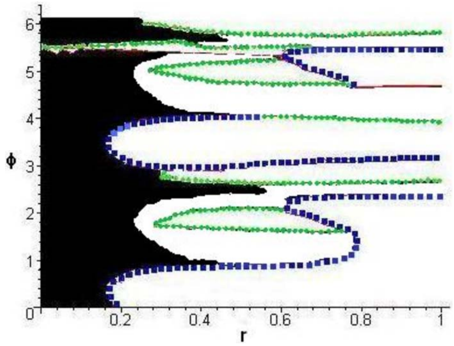

FIG. 2. (Color online) Contours representing the conditions for broadcasting (i) entanglement of the nonlocal states (marked with squares), (ii) physicality of the output local and nonlocal states (marked with dots), and (iii) separability of the output local modes (remaining boundaries of the shaded regions) are plotted in the r-ϕ plane. All the conditions for broadcasting are satisfied in the dark regions.

$$\sigma > 0,$$

$$\nu_-^2 > 1, \tag{24}$$

where ν_- refers to the symplectic eigenvalue of σ. Note that for the CM (21), the above conditions imply

$$\lambda_i > 0,$$

$$\nu_-^2 = [(G+1)(H+1) - E^2 \pm E(G-H)]/4 \geq 1, \tag{25}$$

where λ_i are the standard eigenvalues, and the $\pm$ sign is for the condition on the symplectic eigenvalues referring to $(H-G)$ being positive or negative, respectively.

We next obtain the partial transpose $\tilde{\sigma}_{ib'}^{\text{nonlocal}}(r,\phi)$ of the CM (21) and compute its symplectic eigenvalues. The condition for the entanglement of the nonlocal modes is given by

$$\tilde{\nu}_-^2 = [(G+1)(H+1) + E^2 - E(G+H+2)]/4 < 1. \tag{26}$$

Since the above eigenvalues are functions of the squeezing parameter r and the amplifier phase ϕ, the requirements for entanglement and physicality impose restrictions on the allowed ranges for these parameters for which broadcasting can be implemented.

Our remaining task for implementing broadcasting is to find the conditions under which the local output modes turn out to be separable. After tracing out the ancilla mode on site I and also all the modes on the site J from $\sigma''_{iabja'b'}$, the reduced CM representing the system of the two clones on site I (which is also equal to the corresponding reduced CM on the site J) is given by

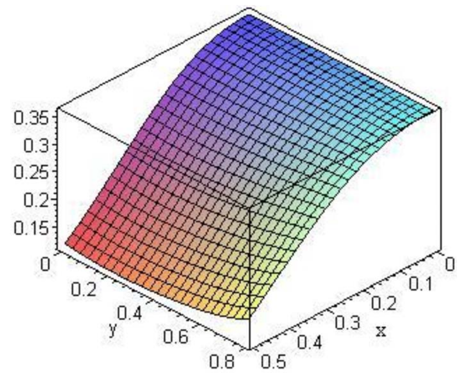

FIG. 3. (Color online) The fidelity of broadcasting F_B is plotted versus the squeezing r (x axis), and the amplifier phase ϕ (y axis).

$$\sigma_{ib}^{\text{local}}(r,\phi) = \sigma_{jb'}^{\text{local}}(r,\phi) = \begin{pmatrix} \frac{G+1}{2} & 0 & \frac{G-1}{2} & 0 \\ 0 & \frac{H+1}{2} & 0 & \frac{H-1}{2} \\ \frac{G-1}{2} & 0 & \frac{G+1}{2} & 0 \\ 0 & \frac{H-1}{2} & 0 & \frac{H+1}{2} \end{pmatrix}. \tag{27}$$

The condition for separability of the modes can be obtained from the generalization of the positivity of partial transposition criterion for continuous-variable systems [12]. For a two-mode state represented by the CM (27), the necessary and sufficient condition for the separability of these modes [10,13] reduces to

$$G \geq 1 \quad (H \geq 1) \tag{28}$$

for $G<H$ $(H<G)$. For local modes to be physical, one must satisfy both the conditions given by Eq. (24), which can be combined with the above separability condition to lead to the requirement of the relation

$$GH \geq 1 \tag{29}$$

being satisfied.

We have thus identified the four conditions: (i) physicality of nonlocal output modes ($\nu_-^2 \geq 1$ and $\lambda_i > 0$); (ii) entanglement of nonlocal modes ($\tilde{\nu}_-^2 < 1$); (iii) physicality of local output modes ($GH \geq 1$), and (iv) separability of local modes [$G \geq 1$ $(H \geq 1)$ for $G<H$ $(H<G)$], for implementing broadcasting. We display in Fig. 2, the contours representing these conditions which demarcate the allowed regions in the space of the squeezing parameter r and the amplifier phase ϕ. One sees that broadcasting is possible (dark shaded regions in the figure) for several values of the above two parameters.

We have shown that broadcasting is possible in our scheme for various combinations of input states and amplifier phases. Finally we compute the fidelity of the broadcasted states, i.e., the entangled nonlocal states (21). The fidelity of broadcasting, F_B is obtained through Eq. (7), in which we substitute the expressions for σ_{in} and σ_{out} from Eqs. (13) and (21), respectively. In Fig. 3 we display F_B as a function of the squeezing parameter r and the amplifier phase ϕ. One sees that this scheme of ours yields a phase- and state-dependent broadcasting fidelity. From the obtained expression for F_B it is possible to see that for $r \to \infty$, $F_B \to 0$, and for $r \to 0$, $F_B \to 0.36$.

IV. CONCLUSIONS

To summarize, we have presented an example for broadcasting of continuous-variable entanglement. We consider an initial two-mode squeezed state of the electromagnetic field which is shared by two distant parties. Both parties apply local cloning machines [6] on their respective modes involving an ancilla state, a linear amplifier, and a beam splitter, which yield two symmetric cloned modes at each end. The initial state is transmitted into a pair of bipartite but less entangled states that is finally shared by the two distant parties. Using the covariance matrix formalism [10] we have shown the entanglement between the pair of nonlocal output modes, as well as the separability of the local output modes. Our protocol for imperfect broadcasting of continuous-variable entanglement can be implemented for various combinations of the squeezing parameter and the phase of the amplifiers that yield physical output states.

We conclude by noting some possible related directions of study. Other cloning protocols [7] could be investigated for the purpose of alleviating the state dependence or phase sensitivity of the present scheme and increasing fidelity of broadcasting. Further, the possibility of generating three-mode quantum channels through local operations [14] useful for communications could be explored with continuous variables using our scheme. Finally, it may be noted that the first experimental demonstration of continuous-variable cloning has been reported recently [15], and with further development it could be feasible to experimentally broadcast entangled states of continuous variables too.

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