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Collisions of three-wave solitons in media with competing quadratic and cubic nonlinearities

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We report results of analysis of collisions between in-phase spatial solitons in a three-wave system, which includes competing $\chi^{(2)}:\chi^{(3)}$ nonlinearities and birefringence. The character of the collisions is quantified by a critical value of the initial tilt of the colliding solitons, Q, which separates quasielastic passage and merger (solitons with phase shift π bounce from each other). If Q is very close to Q_{cr}, the solitons may fuse into a double-peak state. Q_{cr} is smaller for relatively large negative mismatch of the $\chi^{(2)}$ interactions. With the increase of the $\chi^{(3)}/\chi^{(2)}$ ratio, the system quickly transits from the behavior governed by the $\chi^{(2)}$ nonlinearity to a regime dominated by the $\chi^{(3)}$ terms.

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I. INTRODUCTION

Solitons in optical media with the $\chi^{(2)}$ (quadratic) nonlinearity, which combine fundamental-frequency (FF) and second-harmonic (SH) waves, have been a subject of a great deal of work [1]. The formation of solitons is facilitated by the use of the type-II $\chi^{(2)}$ interaction, which involves two FF components with orthogonal polarizations, and a single polarization of the SH, making it possible to use the material birefringence to improve the phase-matching. This mechanism, which gives rise to three-wave (3W) solitary waves, was employed in the first experiment which had produced spatial $\chi^{(2)}$ solitons [2]. The 3W solitons were investigated in $\chi^{(2)}$ models in detail [3].

Generally, $\chi^{(2)}$ acts in combination with the $\chi^{(3)}$ (cubic, alias Kerr) nonlinearity. In the two-wave (type-I) models, the competition between the $\chi^{(2)}$ and $\chi^{(3)}$ nonlinearities has drawn much interest [4]. While in $\chi^{(2)}$ media the Kerr nonlinearity is usually much weaker than its quadratic counterpart, a possibility to induce a strong effective $\chi^{(3)}$ nonlinearity is offered by the quasiphase-matching (QPM) technique [5]. It was proposed as a means to create solitons supported by competing $\chi^{(2)}:\chi^{(3)}$ nonlinearities in ordinary media [6] and in photonic crystals [7]. Another potential source of the strong effective $\chi^{(3)}$ nonlinearity in a $\chi^{(2)}$ medium is provided by optical rectification [8].

As concerns Kerr media, a well-studied topic is the interplay of the $\chi^{(3)}$ nonlinearity and birefringence. If the four-wave mixing (FWM) is taken into regard, the $\chi^{(3)}$ solitons in the so-called slow and fast polarizations are, severally, stable and unstable [9].

A general 3W system that couples two FF components and a single SH one by $\chi^{(2)}$ and $\chi^{(3)}$ terms, taking into regard the birefringence between the FF waves, was introduced in Ref. [10]. Several types of solitons were found in that system, including single-component ones, which are supported solely by the $\chi^{(3)}$ nonlinearity, and generic 3W solitons. Following the variation of the mismatch that controls the $\chi^{(2)}$ interactions, the branch of the 3W solitons bifurcates from the single-wave SH one, and at another bifurcation point it merges into the slow-FF single-component soliton family.

Interactions between $\chi^{(2)}$ solitons have potential applications to the design of all-optical switching schemes [1]. Experimental observations of collisions between spatial solitons [11] and theoretical studies [12] have demonstrated that, depending on the angle between the colliding solitary beams, two outcomes are possible: the quasielastic passage, if the angle exceeds a certain critical value, and fusion into a single beam in the opposite case.

The objective of the present work is to investigate collisions between 3W solitons in the model with the combined $\chi^{(2)}:\chi^{(3)}$ nonlinearity introduced in Ref. [10]. The model makes it possible to explore effects of various physical factors on the inelasticity of the collisions. These factors include the $\chi^{(2)}$ mismatch, birefringence between the two components of the FF wave, and, what seems especially interesting, the relative strength of the $\chi^{(2)}$ and $\chi^{(3)}$ nonlinearities. Simulations of collisions between orthogonally polarized solitons in nonlinear optical fibers with the Kerr nonlinearity also revealed a possibility of inelastic interactions (merger or destruction of the colliding solitons) [13]. However, since the $\chi^{(3)}$ system is relatively close to its integrable counterpart of the Manakov type [14], while the $\chi^{(2)}$ system is inherently nonintegrable, one may expect that the inelasticity is significantly enhanced with the increase of the ratio of $\chi^{(2)}$ and $\chi^{(3)}$ coefficients.

II. MODEL

The general 3W system combining the $\chi^{(2)}$ and $\chi^{(3)}$ nonlinearities which was derived in Ref. [10] includes complex amplitudes $u_\pm$ of the two components of the FF field, and their single SH counterpart, w. Scaled equations for the evolution of these fields along the propagation coordinate, z, with transverse coordinate x, include the usual terms accounting for $\chi^{(2)}$ interaction of type II and the $\chi^{(3)}$ terms that take into regard the self-phase modulation (SPM), cross-phase modulation (XPM), and four-wave mixing (FWM):

$$i(u_\pm)_z + (1/2)(u_\pm)_{xx} + u_\mp^* w + \gamma_1[(1/4)|u_\pm|^2 + (1/6)|u_\mp|^2 + 2|w|^2]u_\pm + (\gamma_1/12)u_\mp^2 u_\pm^* + \gamma_2|w|^2 u_\mp \pm b u_\pm = 0, \quad (1)$$

$$2iw_z + (1/2)w_{xx} + u_+u_- - qw + 2\gamma_1(2|w|^2 + |u_+|^2 + |u_-|^2)w + \gamma_2(u_+u_-^* + u_+^*u_-)w = 0. \quad (2)$$

Here, b is the birefringence coefficient, q is the phase-

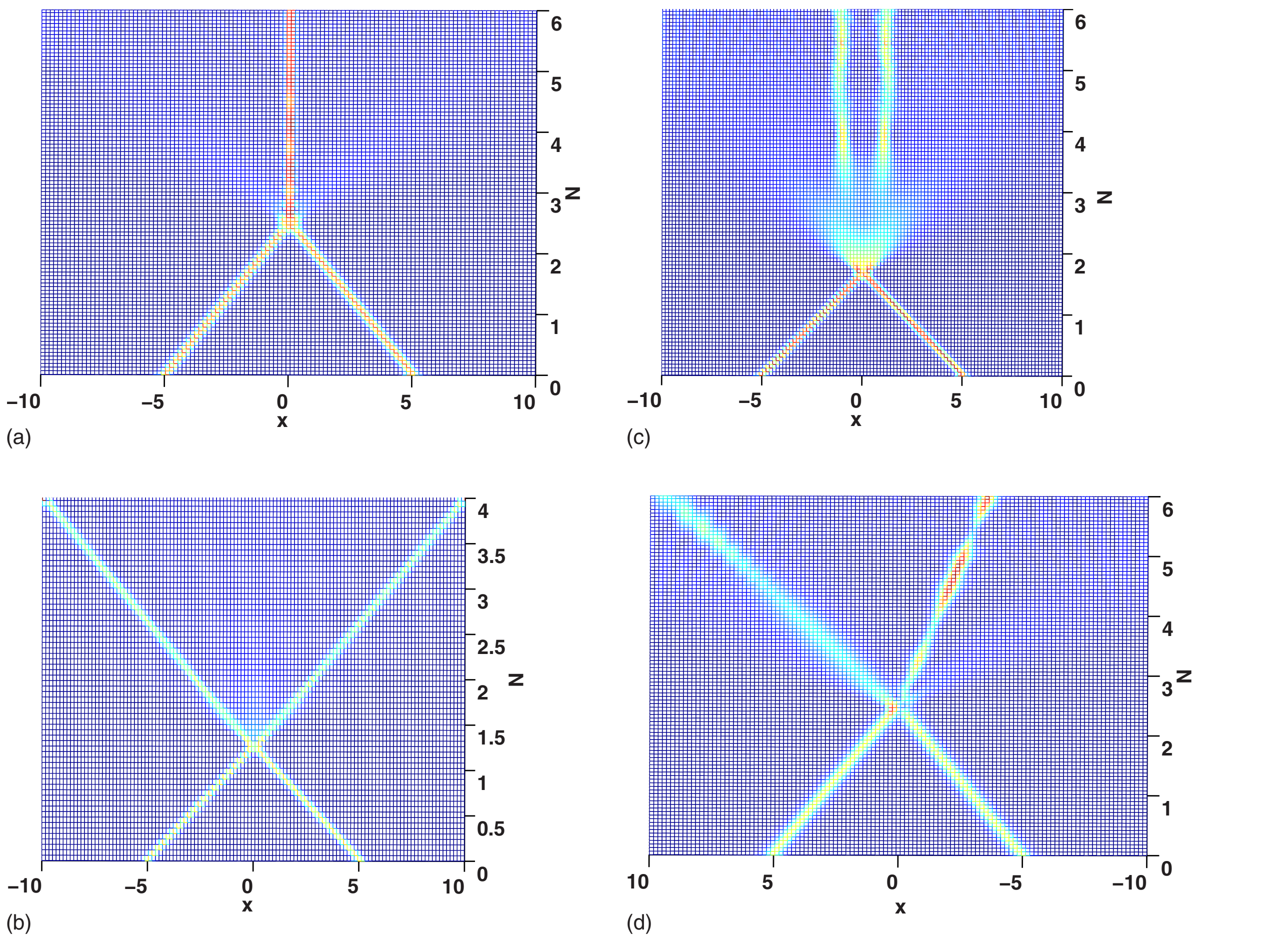

FIG. 1. (Color online) Typical examples of different outcomes of collisions between two identical solitons, tilted with pitch $\pm Q$, are displayed by means of contour plots of $|w(z,x)|$ [pictures for $|u_{\pm}(x,z)|$ are quite similar]: (a) merger, at $Q=2$; (b) quasielastic collision, at $Q=4$; (c) formation of a double-peak state (with intrinsic vibrations), at $Q=2.999$, which is very close to Q_{cr}; and (d) bounce, at $Q=2$, with phase shift $\pi/2$ between the FF components of the solitons. Other parameters are $\gamma_1=1$, $q=-4$, $b=1$, and $k=9$.

mismatch parameter, the $\chi^{(2)}$ coefficient is scaled to be 1, while γ_1 and γ_2 are two $\chi^{(3)}$ coefficients. These constants may be positive or negative, with a constraint that γ_1 and γ_2 usually have the same sign. Below, we are only dealing with the self-focusing Kerr nonlinearity, corresponding to $\gamma_{1,2}>0$, as the 3W solitons tend to be strongly unstable in the system with self-defocusing $\chi^{(3)}$ terms [10]. If the cubic nonlinearity is directly produced by the dielectric material, the $\chi^{(3)}$ coefficients are related as $\gamma_2=\gamma_1/6$. We adopt this relation below, although it may be different if the cubic nonlinearity is artificially created by means of the QPM technique [6,7] (the difference of γ_2 from $\gamma_1/6$ does not tangibly affect the results). The birefringence coefficient in Eq. (1) can be normalized to be $b\equiv 1$ (unless $b=0$), which is fixed below, leaving two independent parameters in the system, γ_1 and q.

Equations (1) and (2) conserve three dynamical invariants, viz., the Hamiltonian, total momentum, $P=i\int_{-\infty}^{+\infty}[(u_+^*)_x u_+ + (u_-^*)_x u_- + 2w_x^* w]dx$, and total power, $W=\int_{-\infty}^{+\infty}(|u_+|^2+|u_+u_-|^2+4|w|^2)dx$. Stationary soliton solutions with propagation constant k are looked for as $u_{\pm}(z,x)=e^{ikz}U_{\pm}(x)$, $w(z,x)=e^{2ikz}W(x)$, where real functions $U_{\pm}$ and W obey equations (recall we set $\gamma_2=\gamma_1/6$)

$$(1/2)U_{\pm}'' + U_{\mp}W + \gamma_1[(1/4)(U_{\pm}^2 + U_{\mp}^2) + 2W^2]U_{\pm} + (\gamma_1/6)W^2U_{\mp} = (k \mp b)U_{\pm}, \tag{3}$$

$$(1/2)W'' + U_+U_- + 2\gamma_1(2W^2 + U_+^2 + U_-^2)W + (\gamma_1/3)U_+U_-W = (4k+q)W. \tag{4}$$

For a special case of $\gamma_1=b=0$, $k=-q/3>0$, an exact 3W solution is available, $U_{\pm}=W=(3k/2)\text{sech}^2(\sqrt{2k}x/2)$. Starting from it, we constructed a family of 3W solitons with positive components (which is the main 3W family [10]) by means of a numerical continuation applied to Eqs. (3) and (4). The stability of the solitons was tested by direct simulations of Eqs. (1) and (2). Only stable solitons were used to collect results reported below.

III. COLLISIONS BETWEEN THREE-WAVE SOLITONS

Equations (1) and (2) are invariant with respect to the Galilean transformation, which means that tilted solitons can be generated from any straight one as $[u_{\pm}(x,z)]_Q=e^{iQx-iQ^2z/2}u_{\pm}(x-Qz,z)$, $w_Q(x,z)=e^{2iQx-iQ^2z}w(x-Qz,z)$,

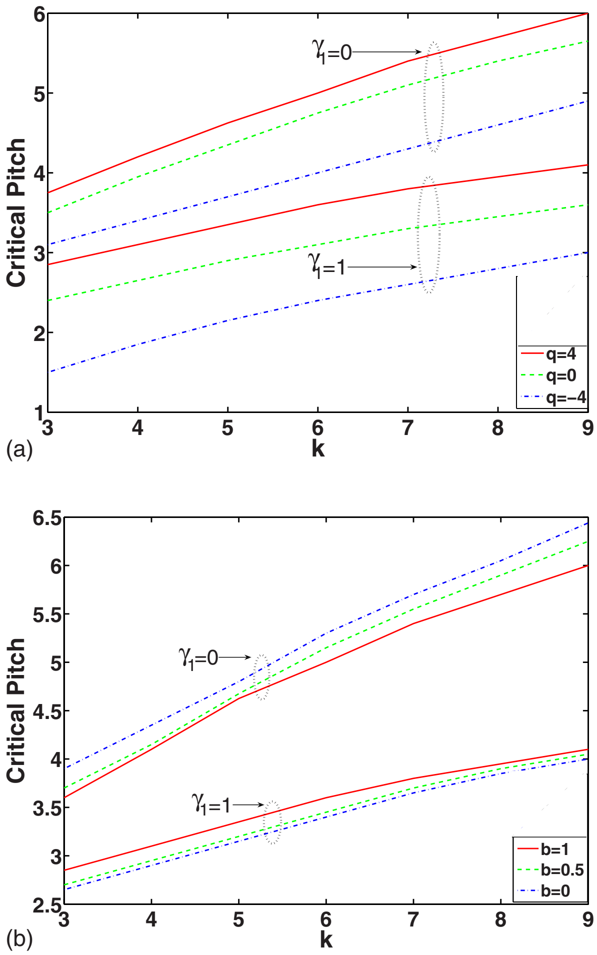

FIG. 2. (Color online) The critical value of the pitch, separating the quasielastic passage and merger of the colliding solitons, versus their propagation constant k, for (a) fixed birefringence, $b=1$, and three different values of the mismatch; and (b) fixed mismatch, $q=4$, and three different values of the birefringence. In each panel, the top and bottom sets of the plots pertain, severally, to the pure $\chi^{(2)}$ system, with $\gamma_1=0$, and to the $\chi^{(2)}:\chi^{(3)}$ system with $\gamma_1=1$.

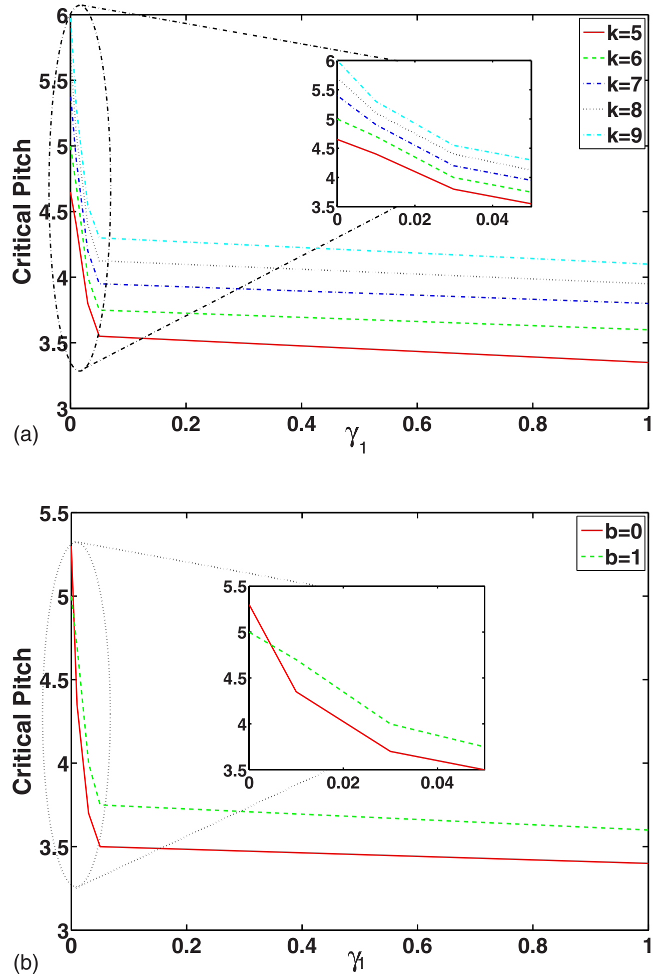

FIG. 3. (Color online) (a) The critical value of the pitch versus the strength of the cubic terms, γ_1, for fixed values of the solitons' propagation constant, k, with birefringence $b=1$ (the plots for $b=0$ are similar). (b) The switch of the dependences with $b=0$ and $b=1$ for $k=6$. Each inset is a blowup of the region where Q_{cr} drops from its value in the pure $\chi^{(2)}$ system to values determined by the domination of the cubic nonlinearity.

where Q is arbitrary pitch that determines the tilt of the soliton beam in the (x,z) plane. For simulations of the collisions, we prepared pairs of far separated identical solitons tilted with pitch $\pm Q$. As illustrated by Fig. 1, for given parameters γ_1, q, and b, and given power of the colliding solitons, a critical pitch, Q_{cr}, can be identified, such that the collision leads to the merger into a single soliton at $Q<Q_{cr}$, and quasielastic passage at $Q>Q_{cr}$. If Q is very close to Q_{cr}, the collision may sometimes lead to the formation of a double-peak state featuring periodic intrinsic vibrations, as shown in Fig. 1(c).

These results pertain to the collisions between solitons with the zero phase shift. We have also simulated collisions with phase shift $\pi/2$ or π between the FF components of the solitons, the corresponding shift between the SH components being π or 2π. In these cases, the solitons bounce from each other in a perturbed form, as shown in Fig. 1(d). The symmetry breaking between the bounced solitons is explained by the mismatch between the "amplitude" and "phase" collision centers [15].

We start the presentation of systematic results by plotting Q_{cr} as a function of propagation constant k. In Fig. 2, the dependences are presented for two different models, one with a relatively strong $\chi^{(3)}$ nonlinearity, $\gamma_1=1$, and the other including the $\chi^{(2)}$ interactions only ($\gamma_1=0$). The collisions are less inelastic (i.e., Q_{cr} is smaller) in the system including the $\chi^{(3)}$ nonlinearity, than in its $\chi^{(2)}$-only counterpart, in accordance with arguments given above.

Figure 2(a) also shows that the collision is considerably less inelastic in the model which combines the $\chi^{(3)}$ nonlinearity and negative mismatch. This feature may be explained if one recalls the cascading limit [1], which corresponds to large $|q|$ and allows one to eliminate the SH field using Eq. (2), $w \approx u_+u_-/q$. The substitution of this in Eq. (1) transforms the $\chi^{(2)}$ terms into additional XPM cubic terms, $q^{-1}|u_{\mp}|^2u_{\pm}$. With $q<0$, they can partly cancel the already present $\chi^{(3)}$ XPM terms, which account for inelastic interactions between colliding solitons. Another peculiarity revealed by Fig. 2(a) is that, while in the system with the mixed nonlinearity the inelasticity increases with the birefringence, in the pure $\chi^{(2)}$ model the trend is the opposite.

To gain further insight into the effect of the competition between the quadratic and cubic nonlinearities on outcomes of collisions between 3W solitons, in Fig. 3 we present a set of plots for Q_{cr} as a function of the $\chi^{(3)}$ coefficient, γ_1, varying from 0 to 1. It is observed that the addition of a rather weak $\chi^{(3)}$ nonlinearity (in the present notation), with $\gamma_1 \simeq 0.05$, leads to a steep drop in Q_{cr} by a factor ~ 1.5 against the pure $\chi^{(2)}$ system, while the further increase of γ_1 causes little additional decrease of Q_{cr}. Thus the cubic nonlinearity dominates starting from $\gamma_1 \simeq 0.05$.

As shown in Fig. 2(b), the pure $\chi^{(2)}$ and mixed $\chi^{(2)}:\chi^{(3)}$ systems exhibit opposite dependences of Q_{cr} on the birefringence. This feature is clarified in Fig. 3(b), which demonstrates that dependences $Q_{\mathrm{cr}}(\gamma_1)$ for $b=1$ and 0 switch at very small values of the $\chi^{(3)}$ coefficient, $\gamma_1 \simeq 0.005$.

IV. CONCLUSION

We have explored effects of various factors in the three-wave system with the mixed $\chi^{(2)}:\chi^{(3)}$ nonlinearity on collisions of spatial solitons. Inelasticity of the collisions was quantified by the critical value of the pitch, Q, which separates quasielastic collisions and the merger. It was found and explained that Q_{cr} is smaller for a relatively large negative mismatch. With the increase of the $\chi^{(3)}/\chi^{(2)}$ ratio, the system quickly switches from the behavior determined by the quadratic nonlinearity to a regime dominated by the cubic terms.

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[1] C. Etrich *et al.*, Prog. Opt. **41**, 483 (2000); A. V. Buryak *et al.*, Phys. Rep. **370**, 63 (2002).

[2] W. E. Torruellas *et al.*, Phys. Rev. Lett. **74**, 5036 (1995).

[3] H. T. Tran, Opt. Commun. **118**, 581 (1995); D. Anderson, M. Lisak, and B. A. Malomed, *ibid.* **126**, 251 (1996); A. V. Buryak, Y. S. Kivshar, and S. Trillo, J. Opt. Soc. Am. B **14**, 3110 (1997); U. Peschel, C. Etrich, F. Lederer, and B. A. Malomed, Phys. Rev. E **55**, 7704 (1997); D. Mihalache, D. Mazilu, L. C. Crasovan, and L. Torner, *ibid.* **56**, R6294 (1997); A. D. Capobianco *et al.*, Opt. Quantum Electron. **30**, 483 (1998); A. Barthélémy *et al.*, *ibid.* **30**, 923 (1998).

[4] M. A. Karpierz, Opt. Lett. **20**, 1677 (1995); A. V. Buryak and Y. S. Kivshar, Phys. Rev. A **51**, R41 (1995); O. Bang, Y. S. Kivshar, and A. V. Buryak, Opt. Lett. **22**, 1680 (1997); A. de Rossi *et al.*, Opt. Commun. **150**, 390 (1998); T. J. Alexander, Y. S. Kivshar, A. V. Buryak, and R. A. Sammut, Phys. Rev. E **61**, 2042 (2000); I. Towers, A. V. Buryak, R. A. Sammut, and B. A. Malomed, *ibid.* **63**, 055601(R) (2001).

[5] P. Di Trapani *et al.*, Phys. Rev. Lett. **87**, 183902 (2001).

[6] C. B. Clausen, O. Bang, and Y. S. Kivshar, Phys. Rev. Lett. **78**, 4749 (1997); O. Bang *et al.*, Opt. Lett. **24**, 1413 (1999).

[7] J. F. Corney and O. Bang, Phys. Rev. E **64**, 047601 (2001).

[8] V. V. Steblina *et al.*, J. Opt. Soc. Am. B **17**, 2026 (2000); U. Peschel *et al.*, Opt. Lett. **26**, 1090 (2001).

[9] K. J. Blow, N. J. Doran, and D. Wood, Opt. Lett. **12**, 202 (1987).

[10] M. Chen, D. J. Kaup, and B. A. Malomed, Phys. Rev. E **69**, 056605 (2004).

[11] Y. Baek *et al.*, Opt. Lett. **22**, 1550 (1997); B. Costantini *et al.*, *ibid.* **23**, 424 (1999).

[12] M. J. Werner and P. D. Drummond, J. Opt. Soc. Am. B **10**, 2390 (1993); D. M. Baboiu, G. I. Stegeman, and L. Torner, Opt. Lett. **20**, 2282 (1995); A. V. Buryak, Y. S. Kivshar, and V. V. Steblina, Phys. Rev. A **52**, 1670 (1995); C. Etrich, U. Peschel, F. Lederer, and B. Malomed, *ibid.* **52**, R3444 (1995); C. Etrich, U. Peschel, F. Lederer, B. A. Malomed, and Y. S. Kivshar, Phys. Rev. E **54**, 4321 (1996); G. Leo, G. Assanto, and W. E. Torruellas, Opt. Lett. **22**, 7 (1997); C. Balslev Clausen, P. L. Christiansen, and L. Torner, Opt. Commun. **136**, 185 (1997); A. A. Sukhorukov, Y. S. Kivshar, and O. Bang, Phys. Rev. E **60**, R41 (1999).

[13] B. A. Malomed and S. Wabnitz, Opt. Lett. **16**, 1388 (1991); B. A. Malomed, J. Opt. Soc. Am. **9**, 2075 (1992).

[14] S. V. Manakov, Zh. Eksp. Teor. Fiz. **65**, 505 (1973) [Sov. Phys. JETP **38**, 248 (1974)].

[15] L. Khaykovich and B. A. Malomed, Phys. Rev. A **74**, 023607 (2006).