Slow light with a doublet structure: Underlying physical processes and basic limitations

R. N. Shakhmuratov[1,2] and J. Odeurs[1]
[1]*Instituut voor Kern- en Stralingsfysica, Katholieke Universiteit Leuven, Celestijnenlaan 200 D, B-3001 Leuven, Belgium*
[2]*Kazan Physical-Technical Institute, Russian Academy of Sciences, 10/7 Sibirsky Trakt, Kazan 420029, Russia*
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Slow light, produced in a medium with two widely spaced absorption resonances, is analytically studied. We show that a pulse with frequency tuned in the middle of such a doublet structure slows down due to its energy storage in the excited atomic states. If the lifetime of the atomic excitation is much longer than the pulse duration, the pulse shape is nicely reproduced at the output of a thick sample for pulses with smooth envelopes. Pulses with sharp edges are fairly reproduced at the output. Each abrupt change of the pulse amplitude is followed by transients, which are delayed together with the pulse. A possibility to slow down a single photon is also considered.

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I. INTRODUCTION

Slow light phenomena attract particular attention because of their great potential in many applications [1]. The first proposals to produce slow light were based on electromagnetically induced transparency (EIT), which employs an auxiliary excitation of an absorbing medium by a coupling field, pumping atoms on an adjacent transition [2]. Later, other mechanisms to produce slow light were proposed, where the main feature is the presence of a transparency window in an absorption band or a single gain (see, for example, Refs. [3–10]). Meanwhile, a transparency window can be even granted by nature if two absorption lines are close in the absorption spectrum of a medium. Then a low group velocity of the radiation field can be obtained without an extra driving field [11–18]. Two main conditions for slow light with doublet structure (SLDS) were formulated in Ref. [15]. The distance between the two resonances must be larger than the pulse bandwidth, and this bandwidth is to be much larger than the width of each absorption line in the doublet. While the first condition is quite natural, the second one needs some clarification. In this paper we show that SLDS has much in common with EIT. The two coherences, induced separately in the simultaneous excitation of the two resonances, play the same role as the low-frequency coherence induced by the two fields in EIT. Therefore, the lifetime of these coherences must be much longer than the pulse duration. Another distinguishing property of SLDS is a negligible pulse broadening. This broadening becomes a serious hindrance in the use of EIT as a delay line or buffer for data synchronization (see, for example, Refs. [19–21] and references therein). In addition, we find in this paper that the third-order-dispersion produces fast transients if the pulse has sharp edges. A remarkable feature of the transients is that they do not propagate with phase velocity c, but they delay together with the pulse, propagating with group velocity $V \ll c$. This is in contradiction with the expectations expressed, for example, in Ref. [14]. It is important to notice that for a rectangular pulse of duration $2t_p$, the transients do not significantly spread the pulse if its bandwidth is smaller than the distortion parameter $2\Delta_{dist}$ defined in Ref. [20]. Under the same condition for a single-photon radiation field, the envelope of the probability amplitude of the photon delays appreciably also with small changes in its shape.

The paper is organized as follows. In Sec. II we show that the pulse becomes slow because the lion share of its energy is stored in the atoms. In Sec. III we develop a general formalism of the description of the response of an atom to a small-amplitude pulse of arbitrary shape. In Secs. IV and V we analyze the pulse propagation in a medium with two widely spaced resonances. In Secs. VI–VIII we consider the propagation of a pulse with Gaussian envelope, rectangular-shaped pulse, and a single photon, respectively.

II. ENERGY STORAGE OF SLOW LIGHT: QUALITATIVE ARGUMENTS

Courtens, using qualitative arguments [22], showed that slow light is formed due to the temporal storage of electromagnetic wave energy in a resonant medium. He obtained for the group velocity

$$V = \frac{c}{1 + U_a/U_{em}}, \tag{1}$$

where U_{em} is the energy density in the laser pulse and U_a is the energy density accumulated in the resonant excitation of atoms. From this equation it follows that the larger the fraction of energy stored in the atoms is, the slower the group velocity of the pulse is.

Later, Grischkowsky [23] showed that if the energy loss is only due to spontaneous emission of the excited atoms with rate $2\gamma = 1/T_1$, the total energy density $U = U_{em} + U_a$ changes with distance z in the medium according to

$$U = U_0 \exp(-\alpha_s z), \tag{2}$$

where $U_0 = U_{em}(0)$ is the energy density of the input pulse. The attenuation coefficient

$$\alpha_s = 2\gamma\left(\frac{1}{V} - \frac{1}{c}\right) \tag{3}$$

also helps to calculate the energy of the pulse at the output of the sample of physical thickness l: $U_{em}(l) = U_0 \exp(-\alpha_s l)$. If the pulse is slow ($V \ll c$) and the effective absorption length

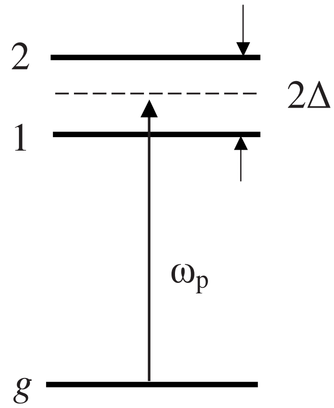

FIG. 1. The excitation scheme of a three-level atom. See details in the text.

$L_s = VT_1$ is long ($L_s \gg l$), the pulse does not lose its energy in a dissipative medium.

In this paper we show that the slow group velocity of the pulse V, given in Eq. (1), and the absorption coefficient α_s in Eq. (3), both derived with the help of intuitive arguments, give a good guideline for the analysis of SLDS.

III. SMALL-AMPLITUDE RADIATION FIELD INTERACTING IN A MEDIUM WITH TWO WIDELY SPACED ABSORPTION RESONANCES

We consider the excitation scheme shown in Fig. 1. A small-amplitude pulse $E(z,t)=E_0(z,t)\exp(-i\omega_p t+ik_p z)$ with slowly varying envelope $E_0(z,t)$ excites simultaneously two transitions $g-1$ and $g-2$ in a three-level atom, where g is the ground state and 1 and 2 are two excited states of the atom. The energy gap between states 1 and 2 is $2\hbar\Delta$. For simplicity, we assume that the carrier frequency of the pulse ω_p is tuned to the middle of this doublet structure, ω_0, such that the resonant detunings for the transitions are $\pm\Delta$. Also we assume that the dipole matrix elements for the transitions d_{g1} and d_{g2} are equal, and hence the coupling parameter $\Psi(z,t)=d_{g1}E_0(z,t)/2\hbar=d_{g2}E_0(z,t)/2\hbar$ is the same for both transitions. The case where they are not equal is considered in Ref. [17]. There, it is shown that the pulse carrier frequency ω_p should be detuned from the middle of the doublet to the frequency $\omega_0+\omega_g$, which corresponds to peak transmission. The detuning is $\omega_g \approx (g_1^{1/3}-g_2^{1/3})\Delta/(g_1^{1/3}+g_2^{1/3})$, where $g_1 \sim |d_{g1}|$ and $g_2 \sim |d_{g2}|$ account for the possibility of different strengths for the two resonances. Meanwhile, for example, for cesium, which has $g_1/g_2=7/9$, the error introduced by assuming $g_1=g_2$ is approximately 0.5% (see Ref. [17] for details of the error estimation in the expansion of the index of refraction).

For a monochromatic cw radiation field with narrow bandwidth, the transmission function can be found from the steady-state solution of the matter equations. If the amplitude of the field is small, one can take the linear response approximation where only the equations for the atomic coherences from the complete set of the matter equations have to be considered:

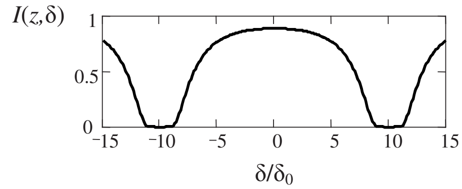

FIG. 2. Frequency dependence of the cw field intensity on the detuning δ from the peak transmission at the output of a thick absorber, Eq. (9). The parameters are $\Delta=10^{-4}\gamma$ and $T=3\times10^6$. The frequency scale is given in units of $\delta_0=\Delta/10$.

$$\dot{\sigma}_{1g} = (-\Gamma + i\Delta + i\delta)\sigma_{1g} + i\Psi_0(\delta), \tag{4}$$

$$\dot{\sigma}_{2g} = (-\Gamma - i\Delta + i\delta)\sigma_{2g} + i\Psi_0(\delta), \tag{5}$$

where $\sigma_{mg}=\rho_{mg}\exp[i(\omega_0+\delta)t-ik_p z]$ are the amplitudes of the nondiagonal components of the three-level atom density matrix ρ_{mg} $(m=1,2)$, δ is the detuning of the field frequency from the peak transmission, and Ψ_0 is the coupling parameter for the cw radiation field. Here Γ is the decay rate of the atomic coherence, which is $\Gamma=\gamma+\Gamma_b$, where Γ_b is a contribution from other broadening mechanisms except natural broadening, which is characterized by γ. In Eqs. (4) and (5) the deviations of the populations ρ_{mm} and ρ_{gg} and the coherence ρ_{12} from nonperturbed values are neglected since their change is proportional to Ψ_0^2 or higher powers of Ψ_0. The contribution of inhomogeneous broadening can be neglected if Δ is greater than the half-width of the inhomogeneous line Δ_{inh} [15,17]. This is valid for any inhomogeneous broadening with a Gaussian distribution of resonant frequencies if the observed absorption line is a convolution of a Lorentzian with a Gaussian line. The result of the convolution is known as the Voigt profile [24,25]. Even if the half-width of the inhomogeneous line Δ_{inh} is much greater than the half-width of the natural line γ, the long tails of the Voigt profile with resonant detunings $|\Delta| > |\Delta_{inh}|$ coincide with the Lorentzian component.

The stationary solution of Eqs. (4) and (5) is

$$\sigma_{mg} = \frac{i\Psi_0(\delta)}{\Gamma - i\delta \pm i\Delta}, \tag{6}$$

where $m=1,2$. The transmission function for the field amplitude can be found from the cw solution of the wave equation and expressed as

$$\Psi_0(z,\delta) = \Psi_0(0,\delta)\exp[-\mathcal{A}(\delta)z], \tag{7}$$

where $\Psi_0(0,\delta)$ is proportional to the field amplitude at the input and

$$\mathcal{A}(\delta) = -\frac{i\alpha}{2}(\sigma_{1g} + \sigma_{2g}). \tag{8}$$

The transmission of the field intensity $I(z,\delta) = \Psi_0(z,\delta)\Psi_0^*(z,\delta)(2\hbar/d_{mg})^2$ is described by the equation

$$I(z,\delta) = I(0,\delta)\exp\{-2\,\mathrm{Re}[\mathcal{A}(\delta)]z\}. \tag{9}$$

Figure 2 shows the frequency dependence of the field intensity for $\Delta=10^{-4}\gamma$ and a nondimensional optical-thickness parameter $T=4\pi\omega_0 N|d_{mg}|^2 l/\hbar\gamma c=3\times 10^6$, where l is the length of the medium and N is the concentration of resonant particles in the medium. For these values of the parameters, the transmission function of the field intensity looks very similar to that shown in Fig. 1 of Ref. [17].

For a pulsed radiation field with small amplitude the matter equations are reduced to

$$\dot{\sigma}_{1g} = -(\Gamma - i\Delta)\sigma_{1g} + i\Psi(z,t), \tag{10}$$

$$\dot{\sigma}_{2g} = -(\Gamma + i\Delta)\sigma_{2g} + i\Psi(z,t), \tag{11}$$

where $\sigma_{mg}=\rho_{mg}\exp(i\omega_p t - ik_p z)$. By means of the Fourier transform

$$F(\nu) = \int_{-\infty}^{+\infty} f(t)e^{i\nu t}dt, \tag{12}$$

Eqs. (10) and (11) are reduced to algebraic equations that can be solved easily. The solutions are

$$\sigma_{mg}(z,\nu) = \frac{i\Psi(z,\nu)}{\Gamma - i\nu \pm i\Delta}, \tag{13}$$

where $\Psi(z,\nu)$ is the Fourier transform of the coupling parameter $\Psi(z,t)$. Here, in the denominator, the plus sign is for $m=2$ and the minus sign is for $m=1$. The solutions (13) can be expanded in the Taylor expansion

$$\sigma_{mg}(z,\nu) = \frac{i\Psi(z,\nu)}{\Gamma \pm i\Delta}\sum_{k=0}^{\infty}\frac{(i\nu)^k}{(\Gamma \pm i\Delta)^k}. \tag{14}$$

If the pulse has a limited duration $[E_p(z,\pm\infty)=0]$, then the inverse transform of (14) is

$$\sigma_{mg}(z,t) = \frac{i}{\Gamma \pm i\Delta}\sum_{k=0}^{\infty}\frac{(-1)^k\partial^k\Psi(z,t)/\partial t^k}{(\Gamma \pm i\Delta)^k}. \tag{15}$$

We consider a widely spaced doublet for which $\Delta \gg \Gamma$. If the pulse bandwidth $2\Delta_{in}$ satisfies the condition $2\Delta_{in}<2\Delta$—i.e., if its bandwidth is smaller than the distance 2Δ between states 1 and 2, which can be considered as the width of the transparency window—the expansion (15) converges and the linear response approximation is valid. Below we will take into account only four terms of the expansion ($k=0$, 1, 2, and 3), which is the adiabatic following approximation [20,26]. We will conclude this section by giving some qualitative arguments, based on a paper by Crisp [26], to support the adiabatic following approximation.

According to the solution (15) the first and main term of the expansion

$$\sigma_{mg}(z,t) \approx \frac{\pm\Delta + i\Gamma}{\Gamma^2 + \Delta^2}\Psi(z,t) \tag{16}$$

has the dominating real part, $\sigma_{mg}(z,t)\approx \pm\Psi(z,t)/\Delta$, if $\Delta \gg \Gamma$. The evolution of the atom, excited by a resonant or near resonant field, is described well by the Bloch-vector model (see, for example, Ref. [27]). According to the conventional notation, the components of the Bloch vector, u, v, and w, are defined as follows: $u-iv=2\sigma_{mg}$ and $w=\rho_{mm}-\rho_{gg}$. It can then be shown that the components of the Bloch vector are approximated by $u\approx \pm 2\Psi(z,t)/\Delta$, $v\approx -2\Gamma\Psi(z,t)/\Delta^2$, and $|u|\gg|v|$. Since the length of the Bloch vector, $|S|=\sqrt{u^2+v^2+w^2}=1$, is conserved, one can obtain that $w=-\sqrt{1-u^2-v^2}\approx -1+u^2/2$. Thus, $|w|\gg|u|\gg|v|$ and the Bloch vector stays mostly in the plane $(u,0,w)$. This means that the Bloch vector follows the changing effective field $(-2\Psi(z,t),0,\pm\Delta)$ adiabatically.

In the atomic response the u component contributes to the real part of the susceptibility $\chi'(\omega_p)$, while the v component is related to its imaginary part $\chi''(\omega_p)$. Therefore, the u component specifies the dispersion of the refractive index and the v component is responsible for the absorption of the pulse and dissipation of its energy (nonadiabatic contribution). Thus, from the Bloch-vector model it follows that the atom is excited adiabatically, its polarization is mostly in phase with the driving field, and the absorption is minimized, if the adiabatic following condition is satisfied. The atomic excitation is fully reversible if $|\Delta|\gg\Delta_{in},\Gamma$. For the atom with two resonances, in the linear response approximation the contribution from each resonance can be described independently. In the next section we consider the influence of the corrections to the main adiabatic term on the pulse propagation in a dense absorptive medium.

IV. PULSE PROPAGATION IN A DENSE ABSORPTIVE MEDIUM

The wave equation for the slowly varying amplitude of the pulse $E_0(z,t)$, propagating in a medium with the doublet structure, is

$$\hat{L}E_0(z,t) = i\hbar\left(\frac{\alpha_1\sigma_{1g}}{d_{1g}} + \frac{\alpha_2\sigma_{2g}}{d_{2g}}\right), \tag{17}$$

where $\hat{L}$ is the differential operator, $\hat{L}=\partial_z + c^{-1}\partial_t$, $\alpha_m=4\pi\omega_p N|d_{mg}|^2/\hbar c$ $(m=1,2)$. If we assume that $\alpha_1=\alpha_2=\alpha$, then the equation for the coupling parameter $\Psi(z,t)$ simplifies as

$$\hat{L}\Psi(z,t) = i\frac{\alpha}{2}(\sigma_{1g}+\sigma_{2g}). \tag{18}$$

Its solution can be found with the Fourier transform (12), which reduces Eq. (18) to

$$\left(\frac{\partial}{\partial z} - \frac{i}{c}\nu + A(\nu)\right)\Psi(z,\nu) = 0, \tag{19}$$

where

$$A(\nu)\Psi(z,\nu) = -\frac{i\alpha}{2}[\sigma_{1g}(z,\nu) + \sigma_{2g}(z,\nu)]. \tag{20}$$

Equation (19) is integrated as

$$\Psi(z,\nu) = \Psi(0,\nu)\exp[(i\nu z/c) - A(\nu)z]. \tag{21}$$

The inverse Fourier transform of Eq. (21) gives the pulse envelope at distance z in the medium if the Fourier components of the pulse at the input of the sample $z=0$—i.e., $\Psi(0,\nu)$—are known. This envelope is

$$\Psi(z,t) = \frac{1}{2\pi}\int_{-\infty}^{+\infty}\Psi(0,\nu)\exp[-i\nu(t-z/c)-A(\nu)z]d\nu. \tag{22}$$

The integral in Eq. (22) can be calculated numerically. Meanwhile, it is possible to find a simple analytical approximation of this integral. In Ref. [20] it was shown that, if the transparency window is wider than the bandwidth of the pulse, it is sufficient to take into account only four terms of the expansion of $A(\nu)$ in a power series near $\nu=0$. For a medium with a doublet structure this approximation is

$$A(\nu)z = T_{trn} - i\nu t_d + \frac{\nu^2}{\Delta_{eff}^2} - i\frac{\nu^3}{3\Delta_{dst}^3}, \tag{23}$$

where

$$T_{trn} = \frac{\alpha z\Gamma}{\Delta^2+\Gamma^2}, \tag{24}$$

$$t_d = \frac{\alpha z(\Delta^2-\Gamma^2)}{(\Delta^2+\Gamma^2)^2} + \frac{z}{c}, \tag{25}$$

$$\Delta_{eff} = \sqrt{\frac{(\Delta^2+\Gamma^2)^3}{\alpha z\Gamma(3\Delta^2-\Gamma^2)}}, \tag{26}$$

$$\Delta_{dst} = \sqrt[3]{\frac{(\Delta^2+\Gamma^2)^4}{3\alpha z[(\Delta^2-\Gamma^2)^2-4\Delta^2\Gamma^2]}}. \tag{27}$$

The two first terms of the expansion (23) give a reduction of the pulse amplitude and a time delay:

$$\Psi_1(z,t) = e^{-T_{trs}}\Psi(0,t-t_d). \tag{28}$$

The delay of the pulse is caused by the reduction of its group velocity to the value

$$V_g = \left[c^{-1} + \frac{\alpha(\Delta^2-\Gamma^2)}{(\Delta^2+\Gamma^2)^2}\right]^{-1}. \tag{29}$$

The third term of the expansion (23) gives a time broadening of the pulse [20,28]:

$$\Psi_2(z,t) = \frac{\Delta_{eff}}{2\sqrt{\pi}}\int_{-\infty}^{+\infty}\Psi_1(z,\tau)e^{-(\Delta_{eff}/2)^2(t-\tau)^2}d\tau. \tag{30}$$

For example, a pulse with a Gaussian envelope, $\Psi_G(0,t) = \Psi_0\exp[-(\Delta_{in}t/2)^2]$, transforms with distance as

$$\Psi_{G2}(z,t) = \frac{\Delta_{out}}{\Delta_{in}}\Psi_0\exp\left[-T_{trn} - \frac{1}{4}\Delta_{out}^2(t-t_d)^2\right], \tag{31}$$

where Δ_{in} and Ψ_0 are the half-width and the amplitude of the pulse at the input, respectively, and Δ_{out} is the half-width of the pulse at distance z. Δ_{out} is defined as

$$\Delta_{out} = \frac{\Delta_{in}}{\sqrt{1+(\Delta_{in}/\Delta_{eff})^2}}. \tag{32}$$

The fourth term of the expansion (23) produces a distortion of the pulse envelope [20]:

$$\Psi_3(z,t) = \Delta_{dst}\int_{-\infty}^{+\infty}\Psi_2(z,t-\tau)\mathrm{Ai}(-\Delta_{dst}\tau)d\tau, \tag{33}$$

where $\mathrm{Ai}(x)$ is the Airy function. This result can be obtained from the integral representation of the Airy function [29]:

$$\Delta_{dst}\mathrm{Ai}(\pm\Delta_{dst}\tau) = \frac{1}{2\pi}\int_{-\infty}^{+\infty}\exp\left[-i\left(\nu\tau \pm \frac{\nu^3}{3\Delta_{dst}^3}\right)\right]d\tau. \tag{34}$$

V. ANALYSIS OF THE SOLUTION OF THE WAVE EQUATION

If the distance 2Δ between the excited states 1 and 2 is much larger than the linewidth 2Γ—i.e., $\Delta \gg \Gamma$—the parameters T_{trn} and t_d are approximated as follows: $T_{trn} \approx \alpha z\Gamma/\Delta^2$ and $t_d \approx \alpha z/\Delta^2$. Here, we neglect the contribution from z/c, which is very small. For simplicity we disregard the difference between γ and Γ and we introduce a nondimensional optical-thickness parameter, which is $T=\alpha z/\gamma$. Then we have $T_{trn} \approx T\gamma^2/\Delta^2$ and $t_d \approx T\gamma/\Delta^2 = T_{trn}/\gamma$. These parameters have some similarities with the parameters of the conventional EIT, which are $T_{trn} \approx T\Gamma_p\gamma_{lf}/\Omega^2$ and $t_d \approx T\Gamma_p/\Omega^2 = T_{trn}/\gamma_{lf}$, where Γ_p is the decay rate of the optical coherence, induced by the probe pulse, γ_{lf} is the decay rate of the low-frequency coherence, induced by the probe and coupling fields in a two-quantum process, Ω is the Rabi frequency for the coupling field, and $T=\alpha z/\Gamma_p$. For both SLDS and EIT, we have a reduction of the absorption due to the decrease of the optical thickness from T to T_{trn}. In the case of EIT we usually have $\Gamma_p > \Omega > \gamma_{lf}$ and the half-width of the transparency window is $\Delta_{eit}=\Omega^2/\Gamma$. The half-width of the spectrum of the input pulse, Δ_{in}, must be smaller than the half-width of the transparency window—i.e., $\Delta_{in} < \Delta_{eit}$. The pulse is appreciably delayed in an EIT medium if $\Delta_{in}t_d \gg 1$. Substituting t_d in this inequality, we obtain $T \gg \Delta_{eit}/\Delta_{in} > 1$. At the same time the absorption of the pulse should be kept small. This condition is satisfied if $T_{trn} \approx T\gamma_{lf}/\Delta_{eit} < 1$. Since the pulse-delay condition demands $T/\Delta_{eit} \gg 1/\Delta_{in}$, a small absorption is realized if $\gamma_{lf}/\Delta_{in} \ll T_{trn} < 1$. Therefore, to have a large delay of the pulse without appreciable absorption, the pulse bandwidth must satisfy the condition $\Delta_{in} \gg \gamma_{lf}$.

For SLDS there is no low-frequency coherence with small decay rate γ_{lf}. The pulse induces only the coherences σ_{g1} and σ_{g2} with a common decay rate γ. To have a large pulse delay $\Delta_{in}t_d \gg 1$ in a medium with two widely spaced resonances, we have to satisfy the condition $\Delta_{in}T\gamma/\Delta^2 \gg 1$. A small absorption of the pulse is realized if $T_{trn}=T\gamma^2/\Delta^2 < 1$. Combining both conditions we find $\gamma/\Delta_{in} \ll T\gamma^2/\Delta^2 < 1$. Thus, $\gamma \ll \Delta_{in}$ and SLDS takes place if the pulse duration is much shorter than the decay time T_2 of the optical coherence σ_{gm}. This is consistent with the concept of energy storage of slow light in the atomic excitation; see Sec. II. The attenuation coefficient $\alpha_s \approx 2\gamma/V_g$, Eq. (3), derived with the help of intuitive arguments, coincides with the analytically found expression, since $\alpha_s z = 2T_{trn}$. The factor of 2 in this relation comes from the fact that α_s is defined for the intensity and T_{trn} for the amplitude of the pulse.

There is one remarkable feature of SLDS, distinguishing it from EIT. For EIT the half-width of the transparency window for a single atom is defined as $\Delta_{eit}=\Omega^2/\Gamma$. It narrows with distance as $\Delta_{eff}=\Delta_{eit}/\sqrt{T}$ [20]. For SLDS the half-width of the transparency window can be defined as Δ. However, according to Eq. (26) we have $\Delta_{eff}\approx\Delta^2/(\gamma\sqrt{3T})$. Then the ratio $\Delta_{in}/\Delta_{eff}=\sqrt{3T_{trn}}(\Delta_{in}/\Delta)$ is smaller than 1 if $\Delta_{in}<\Delta/\sqrt{3T_{trn}}$. This means that time broadening of the pulse is small for SLDS.

Pulse distortion for SLDS can play an essential role. It was shown in Ref. [20] that the distortion of the pulse in an EIT medium is negligible if $\Delta_{dst}>\Delta_{in}$. We can approximate $\Delta_{dst}\approx\sqrt[3]{\Delta^4/(3T\gamma)}$. Then this condition reads as $\Delta^4/(3T\gamma)>\Delta_{in}^3$. It can be expressed as follows: $\Delta^2/3\Delta_{in}^2>\Delta_{in}t_d$. It shows that there is a limit for a maximum pulse delay without distortion of its shape. Thus, to have a large fractional delay, $\Delta_{in}t_d=n_d\gg 1$, without breakup of the pulse, the bandwidth of the pulse must satisfy the condition $\Delta_{in}<\Delta/\sqrt{3n_d}$.

Concluding the comparison of SLDS with EIT due to the presence of a narrow hole in a broad absorption line, we should point out that for EIT the pulse spectrum narrows with distance as $\sim 1/\sqrt{T}$, while the distortion parameter decreases as $\sim 1/\sqrt[3]{T}$ [20]. If the bandwidth of the input pulse is narrower than the transparency window Δ_{eit}, then at any distance the pulse spectrum narrows more than the distortion parameter, and hence the condition $\Delta_{eff}<\Delta_{dist}$ holds at any distance. Therefore, the pulse is not distorted. For SLDS there is a limit for the pulse propagation without distortion. The fractional delay of the pulse, $n_d=\Delta_{in}t_d$, is limited by the condition $n_d<\Delta^2/3\Delta_{in}^2$, and hence the thickness is limited as $T<\Delta^4/(3\Delta_{in}^3\gamma)$. Below we consider three examples of SLDS for pulses with different envelops.

VI. GAUSSIAN PULSE

First, we consider the propagation of a Gaussian pulse in a medium with a doublet structure in the absorption spectrum. We take $\Psi(0,t)=\Psi_0\exp[-(\Delta_{in}t/2)^2]$ for the pulse at the input. For convenience, we adopt the values $\Delta=10\Delta_{in}$ and $\gamma=10^{-3}\Delta_{in}$, which are realized in the experiment [17]. With these values the fractional delay of the pulse is $\Delta_{in}t_d=10^{-5}$ T. To have a fractional delay of the pulse equal to 10 ($\Delta_{in}t_d=10$), we take $T=10^6$. In this case the parameter $\Delta_{eff}=(10^2/\sqrt{3})\Delta_{in}$ is 58 times larger than the half-width of the spectrum of the incoming pulse. Therefore, the pulse broadening is negligible. Besides, we have $T_{trn}=10^{-2}$ and, hence, the attenuation of the pulse amplitude is also negligible. The distortion parameter is $\Delta_{dst}=\sqrt[3]{10/3}\Delta_{in}$; i.e., it is larger than the half-width of the pulse spectrum. One can expect that in this case the shape of the pulse is only very slightly distorted since $\sqrt[3]{10/3}\approx 1.5$ and hence the bandwidth of the pulse, $2\Delta_{in}$, is smaller than the distortion bandwidth $2\Delta_{dst}$. Figure 3 represents the time evolution of the pulse envelope, which is well approximated by the function $\Psi(z,t)=\Psi_0\exp\{-[\Delta_{in}(t-t_d)/2]^2\}$.

To demonstrate the case if the pulse is distorted, we consider the Gaussian pulse propagation in the medium with $T=5\times10^6$. Then the pulse delay is $\Delta_{in}t_d=50$, the attenuation parameter is small, $T_{trn}=5\times10^{-2}$, the pulse-broadening parameter is large, $\Delta_{eff}=26\Delta_{in}$, and the pulse distortion parameter is $\Delta_{dst}=0.87\Delta_{in}$. The pulse broadening can be again disregarded since $\Delta_{in}\ll\Delta_{eff}$, while the pulse distortion must be taken into account because $\Delta_{in}>\Delta_{dst}$. In this case we can approximate the output pulse as

$$\Psi(z,t)=e^{-T_{trn}}\Delta_{dst}\int_{-\infty}^{+\infty}\Psi(0,t-t_d-\tau)\mathrm{Ai}(-\Delta_{dst}\tau)d\tau. \tag{35}$$

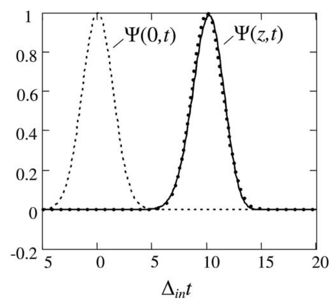

FIG. 3. Time evolution of a Gaussian pulse in a dense absorptive medium with two widely spaced resonances. The solid line is the result of a numerical calculation of the integral in Eq. (22). The dots are the approximation given by Eq. (28) where the contribution of T_{trn} is disregarded ($T_{trn}=0$). The dashed line represents the pulse envelope at the input ($z=0$). Time scale is in units of $1/\Delta_{in}$, $T=10^6$, and the other parameters are defined in the text. The pulse amplitude is normalized to Ψ_0.

The Airy function is available in MATHCAD in the list of built-in functions. An example of its time dependence for $\Delta_{dst}=0.87\Delta_{in}$ is shown in Fig. 4(a). The result of the convolution of the input pulse with the Airy function, Eq. (35), is shown in Fig. 4(b). The pulse is distorted and it acquires oscillatory features in its tail.

VII. RECTANGULAR PULSE

In this section we consider the propagation of the rectangular pulse,

$$\Psi(0,t)=\Psi_0[\theta(t+t_p)-\theta(t-t_p)], \tag{36}$$

in an optically dense medium with the doublet structure in the absorption spectrum. Here $\theta(t)$ is the Heaviside step function and $2t_p$ is the pulse duration. The spectrum of the pulse is

$$\Psi(0,\nu)=2\Psi_0\frac{\sin(\nu t_p)}{\nu}. \tag{37}$$

It has long tails, decreasing as $\sim 1/\nu$. This is the result of the instantaneous rise and drop of the front and trailing edges of the pulse. One can expect that even the presence of the transparency window of width 2Δ, which is much broader than the central part of the pulse spectrum $\sim 2/t_p$, Eq. (37), will not allow transmission of the pulse without corruption. The long tails of its spectrum will interact with the absorption resonances. To verify this expectation, we numerically calcu-

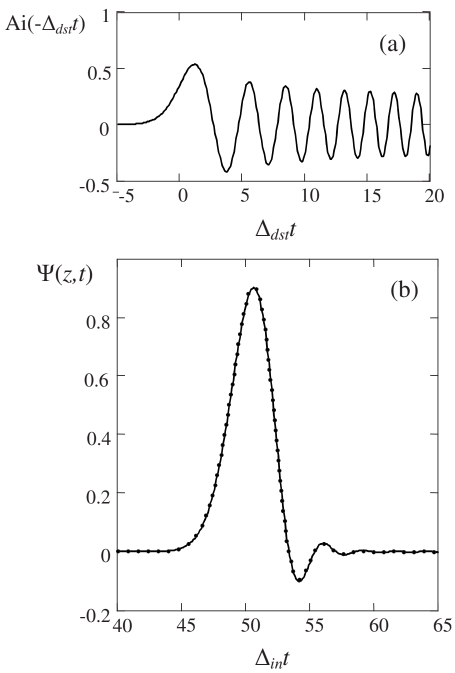

FIG. 4. (a) Time dependence of the Airy function. The value of the distortion parameter is $\Delta_{dst}=0.87\Delta_{in}$. (b) Time evolution of the pulse envelope at the output of the sample with thickness $T=5\times10^6$. The solid line is the result of the numerical calculation of the integral in Eq. (22), and the dots are the approximation given by Eq. (35). The other parameters are the same as in Fig. 4.

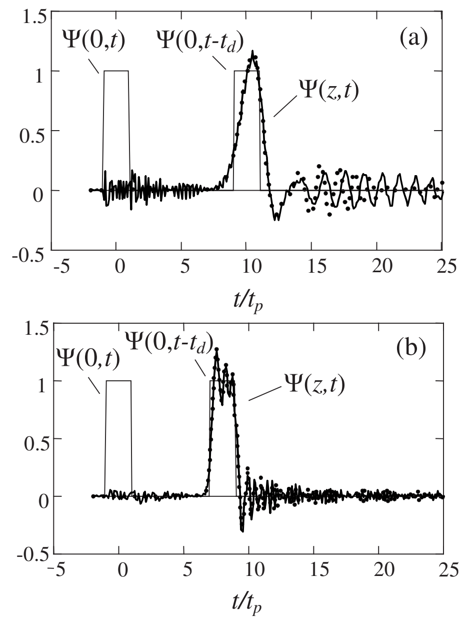

FIG. 5. The time evolution of the rectangular pulse in a dense medium is shown by the thick solid line. It is obtained by the numerical calculation of the solution of the wave equation (22). The analytical approximation, Eq. (38), is shown by dots. The pulse at the input and the uncorrupted delayed pulse are shown by the thin solid line. The parameters are $T=10^6$, $\Delta=10^{-4}\gamma$ for plot (a) and $T=2\times10^7$, $\Delta=5\times10^{-4}\gamma$ for plot (b).

lated the integral in Eq. (22), which describes the pulse propagation in a dense medium. We took the same optical thickness $T=10^6$ as in a previous section. Roughly, the half-width of the central part of the pulse spectrum, Eq. (37), is $\Delta_{in}=2/t_p$. If $\Delta=5\Delta_{in}$ and if we take the same ratio of splitting Δ and the linewidth γ as in the previous section—i.e., $\gamma=10^{-4}\Delta$—then $t_d=10t_p$ and the attenuation coefficient is $T_{trn}=10^{-2}$. The pulse-broadening parameter is $\Delta_{eff}=28.8\Delta_{in}$; i.e., it is much larger than the spectrum half-width Δ_{in}. Therefore, we cannot expect pulse broadening. Meanwhile, the distortion parameter is $\Delta_{dst}=0.75\Delta_{in}$—i.e., $\Delta_{in}>\Delta_{dst}$—which means that the pulse must be corrupted. According to these arguments, the output pulse should be well described by taking into account only the second and fourth terms of the adiabatic expansion, Eq. (23), which gives

$$\Psi(z,t)=\Psi_0\int_{\Delta_{dst}(t-t_d-t_p)}^{\Delta_{dst}(t-t_d+t_p)}\mathrm{Ai}(-\tau)d\tau. \qquad (38)$$

The results of the calculation are shown in Fig. 5(a). Expression (22), shown by the thick solid line, gives fast-oscillating, small-amplitude transients, which propagate fast with a small delay. The large-amplitude pulse with slow oscillations is delayed by t_d. The delayed part is well described by Eq. (38), except for a small mismatch in the period of the transients, following the first bump and dip in the temporal profile of the pulse.

One could expect that, if the parameters of the medium are chosen such that $\Delta_{in}<\Delta_{dst}$, the delayed pulse envelope would not be much corrupted. To clarify this point, we take the following set of parameters: $T=2\times10^7$, $\Delta=5\times10^{-4}\gamma$, and $\Delta_{in}=\Delta/25$. Then the pulse broadening and attenuation are again small since $T_{trn}=8\times10^{-3}$ and $\Delta_{eff}=161\Delta_{in}$—i.e., $\Delta_{eff}\gg\Delta_{in}$. Meanwhile, the distortion parameter is $\Delta_{dst}=2.35\Delta_{in}$—i.e., $\Delta_{in}<\Delta_{dst}$. The pulse evolution for these values of the parameters is shown in Fig. 5(b). In this case the sharp edges are fairly well reproduced, but they are followed by decaying, fast-oscillating transients.

The propagation of the rectangular pulse in a SLDS medium is very different from that in an EIT medium with a narrow dip in a broad absorption line. The latter was considered in Ref. [14]. A rectangular pulse in an EIT medium is split into fast and slow components. The slow component propagates with group velocity V_g and acquires a shape close to a Gaussian one. No further distortion of the slow component is present in EIT, except temporal broadening with propagation distance. In Refs. [20,28] it is shown that the shape of the slow component is the convolution of the shape of the input pulse with a Gaussian of half-width Δ_{eff}, which narrows with propagation distance [see Eq. (30)]. The fast components propagate with group velocity c. They are associated with the discontinuities in the front and trailing edges of the input pulse. Both are transformed to short pulses separated in time by $2t_p$ and followed by fast transients [14]. As shown in Refs. [20,28], the oscillation frequency and decay rate of these transients are defined by the parameters of the broad absorption line and optical thickness of the medium without EIT dip.

For SLDS each atom is characterized by the spectral function $A(\nu)$, Eq. (20), which consists of two absorption reso-

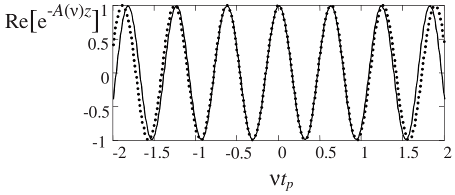

FIG. 6. Frequency dependence of the function $\mathrm{Re}\{\exp[-A(\nu)z]\}$ (solid line) for $T=10^6$ and $\Delta=10^{-4}\gamma$. Frequency scale is normalized to $t_p=\Delta/10$. The function $\cos(\nu t_d)$ is shown by dots for comparison.

nances. Meanwhile, the ensemble of atoms is characterized by the function $\exp[-A(\nu)z]$. If the medium is thick (z is large), the function $\exp[-A(\nu)z]$ becomes periodic; see Fig. 6. This function should not be confused with the transmission function for the intensity of a monochromatic field, Eq. (9). The period of the function $\exp[-A(\nu)z]$ in the close vicinity of its center ($\nu=0$) is $2\pi\Delta^2/(\gamma T)=2\pi/t_d$. If the function $\exp[-A(\nu)z]$ is perfectly periodic in a wide frequency domain, a pulse of any shape will be delayed without corruption and described by Eq. (28). Because of the periodic structure, shown in Fig. 6, the different frequency components of the pulse acquire different phases. Thus, the pulse delay can be explained by the interference of the spectral components of the pulse.

In reality, the function $\exp[-A(\nu)z]$ is slightly nonperiodic; see the comparison of this function with $\cos(\nu t_d)$ shown by dots in Fig. 6. This slightly inharmonic dependence is caused by the term $\sim i\nu^3/(3\Delta_{dst}^3)$ in the expansion (23) of $A(\nu)z$, which produces transients described by Eq. (38). Analysis of Eq. (38) [see Ref. [29], where the asymptotic expansion of the integral in Eq. (38) is given for large values of its limits] shows that, for example, the transients following the leading edge of the rectangular pulse are well described by expression

$$\Psi(z,t)=\Psi_0\left\{1-\frac{\cos\left\{\frac{2}{3}[\Delta_{dst}(t-t_d+t_p)]^{3/2}+\frac{\pi}{4}\right\}}{\sqrt{\pi}[\Delta_{dst}(t-t_d+t_p)]^{3/4}}\right\} \tag{39}$$

for $\Delta_{dst}(t+t_p-t_d)>1$. This expression is valid in the time interval $(t_d-t_p)<t<(t_d+t_p)$ excluding domains $\sim 1/\Delta_{dst}$ in the close vicinity of the edges of the delayed pulse, $t_d\pm t_p$.

VIII. SINGLE-PHOTON PROPAGATION

In this section we consider the SLDS propagation of a photon emitted by a single-photon source of the first kind [28]. This source emits a single photon in free-space vacuum modes after the formation of an excited state of a single particle by a short laser pulse or in an atomic (or nuclear) cascade of successive emission of two photons. The probability amplitude of such a photon has a sharply rising leading edge and an exponentially decaying tail (see Fig. 7). The sharp leading edge is a result of causality: no photon can be emitted before the formation of an excited state. It has maximum probability when the excited state is formed. Then the probability amplitude decays exponentially with time. The probability amplitude of such a photon is

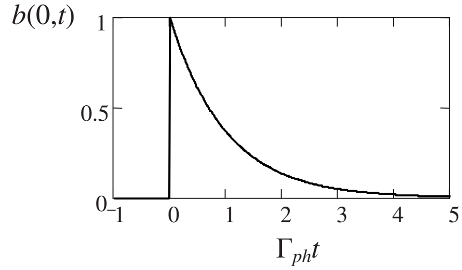

FIG. 7. Time evolution of the probability amplitude of a single photon at the input of an absorber.

$$b_{ph}(z,t)=b_0e^{-(i\omega_p+\Gamma_{ph})(t-z/c)}\theta(t-z/c), \tag{40}$$

where ω_p is the carrier frequency of the photon, Γ_{ph} is the radiative decay rate of the emitting state, and z is the distance from the source. Below we disregard the value z/c with respect to the time scale of the radiative decay and we normalize the maximum of the photon probability b_0 to 1. We consider the evolution of the time envelope of the probability amplitude $b_{ph}(z,t)$ in a thick sample with two widely spaced resonances, assuming that at the sample input $z=0$ this envelope is described by the function

$$b(0,t)=e^{-\Gamma_{ph}t}\theta(t). \tag{41}$$

The Fourier transform of this function is

$$b(0,\nu)=\frac{1}{\Gamma_{ph}-i\nu}. \tag{42}$$

Then the time evolution of the envelope of the photon-probability amplitude at the output of the sample is described by Eq. (22), where $\Psi(0,\nu)$ is substituted by $b(0,\nu)$. First, we take the same parameters of the sample as for Gaussian and rectangular pulses in Fig. 5(a)—i.e., $T=10^6$ and $\gamma=10^{-4}\Delta$. The half-width of the photon spectrum is Γ_{ph}, and we take the same relation between Δ and Γ_{ph} for the photon as for the Gaussian pulse in Fig. 3—i.e., $\Delta=10\Gamma_{ph}$. The time evolution of the photon probability amplitude at the output of the sample is shown in Fig. 8(a). The propagation parameters in this case are $\Gamma_{ph}t_d=10$, $T_{trn}=10^{-2}$, $\Delta_{eff}=57.7\Gamma_{ph}$, and $\Delta_{dst}=1.5\Gamma_{ph}$. As seen from the plots, the probability amplitude $b(z,t)$, Eq. (22) (shown by the thick solid line), is approximated reasonably well by the function

$$b(z,t)=\Delta_{dst}\int_{-\infty}^{t-t_d}b(0,t-t_d-\tau)\mathrm{Ai}(-\Delta_{dst}\tau)d\tau, \tag{43}$$

shown by dots. Here, in spite of the condition $\Delta_{dst}>\Gamma_{ph}$, the time envelope of the probability amplitude is corrupted.

We increased the ratio Δ_{dst}/Γ_{ph} three times by taking $\Delta=50\Gamma_{ph}$ and $T=2\times10^7$ [see Fig. 8(b)]. Then the parameters are $\Gamma_{ph}t_d=8$, $T_{trn}=8\times10^{-3}$, $\Delta_{eff}=322.7\Gamma_{ph}$, and $\Delta_{dst}=4.7\Gamma_{ph}$. Then the photon envelope is almost reproduced, but it is followed by fast transients. For large arguments of the

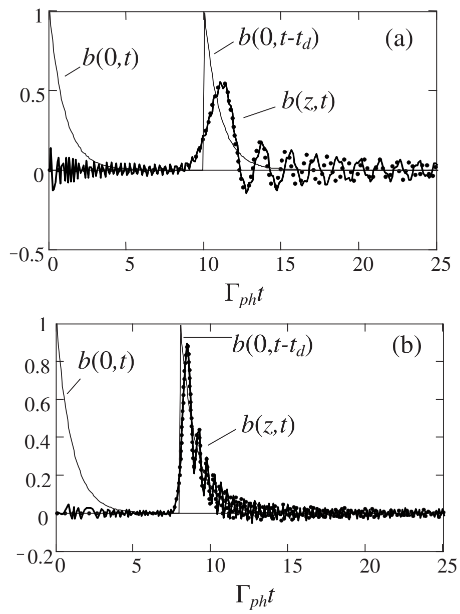

FIG. 8. The time evolution of the probability amplitude of a single photon in a dense medium is shown by the thick solid line. It is obtained by the numerical calculation of the solution of the wave equation (22). The analytical approximation, Eq. (43), is shown by dots. The probability amplitude at the input and the uncorrupted delayed probability amplitude are shown by the thin solid line. The parameters are $T=10^6$, $\Delta=10^{-4}\gamma$ for plot (a) and $T=2\times10^7$, $\Delta=5\times10^{-4}\gamma$ for plot (b). $\Gamma=10^{-3}\gamma$ for both plots.

Airy function, $\Delta_{dst}(t-t_d)>1$, the oscillating part of the probability amplitude can be approximated by the expression

$$b(z,t)=e^{-\gamma(t-t_d)}-\frac{\cos\left\{\frac{2}{3}[\Delta_{dst}(t-t_d)]^{3/2}+\frac{\pi}{4}\right\}}{\sqrt{\pi}[\Delta_{dst}(t-t_d)]^{3/4}}. \quad (44)$$

The propagation of a single photon in a SLDS medium is also very different from that in an EIT medium with a narrow dip in a broad absorption line. In an EIT medium, the single photon is split into a slow and a fast components [28]. The slow component propagates with small group velocity V_g and broadens in time, acquiring almost a Gaussian shape. The fast component propagates with group velocity c and transforms into a sharp pulse followed by oscillating transients. In Ref. [28] we considered time filtering of the photon emitted by a single-photon source of the first kind with EIT. This filtering is necessary to improve the quality of such a photon (see Ref. [28] for details). In this section we demonstrated that with SLDS a similar filtering of a single photon is impossible. However, it is possible to keep the photon quite a long time t_d in the SLDS medium with only a small corruption of the time envelope of the photon wave packet.

IX. CONCLUSION

We analyzed the main physical processes forming slow light in a resonant medium with two widely spaced resonances. The light pulse becomes slow due to its energy storage in the excited-state atoms. The process of energy exchange between the atoms and the pulse (excitation and deexcitation) is adiabatic if the distance between the two resonances (2Δ) is larger than the pulse bandwidth ($2\Delta_{in}$). The pulse experiences a long fractional delay ($\Delta_{in}t_d\gg1$) if the pulse duration is much shorter than the decay time of optical coherence—i.e., $t_p\ll T_2$. In contrast to EIT, the slow pulse due to the doublet structure is not broadened in time. However, there is a limit for the uncorrupted pulse propagation, which is set by the third-order dispersion. These conclusions are valid for pulses with smooth envelops. Pulses with sharp edges are always corrupted since their spectral wings fall off slowly as $\sim1/\nu$ and, hence, they overlap with distant resonances. We formulated conditions when the sharp edges of these pulses are also delayed. However, they are always followed by fast-oscillating transients.

SLDS was observed in Rb vapor [15] and Cs vapor [16,17]. We propose to make experiments in a solid. For example, ruby has two doublets. One is in the ground state $^4A_2$, which is separated by 0.382 cm^{-1} (see, for example, Ref. [30]). At liquid helium temperature the homogeneous decay time of the optical coherence for the transition $^4A_2$-$^2E(\bar{E})$ is a few hundred ns. This decay rate depends strongly on an externally applied dc magnetic field and its alignment with respect to the crystal axis (see, for example, Ref. [31]). Another doublet is formed by the two excited states $^2E(\bar{E})$ and $^2E(2\bar{A})$, separated by 29 cm^{-1} (see, for example, Refs. [30,32]). Transitions from the ground state $^4A_2$ to these states form the R_1 and R_2 lines, which are separated even at room temperature since their linewidth is 11 cm^{-1} at 300 K. We should notice that the delay time of slow light, t_d, almost does not depend on the decay rate of the optical coherence, Γ, if the doublet splitting 2Δ is larger than the linewidth 2Γ; see Eq. (25). Other parameters, such as T_{trn}, Δ_{eff}, and Δ_{dst}, which are the depth of the transparency window, the pulse-broadening parameter, and the pulse distortion parameter, respectively, depend on the decay rate of the optical coherence. We assume that the two widely spaced absorption resonances, the R_1 and R_2 lines in ruby, are good candidates for SLDS for short pulses and the doublet structure in the ground state for long pulses.

ACKNOWLEDGMENTS

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