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Determination of the equation of state of a polarized Fermi gas at unitarity

Yong-il Shin*

Department of Physics, MIT-Harvard Center for Ultracold Atoms, and Research Laboratory of Electronics, Massachusetts Institute of Technology, Cambridge, Massachusetts 02139, USA

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We report on the measurement of the equation of state of a two-component Fermi gas of ^{6}Li atoms with resonant interactions. By analyzing the *in situ* density distributions of a population-imbalanced Fermi mixture reported in a recent experiment [Y. Shin *et al.*, Nature **451**, 689 (2008)], we determine the energy density of a resonantly interacting Fermi gas as a function of the densities of the two components. We present a method to determine the equation of state directly from the shape of the trapped cloud, where the fully polarized, noninteracting ideal Fermi gas in the outer region provides the absolute calibration of particle density. From the density profiles obtained at the lowest temperature, we estimate the zero-temperature equation of state.

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Interacting fermions are a paradigm of modern physics, encompassing superconductivity and superfluidity. One interesting situation arises when the constituents interact resonantly, i.e., the scattering length for the free fermions diverges. At this so-called unitarity limit, the behavior of the system becomes universal, being independent of the nature of the interactions. Ultracold atomic Fermi gases near Feshbach collisional resonance present a model system for studying strongly interacting fermions [1]. Recently, the phase diagram of a two-component Fermi gas with resonant interactions has been experimentally established [2], showing that at zero temperature the system undergoes a first-order quantum phase transition from a fully paired superfluid to a partially polarized normal gas when the imbalance between the two spin components exceeds a critical value, called the Chandrasekhar-Clogston limit of superfluidity [3,4].

The nature of the partially polarized normal phase, however, is still a subject of investigation. The spectral shift observed in the minority rf excitation spectrum has been interpreted as the existence of "pairing" in the normal phase [5], but several theoretical studies for a highly polarized system, e.g. a single minority atom in a majority Fermi sea, suggest that the system is well-described as a normal Fermi liquid, where the minority atoms are associated with weakly interacting quasiparticles [6–8]. This picture seems to be supported by the experimental observation that the shape of the minority cloud in the normal phase is similar to a free Fermi gas [2,9]. It has been speculated that exotic pairing states might exist in the partially polarized phase [10].

In this paper, we determine the equation of state of a polarized Fermi gas at unitarity from the *in situ* density profiles of a population-imbalanced Fermi mixture confined in a harmonic trap. Since the variation of the external trapping potential across the sample scans the chemical potential, in principle, the density information of a single sample contains the whole equation of state [11,12]. We present a method to determine the equation of state directly from the shape of the trapped cloud. Because of its exactly known thermal properties, a fully polarized, noninteracting ideal Fermi gas in the outer region provides the absolute density calibration. The equation of state of a polarized Fermi gas can be parametrized using a normal Fermi liquid description, which includes the binding energy of a single minority atom resonantly interacting with a majority Fermi sea, the effective mass of the quasiparticles, and its correction. This work is a quantitative study of the thermodynamic properties of the polarized normal state with strong interactions, finding reasonable agreement with recent calculations [6–8,11,12].

For infinite scattering length, the unitarity limit implies that all interaction energies scale with the Fermi energies of the components $\varepsilon_{Fi}=\hbar^2/2m(6\pi^2 n_i)^{2/3}$ [13], where $\hbar$ is the Planck constant divided by 2π, m is the particle mass, n_i is the density of component i, and $i=1,2$. As a result, a simple dimensional scaling argument implies that the energy density $\mathcal{E}(n_1,n_2)$ of a two-component Fermi gas can be parametrized as

$$\mathcal{E}(n_1,n_2)=\frac{3}{5}\alpha[n_1 g(x)]^{5/3}, \tag{1}$$

introducing a dimensionless universal function $g(x)$ [12], where $\alpha=(6\pi^2)^{2/3}\hbar^2/2m$ and $x=n_2/n_1$ is the density ratio of the two components. Without loss of generality, $0\le x\le 1$ due to the symmetry of the two components. From the chemical potential relation $\mu_i=\partial\mathcal{E}/\partial n_i$, the universal function can be expressed as

$$g(x)^{5/3}=\frac{\mu_1+x\mu_2}{\alpha n_1^{2/3}}=\frac{\mu_1}{\varepsilon_{F1}}(1+xy), \tag{2}$$

where $y=\mu_2/\mu_1$ is the chemical potential ratio of the two components.

The main result of this paper is the measurement of the universal function $g(x)$ for a resonantly interacting Fermi gas. When a Fermi mixture is confined in a harmonic trap, $V(r)\propto r^2$, the local chemical potential is given as $\mu_i(r)=\mu_{i0}-V(r)$, where μ_{i0} is the global chemical potential with respect to the bottom of the trapping potential. The global chemical potential of the majority component μ_{10} is determined from the radius of the majority cloud R_1, i.e., $\mu_{10}=V(R_1)$. Then, the chemical potential ratio $y(r)=\mu_2(r)/\mu_1(r)$ is given as

*yishin@mit.edu

$$y(r) = \frac{y_0 - r^2/R_1^2}{1 - r^2/R_1^2}, \tag{3}$$

where $y_0 = \mu_{20}/\mu_{10}$. With population imbalance, i.e. $y_0 < 1$, $x(r)$ and $y(r)$ vary over the sample. From Eq. (2), the spatial correlation of the local particle densities and the local chemical potentials in a trapped sample determines $g(x)$ under the local density approximation.

We estimate the zero-temperature equation of state by analyzing the density profiles obtained at the lowest temperature ($T/T_F \approx 0.03$ where T_F is the Fermi temperature of the majority component) in Ref. [2]. A spin mixture of the two lowest hyperfine states of ^{6}Li atoms was prepared in a three-dimensional (3D) harmonic trap on a broad Feshbach resonance, located at a magnetic field of 834 G [14], resonantly enhancing the interactions between the two spin states. The detailed description of the experimental procedure for the sample preparation and the signal processing has been provided in Ref. [2].

Figure 1 shows the spatial structure of a resonantly interacting Fermi mixture in a harmonic trap. According to the zero-temperature phase diagram [2], three distinctive spatial regions can be identified in the inhomogeneous sample with population imbalance ($y_0 < 1$). When the chemical potential ratio at center y_0 is larger than a critical value y_c for the superfluid-to-normal phase transition, the sample has (I) a fully paired superfluid core ($x=1$, $y > y_c$) surrounded by (II) a partially polarized normal gas ($0 < x \le x_c, y < y_c$), showing a discontinuity in the density ratio x at the phase boundary. In the outer region where the minority component is completely depleted, (III) a fully polarized noninteracting Fermi gas forms ($x=0$, $y < y_m < y_c$). The radii, R_c and R_2 are the I-II and II-III transition points, defining $y(R_c) = y_c$ and $y(R_2) = y_m$, respectively.

The noninteracting ideal Fermi gas in the outer ($r > R_2$) region provides a reliable method to measure the local chemical potential $\mu_1(r)$ in the strongly interacting, inner ($r < R_2$) region. Since $\varepsilon_{F1} = \mu_1$ in the outer region, the extension of the noninteracting ideal Fermi distribution $n_0(r)$, fit to the outer region, into the inner region gives the local chemical potential as $\mu_1(r) = \alpha n_0^{2/3}$, consequently $\mu_1/\varepsilon_{F1} = (n_0/n_1)^{2/3}$. This method allows us to measure the equation of state directly from the shape of the cloud without any absolute calibration for particle density. Furthermore, when a sample has a superfluid core, i.e. the whole range of the density ratio $0 \le x \le 1$, a single shot image of the sample can provide all information for the determination of the equation of state.

In the experiments, the total population imbalance was controlled to be $\delta = (N_1 - N_2)/(N_1 + N_2) = 44(4)\%$ less than the critical imbalance $\delta_c (\approx 75\%)$ [15,16] to have a superfluid core, i.e. $y_0 > y_c$, where N_i is the total atom number of component i. The phase boundary R_c, located by the kink and peak position in the column density difference profile, was measured to be $R_c/R_1 = 0.430(3)$ and the critical density ratio was measured to be $x_c = 0.53(5)$. The reference density $n_0(r)$ and the radius $R_1(R_2)$ were determined from the fit of the outer region, $r > R_2 (r > R_c)$ of the majority(minority) column density profile to a zero-temperature Thomas-Fermi (TF) distribution.

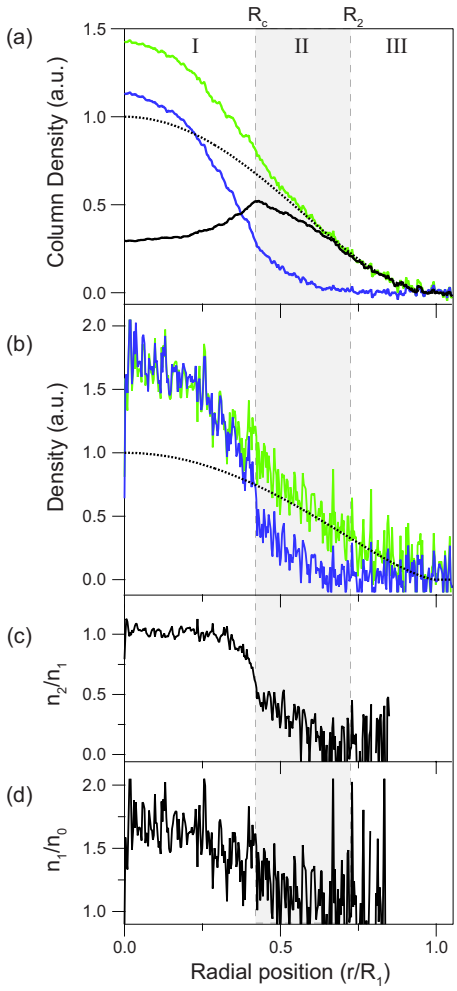

FIG. 1. (Color online) Spatial profiles of a population-imbalanced Fermi mixture confined in a harmonic trap at unitarity. (a) The averaged column density profiles and (b) the reconstructed 3D profiles at the lowest temperature from Ref. [2] [green (gray): majority; blue (dark gray): minority; black: difference]. R_1 and R_2 are the radii of the majority (spin 1) and the minority (spin 2) cloud, respectively. The core radius R_c was determined as the kink and peak position in the column density difference. The black dotted line in (a) is a zero-temperature Thomas-Fermi (TF) distribution fit to the outer region ($r > R_2$) of the majority column density profile and the black dotted line in (b) is the corresponding 3D distribution, n_0. The sample has three regions: (I) The superfluid core region ($0 \le r < R_c$), (II) the partially polarized, intermediate region ($R_c < r < R_2$), and (III) a fully polarized noninteracting outer region ($R_2 < r < R_1$). (c) The density ratio of the two components, $x(r) = n_2/n_1$. (d) The majority density normalized by the reference density, n_0.

The well-known zero-temperature thermodynamics of a balanced superfluid and a noninteracting Fermi gas provide physical constraints on the form of the universal function. Since the chemical potential of a fully paired ($x=1$) superfluid is proportional to the Fermi energy, i.e. $\mu_s = (\mu_1 + \mu_2)/2 = \xi\varepsilon_{F1}$, we have $\mu_1/\varepsilon_{F1} = 2\xi/(1+y)$ and $g(1) = (2\xi)^{3/5}$. On the other hand, a fully polarized ($x=0$) noninteracting Fermi gas has $\mu_1 = \varepsilon_{F1}$ so that $g(0) = 1$. The universal parameter ξ can be independently determined from the

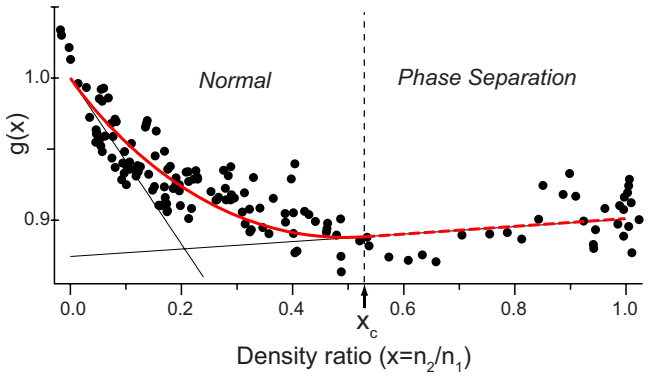

FIG. 2. (Color online) Thermodynamic potential at unitarity. The universal function $g(x)$ for energy density was constructed as a function of the density ratio x of the two components, from five independent measurements of the density profiles (Fig. 1). The critical chemical potential ratio and the critical density ratio were measured to be $y_c=0.03(2)$ and $x_c=0.53(5)$, respectively (see text for determination method). The red (dark gray) solid line is obtained by fitting the model equation Eq. (4) with $\{\xi,y_c,y_m,x_c\}=\{0.42,0.03,-0.58,0.53\}$ to the normal region ($x<x_c$) of $g(x)$. A red (dark gray) dashed line in the phase separation region ($x_c<x<1$) connects the two points $[x=1,g(1)=(2\xi)^{3/5}]$ and $[x=x_c,g(x_c)]$.

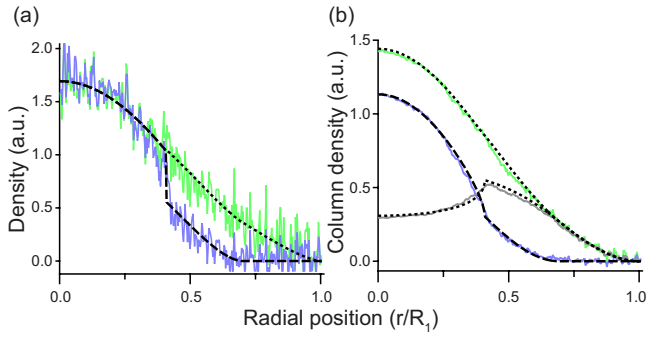

FIG. 3. (Color online) Comparison between the experimental data and the model. (a) The 3D density profiles and (b) the column density profiles for $\delta=44\%$ (dotted-dashed: majority; dashed: minority; dotted: difference) generated from the model equation of state [the red (dark gray) line in Fig. 2]. Experimental data in Fig. 1 are plotted together for comparison (same color usage).

majority profile by comparing the curvature of the Fermi energy distribution $\varepsilon_{F1}(r)\propto n_1(r)^{2/3}$ in the core region and in the outer region, as $\xi=(d^2\varepsilon_{F1}/dr^2)_{r>R_2}/(d^2\varepsilon_{F1}/dr^2)_{r<R_c}$. This determination is, however, technically limited due to the low signal-to-noise ratio. In the following analysis we use the theoretically predicted value $\xi_{th}=0.42(1)$[17,18], confirmed in previous measurements [19–26].

Figure 2 displays the universal function $g(x)$ constructed from the density profiles at the lowest temperature. The critical value y_c was determined to be $y_c=0.03(2)$ such that the average value of $g(x)$ for $x>0.9$ gives $(2\xi_{th})^{3/5}=0.90$. This critical value y_c has been discussed in Ref. [2] to demonstrate the stability of a fully paired superfluid state at zero temperature. At zero temperature, $g(x)$ is not defined for a homogeneous system with $x_c<x<1$. The sparse population of the data points in the region of $x_c<x<1$ indicates the phase separation in the sample, associated with the first-order phase transition.

The other critical value y_m represents the binding energy E_b of a single minority resonantly interacting with a majority Fermi sea as $E_b=\lim_{x\to 0^+}\mu_2=y_m\varepsilon_{F1}$. $y=g'(x)/[g(x)-xg'(x)]$ and $g(0)=1$ give $y_m=g'(0)$. By fitting $1+y_mx$ to $g(x)$ for $x<0.1$, we estimated $y_m=-0.58(5)$, which is in good agreement with the recent theoretical results of -0.6 [11], $-0.58(1)$ [6], -0.6066 [7], $-0.54(4)$ [12], and -0.618 [8]. From the definitions, $y_c=y(R_c)$ and $y_m=y(R_2)$, the estimated critical values $\{y_c,y_m\}=\{0.03(2),-0.58(5)\}$ suggest $R_2=0.707(20)R_1$ for $R_c=0.43R_1$. However, the minority radius was measured to be $R_2/R_1=0.73(1)$, suggesting $y_m=-0.69(8)$. We attribute this discrepancy to the fact that the shape of the minority density profile in the intermediate region cannot be completely captured with a zero-temperature TF distribution [27].

Following a normal Fermi liquid description [6–8], we consider a model for the equation of state of a partially polarized Fermi gas in the following form:

$$g(x)^{5/3}=1+\frac{5}{3}y_mx+\frac{1+cx}{m^*}x^\gamma \quad \text{for } x<x_c, \tag{4}$$

satisfying the boundary conditions $g(0)=1$ and $g'(0)=y_m$ at $x=0$. The second term corresponds to the momentum-independent binding energy for the minority atoms and the third term describes the deviation from the free particle behavior, regarding m^* as the effective mass and c as its correction [for the noninteracting case, $g_0(x)^{5/3}=1+x^{5/3}$]. The equilibrium condition for the coexistence of two spatially separate phases requires that the two phases have the same chemical potential and pressure at the critical point [28], imposing the boundary conditions at $x=x_c$ such as $g(x_c)=[(1+x_cy_c)/(1+y_c)]g(1)$ and $g'(x_c)=[y_c/(1+y_c)]g(1)$. Then, for given values $\{\xi,y_c,y_m,x_c\}$, this model relies on only one free parameter. With $\{\xi,y_c,y_m,x_c\}=\{0.42,0.03,-0.58,0.53\}$, the fit of Eq. (4) to the intermediate region ($x<x_c$) gives $\gamma=1.60(13)$, having $m^*=1.06$ and $c=-0.019$. Quantum Monte Carlo calculations for small x predicts $m^*=1.04(3)$ with $\gamma=5/3$ [6], which is very close to the observed behavior. Figure 3 displays the density profiles generated from the model equation of state, together with the experimental data.

Our observation of $\gamma\approx 5/3$ and the small change in the effective mass suggests that a polarized Fermi gas with resonant interactions can be described as a normal Fermi liquid with weakly interacting quasiparticles. However, it is an open question whether the Fermi liquid description is still valid for high minority concentrations, where the Pauli blocking effect of the minority Fermi sea might play an important role. The possibility of the exotic ground state of a partially polarized system has been suggested by the recent observation of the temperature-dependent spectral shift in the minority rf excitation spectrum [5]. We note that it is not clear how to distinguish possible exotic states [10] via the equation of state. More experimental studies for microscopic properties of the system, e.g. majority rf spectroscopy, are necessary to clarify the issue.

In conclusion, we measure the equation of state of a two-

component Fermi gas with resonant interactions by analyzing the *in situ* density distributions of the trapped sample. In a similar way, the density profiles at finite temperature may reveal the excitation spectrum of the system [29].

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