

Oscillations of the purity in the repeated-measurement-based generation of quantum states

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Repeated observations of a quantum system interacting with another one can drive the latter toward a particular quantum state, irrespective of its initial condition, because of an effective nonunitary evolution. If the target state is a pure one, the degree of purity of the system approaches unity, even when the initial condition of the system is a mixed state. In this paper we study the behavior of the purity from the initial value to the final one, that is, unity. Depending on the parameters, after a finite number of measurements, the purity exhibits oscillations that brings about a lower purity than that of the initial state, which is a point to be taken care of in concrete applications.

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I. INTRODUCTION

The initialization of a physical system is a fundamental task in connection with applications in nanotechnology and quantum information [1]. In particular, the purification of a quantum state is an important issue in quantum physics [2]. Recently, a state generation strategy based on the extraction of a state through repeated measurements has been proposed [3,4]. Since in most cases this procedure allows one to extract a pure state from a mixed one, it has been referred to as a “purification.” Moreover, on the basis of this idea, many applications have been proposed: the extraction of entanglement [4,5] and the initialization of multiple qubits would be useful for quantum computation [4,6]; extensions of the scheme enable us to establish entanglement between two spatially separated systems via repeated measurements on an entanglement mediator [7]; in single trapped ions, the extraction of angular-momentum quantum superposition (“Schrödinger cat”) states has been proposed [8] and the possibility of steering the extraction of pure states through quantum Zeno effect has been predicted [9]. The effect of the environment during the process has been deeply analyzed [10]. The scheme for the extraction of quantum states is based on the following idea. When a quantum system is put in interaction with a periodically measured one, the initial state of the former system $\rho(0)$ turns out to be mapped into the state $\mathcal{N}_k \hat{V}^k(\tau) \rho(0) [\hat{V}(\tau)^\dagger]^k$ with $\hat{V}(\tau) = \langle \phi_0 | e^{-i\hat{H}\tau} | \phi_0 \rangle$, where \hat{H} is the Hamiltonian ($\hbar=1$) of the whole system (i.e., the two interacting quantum systems one of which is repeatedly measured), τ is the time between two measurements, $|\phi_0\rangle$ is the state of the subsystem projected by the measurements, assuming that it is always the same, k is the number of observations, and \mathcal{N}_k is the normalization constant. The evolution described by the linear map $\hat{V}(\tau)$ is a conditional one [3]. After a large number of measurements, the system is driven toward a subspace which is given by the (right) eigenspaces of $\hat{V}(\tau)$ corresponding to the maximum (in

modulus) eigenvalues in its spectrum. If the extracted subspace is one dimensional, the final state (ideally reached after an infinite number of measurements, in practical situations approached after a finite and not too large number of steps) is pure. A first expectation one can have about the behavior of purity is that, starting from the initial value $\text{tr}[\rho(0)^2]$, it reaches the value 1 monotonically. In reality, as will become clear later, this is not always the case. In fact, in many situations the purity of the system oscillates and, passing through local minima and maxima, reaches the final asymptotic value. This is an important point to be understood since if one does not perform a sufficient number of measurements the state which one gets is not only different from the desired one, but also a state with lower purity than the initial one.

In this paper we analyze the behavior of the purity of the state of a two-level system interacting with a repeatedly measured one. In the next section we derive an expression for the purity after the k th measurement and find suitable conditions that characterize monotonic and nonmonotonic behavior of this quantity. In Sec. III we apply the general formalism to a simple case, which could be of practical interest. Finally, in Sec. IV we give a summary of the results.

II. THE PURITY AFTER A FINITE NUMBER OF MEASUREMENTS

Let us consider a two-level system (S) interacting with another system (X) which is repeatedly measured. Its state after k measurements of X [11] is expressible as the k th power of the operator $\hat{V}(\tau)$, or simply \hat{V} , applied to the initial state $\rho_k = \mathcal{N}_k \hat{V}^k(\tau) \rho(0) [\hat{V}(\tau)^\dagger]^k$. Assume that \hat{V} is diagonalizable, i.e., $\hat{V} = \lambda_1 |u_1\rangle\langle v_1| + \lambda_2 |u_2\rangle\langle v_2|$, with $|\lambda_1| > |\lambda_2|$ the two eigenvalues, $\langle v_1|$ and $\langle v_2|$ the two left eigenvectors, and $|u_1\rangle$ and $|u_2\rangle$ the two independent (in general, nonorthogonal) normalized ($\langle u_j | u_j \rangle = 1$) right eigenvectors. The left eigenvectors constitute a biorthonormal basis with the right eigenvectors, i.e., $\langle v_i | u_j \rangle = \delta_{ij}$, so that the identity operator related to the Hilbert space of system S can be expressed as $\mathbb{I}_S = \sum_{k=1,2} |u_k\rangle\langle v_k|$. Therefore, the initial state of system S , $\rho_0 = \rho(0)$, can be expanded as

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$$\rho_0 = a|u_1\rangle\langle u_1| + b|u_2\rangle\langle u_2| + c|u_1\rangle\langle u_2| + c^*|u_2\rangle\langle u_1|, \quad (1a)$$

with

$$a = \langle v_1 | \rho_0 | v_1 \rangle \geq 0, \quad (1b)$$

$$b = \langle v_2 | \rho_0 | v_2 \rangle \geq 0, \quad (1c)$$

$$c = \langle v_1 | \rho_0 | v_2 \rangle. \quad (1d)$$

Its determinant is expressible as

$$\det \rho_0 = (ab - |c|^2) |\langle u_2 | u_1^\perp \rangle|^2, \quad (2)$$

with $\langle u_1 | u_1^\perp \rangle = 0$ and $\langle u_1^\perp | u_1^\perp \rangle = 1$. After k steps, the initial state is mapped into

$$\rho_k = \mathcal{N}_k [a|\lambda_1|^{2k}|u_1\rangle\langle u_1| + b|\lambda_2|^{2k}|u_2\rangle\langle u_2| + c(\lambda_1\lambda_2^*)^k|u_1\rangle\langle u_2| + c^*(\lambda_1^*\lambda_2)^k|u_2\rangle\langle u_1|], \quad (3a)$$

with

$$\mathcal{N}_k^{-1} = a|\lambda_1|^{2k} + b|\lambda_2|^{2k} + [c(\lambda_1\lambda_2^*)^k\langle u_2 | u_1 \rangle + \text{c.c.}]. \quad (3b)$$

The purity of this state is

$$P[\rho_k] := \text{tr } \rho_k^2 = 1 - \frac{2g^{2k} \det \rho_0}{(a + bg^{2k} + 2\tilde{c}g^k \cos \alpha_k)^2}, \quad (4a)$$

with

$$g = \left| \frac{\lambda_2}{\lambda_1} \right| < 1, \quad (4b)$$

$$\tilde{c} = |c\langle u_2 | u_1 \rangle| \geq 0, \quad (4c)$$

$$\alpha_k = \arg [c\langle u_2 | u_1 \rangle (\lambda_1\lambda_2^*)^k]. \quad (4d)$$

If $g \neq 0$, Eq. (4a) gives also the purity of the initial state for $k=0$. One immediately sees that if the initial state is pure $P[\rho_0]=1 \Leftrightarrow \det \rho_0=0$, then the purity of the state ρ_k is always 1. Instead, if the initial state is not a pure one, oscillations of purity are possible, depending on the parameters.

Local minima. In some cases, the action of the operator \hat{V} can diminish the purity of the state. To find the relevant condition, let us impose $P[\rho_1] < P[\rho_0]$, which immediately leads to

$$\frac{g^2}{(a + bg^2 + 2g\tilde{c} \cos \alpha_1)^2} > \frac{1}{(a + b + 2\tilde{c} \cos \alpha_0)^2}.$$

Taking into account that the denominators are positive [12], one eventually gets the condition

$$a < bg - \frac{2g\tilde{c}}{1-g} (\cos \alpha_1 - \cos \alpha_0). \quad (5)$$

Therefore, if Eq. (5) is satisfied and the initial state is not pure, the purity of the state decreases after the first measurement. Since the purity eventually reaches the value 1, the existence of a minimum of the purity is guaranteed.

The relevant mechanism can be understood considering the very special case where \hat{V} is diagonalizable in the usual

sense, i.e., $|v_j\rangle = |u_j\rangle$, and the initial state is a mixture of its two eigenstates, i.e., $\rho(0) = \rho_{11}(0)|u_1\rangle\langle u_1| + \rho_{22}(0)|u_2\rangle\langle u_2|$, so that $a = \rho_{11}(0)$ and $b = \rho_{22}(0)$. In such a case, condition (5) just becomes $\rho_{11}(0) < g\rho_{22}(0)$, which means that the population of the state to be extracted should be smaller than that of the other one times the parameter which determines how fast the process of extraction is (indeed, note that the smaller g is, the smaller the number of measurements required to extract the target state is). In fact, since the state that will be extracted is $|u_1\rangle$, the extraction process lowers down the population of $|u_2\rangle$ and increases that of $|u_1\rangle$. Depending on how “slow” the process is (i.e., how many measurements are to be performed to extract the target state), at some step the two populations will reach closer values, which corresponds to a state closer to the maximally mixed one than the initial state, and therefore corresponds to a smaller value of the purity.

Local maxima. The two conditions

$$P[\rho_1] > P[\rho_0], \quad P[\rho_1] > P[\rho_2] \quad (6)$$

can be achieved simultaneously, determining the presence of a local maximum at the first measurement. These two conditions are compatible only if

$$b < \frac{2\tilde{c}}{(1-g)^2(1+g)} [\cos \alpha_1 - \cos \alpha_0 + g(\cos \alpha_1 - \cos \alpha_2)]. \quad (7)$$

The presence of local maxima is less intuitive to understand, even though it can be forecasted mathematically. It is worth to note that in the case wherein the operator \hat{V} can be diagonalized in the usual sense, local maxima are impossible, since in such a case $\langle u_2 | u_1 \rangle = 0$ implies $\tilde{c} = 0$ and eventually $b < 0$, which is incompatible with Eq. (1c).

Condition for monotonic behavior. Here we give general conditions to ensure that the purity increases toward the target value, 1. The condition expressing the monotonic behavior of the purity from the k_0 th measurement on is

$$P[\rho_k] \geq P[\rho_{k-1}], \quad \forall k \geq k_0, \quad (8)$$

which, after elementary manipulations, gives the necessary and sufficient condition

$$bg^{2k-1} - \frac{2g\tilde{c}}{1-g} (\cos \alpha_k - \cos \alpha_{k-1}) - a \leq 0, \quad \forall k \geq k_0. \quad (9)$$

Since the absolute value of the second term on the left-hand side is always not larger than $4\tilde{c}g^k/(1-g)$ whatever $k \geq 1$ is, a sufficient condition is given by

$$g^{2k} + \frac{4g\tilde{c}}{(1-g)b} g^k - \frac{ag}{b} \leq 0, \quad \forall k \geq k_0, \quad (10)$$

which can be rewritten as

$$G_- \leq g^k \leq G_+, \quad k \geq k_0, \quad (11)$$

$$G_{\pm} = -\frac{2g\tilde{c}}{(1-g)b} \pm \sqrt{\frac{4g^2\tilde{c}^2}{(1-g)^2b^2} + \frac{ag}{b}}.$$

Taking into account that $G_- \leq 0$ and $0 \leq g < 1$, that implies $g^{k+1} \leq g^k$, Eq. (11) turns out to be equivalent to

$$g^{k_0} \leq \sqrt{\frac{4g^2\tilde{c}^2}{(1-g)^2b^2} + \frac{ag}{b}} - \frac{2g\tilde{c}}{(1-g)b}, \quad (12)$$

which makes it easy to find the minimum number of measurements sufficient to get monotonicity:

$$k \geq \ln \left[\sqrt{\frac{4g^2\tilde{c}^2}{(1-g)^2b^2} + \frac{ag}{b}} - \frac{2g\tilde{c}}{(1-g)b} \right] / \ln g. \quad (13)$$

In other words, after a number of measurements not smaller than the right-hand side of Eq. (13) no oscillation of purity is possible and the purity of the state of the system monotonically reaches the asymptotic value 1. When it happens that $\tilde{c} \ll 0$ or $g \ll 1$, condition (13) reduces to

$$k \geq \frac{1}{2} \left(1 + \ln \frac{a}{b} \right) / \ln g, \quad (14)$$

which is a simplified expression allowing to obtain qualitative estimations of the number of measurements sufficient to have only increasing purity. The same simplified condition comes out directly and exactly from Eq. (9) in the special case $\lambda_1 \lambda_2^* \in \mathbb{R}^+$, which implies $\alpha_k = \alpha_{k-1}$, or if \hat{V} can be diagonalized in the usual sense, which implies $\tilde{c} = 0$ whatever the initial state is. In such cases, condition (14) is equivalent to Eq. (9), and therefore becomes a necessary and sufficient condition.

III. A SIMPLE PHYSICAL SYSTEM

As a specific example, let us consider a system extensively studied consisting of two qubits subjected to an interaction preserving the number of excitations. A possible realization is given by two two-level atoms subjected to a dipole-dipole interaction. Assuming that the matrix elements of the dipole operators are real, and neglecting the counter-rotating terms, one reaches the following Hamiltonian (for details, see Refs. [4,13]):

$$\hat{H}_{\text{tot}} = \sum_{i=S,X} \frac{\Omega}{2} (1 + \hat{\sigma}_z^{(i)}) + \epsilon (\hat{\sigma}_+^{(S)} \hat{\sigma}_-^{(X)} + \hat{\sigma}_-^{(S)} \hat{\sigma}_+^{(X)}), \quad (15)$$

where $\hat{\sigma}_z^{(i)} = |\uparrow\rangle_i \langle \uparrow| - |\downarrow\rangle_i \langle \downarrow|$, $\hat{\sigma}_\pm^{(i)} = |\uparrow\rangle_i \langle \downarrow| = (\hat{\sigma}_\mp^{(i)})^\dagger$, Ω is the Bohr frequency of the two-level system, and ϵ is the coupling constant.

The eigenstates of the Hamiltonian are the triplet and singlet two-spin states

$$|2\rangle_{\text{tot}} = |\uparrow\rangle_S |\uparrow\rangle_X, \quad (16a)$$

$$|1\rangle_{\text{tot}} = \frac{1}{\sqrt{2}} [|\uparrow\rangle_S |\downarrow\rangle_X + |\downarrow\rangle_S |\uparrow\rangle_X], \quad (16b)$$

$$|0\rangle_{\text{tot}} = |\downarrow\rangle_S |\downarrow\rangle_X, \quad (16c)$$

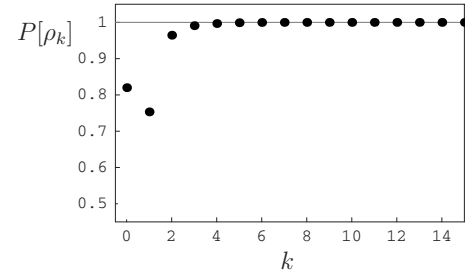


FIG. 1. The purity as a function of the number of measurements, starting with $\rho(0) = 0.1|\downarrow\rangle_S \langle \downarrow| + 0.9|\uparrow\rangle_S \langle \uparrow|$, in the parameter region characterized by $\epsilon\tau = 7.82$, $\theta = 2.25$, $\epsilon/\Omega = 0.1$.

$$|s\rangle_{\text{tot}} = \frac{1}{\sqrt{2}} [|\uparrow\rangle_S |\downarrow\rangle_X - |\downarrow\rangle_S |\uparrow\rangle_X]. \quad (16d)$$

The corresponding eigenenergies are 2Ω , $\Omega + \epsilon$, 0 , and $-\epsilon$, respectively. In the case $\Omega > \epsilon$, the state $|0\rangle$ is the ground state. The state $|s\rangle$ is stable, not being coupled to any of the other three states.

Repeatedly measuring the system X , and finding it in the state $|\theta\rangle_X = \cos\frac{\theta}{2}|\uparrow\rangle_X + \sin\frac{\theta}{2}|\downarrow\rangle_X$ (the most general state should include a phase factor, here assumed equal to zero for simplicity), the system S , is subjected to a nonunitary evolution governed by the operator [4]

$$\begin{aligned} \hat{V} = & \left(e^{-2i\Omega t} \cos^2\frac{\theta}{2} + e^{-i\Omega t} \cos\epsilon t \sin^2\frac{\theta}{2} \right) |\uparrow\rangle_S \langle \uparrow| \\ & + \left(\sin^2\frac{\theta}{2} + e^{-i\Omega t} \cos\epsilon t \cos^2\frac{\theta}{2} \right) |\downarrow\rangle_S \langle \downarrow| \\ & - i e^{-i\Omega t} \sin\epsilon t \sin\frac{\theta}{2} \cos\frac{\theta}{2} (|\uparrow\rangle_S \langle \downarrow| + |\downarrow\rangle_S \langle \uparrow|), \end{aligned} \quad (17)$$

where we have omitted the subscript S of the states.

This very simple physical situation exhibits nonmonotonic behavior associated with the presence of minima and maxima of the purity. Consider the following situations described in the figures. In Fig. 1 we see that starting from $\rho(0) = 0.1|\downarrow\rangle_S \langle \downarrow| + 0.9|\uparrow\rangle_S \langle \uparrow|$, repeatedly measuring the state of X , characterized by $\theta = 2.25$ with time interval $\tau = 7.82/\epsilon$, a minimum of the purity occurs at the first measurement. Instead, Fig. 2 shows that starting from the maximally mixed state and measuring the state of the system X , characterized

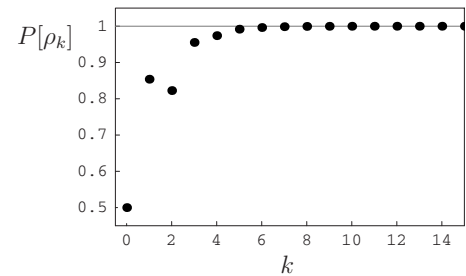


FIG. 2. The purity as a function of the number of measurements, starting with $\rho(0) = 0.5|\downarrow\rangle_S \langle \downarrow| + 0.5|\uparrow\rangle_S \langle \uparrow|$, in the parameter region characterized by $\epsilon\tau = 2.50$, $\theta = 1.0$, $\epsilon/\Omega = 0.1$.

by $\theta=1.0$ with time interval $\tau=2.50/\epsilon$, the purity exhibits a local maximum at the first measurement and a local minimum at the second measurement. From the point of view of applications, it could be important to give the conditions under which nonmonotonic behavior of purity is avoided.

Measuring the state $|\uparrow\rangle_X$. As a very special case, assume that the system X , is repeatedly measured (after each τ such that $\epsilon\tau \neq q\pi$, with $q \in \mathbb{Z}$, to avoid the case $g=1$) and found in the up-state $|\uparrow\rangle_X$, so that

$$\hat{V} = e^{-2i\Omega t} |\uparrow\rangle\langle\uparrow| + e^{-i\Omega t} \cos \epsilon t |\downarrow\rangle\langle\downarrow|, \quad (18)$$

in correspondence to which we get $g = |\cos \epsilon t|$, while the eigenvectors are $|u_1\rangle = |v_1\rangle = |\uparrow\rangle$ and $|u_2\rangle = |v_2\rangle = |\downarrow\rangle$. In such a situation, the coefficients a , b , and c are the usual matrix elements of the density operator $\rho(0)$ with respect to an orthonormal basis. Starting from the general state $\rho(0) = p_{\uparrow\uparrow} |\uparrow\rangle\langle\uparrow| + (1-p_{\uparrow\uparrow}) |\downarrow\rangle\langle\downarrow| + c |\uparrow\rangle\langle\downarrow| + c^* |\downarrow\rangle\langle\uparrow|$, we have $a = p_{\uparrow\uparrow}$, $b = 1-p_{\uparrow\uparrow}$, and $\tilde{c} = 0$, so that the (necessary and sufficient) condition for monotonicity is

$$k \geq \frac{1}{2} \left(1 + \ln \frac{p_{\uparrow\uparrow}}{1-p_{\uparrow\uparrow}} \bigg/ \ln |\cos \epsilon \tau| \right). \quad (19)$$

If $p_{\uparrow\uparrow} > 0.5$, the right-hand side is smaller than unity and only monotonic increase of the purity is possible. On the contrary, if $p_{\uparrow\uparrow}$ is small enough to have $\ln[p_{\uparrow\uparrow}/(1-p_{\uparrow\uparrow})] < \ln |\cos \epsilon \tau|$, nonmonotonic behavior is possible.

Figure 3 shows the threshold for the number of measurements necessary to guarantee monotonicity as a function of the population of the target state in the initial condition ($p_{\uparrow\uparrow}$) and of the time τ . Since the right-hand side of Eq. (19) can become negative (in which case we get monotonic behavior, as well as for any value smaller than unity), we consider its nonnegative counterpart, i.e., $\eta = \max(0, \frac{1}{2} \{1 + \ln[p_{\uparrow\uparrow}/(1-p_{\uparrow\uparrow})]/\ln |\cos \epsilon \tau|\})$, without loss of information. It is well visible that for most of the values of the parameters (on the right-hand side of the figure, beyond the curve $\eta=1$) the threshold is lower than unity, meaning that only monotonic increase of the purity occurs. Instead, for low values of $p_{\uparrow\uparrow}$ and τ the threshold becomes higher than unity, meaning that nonmonotonic behavior occurs. If the state $|\downarrow\rangle_X$ is measured

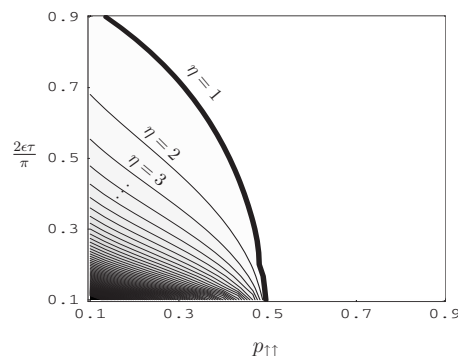


FIG. 3. Contour plot of the quantity $\eta = \max(0, \frac{1}{2} \{1 + \ln[p_{\uparrow\uparrow}/(1-p_{\uparrow\uparrow})]/\ln |\cos \epsilon \tau|\})$ vs the population of $|\uparrow\rangle$, $p_{\uparrow\uparrow}$, and the dimensionless time between two measurements $2\epsilon\tau/\pi$ both in the range $[0.1, 0.9]$. The bold line corresponds to $\eta=1$, which discriminates between monotonic behavior of the purity ($\eta < 1$ on the right-hand side of the line) and oscillations ($\eta > 1$ in the left-hand side). In the dark region (in the bottom of the left-hand side) the value of η is very high, meaning that one needs a large number of measurements to guarantee monotonicity.

we get essentially the same results, even the same expression of the threshold, provided the replacement $p_{\uparrow\uparrow} \rightarrow 1-p_{\uparrow\uparrow}$ has been done.

IV. SUMMARY

Summing up, we have analyzed the problem of nonmonotonic behavior of the purity of the state of a physical system when it is subjected to a process of extraction of pure states by repeatedly measuring a quantum system interacting with the former one. The knowledge of this phenomenon is of fundamental importance, since it brings to the light interesting features of the transient from the initial state to the final one. Moreover, in the applications it is crucial to determine the minimum number of steps necessary to get the monotonic increase of purity. Otherwise, the purification process could be counterproductive, resulting in a diminishing of the purity. The general results we have found for a two-level system, reported in Sec. II and then applied to a specific physical system in Sec. III, allow us to overcome this problem and could be of interest in practical applications of this scheme for the generation of pure states.

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