Spatially dependent spontaneous emission and Rabi oscillations of atoms in an open cavity with nonorthogonal eigenmodes

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The spatial dependence of the cavity modified spontaneous emission of an atom in a cavity with finite output coupling loss is analyzed. The analysis is based on the quantized nonorthogonal eigenmodes defined by the open cavity boundary condition. The spatial dependence of the Rabi oscillation frequency in the strong coupling regime is also analyzed. For spontaneous decay, the spatial dependence is a sinusoidal modulation on top of a dc value, where the interaction with all cavity modes is considered. For Rabi oscillations, where the atom interacts mainly with only one cavity mode, the oscillation frequency also has a sinusoidal spatial dependence.

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The cavity modified atomic radiation has attracted long lasting research interests. The enhancement and suppression of spontaneous emission, Rabi oscillations, and its collapse and revival have been studied theoretically [1,2] and experimentally [3–5]. With the recent fabrication progress in photonic crystals and quantum dots, it has stimulated further interests [6–9] due to its importance in all solid state quantum optical devices. The spatial dependence of the modification together with the open nature of the cavity, however, still needs more detailed attention.

A rigorous quantum analysis requires the quantization of cavity eigenmodes. In quantum optics, the cavity field is normally quantized based on the orthogonal eigenmodes of a closed cavity. For the atom-cavity-field interaction problem, the cavity often has a finite output coupling loss for it to be a useful device in practice. There have been different approaches for analyzing such a problem [10]. The choice of modes for quantization is a bit more subtle. The cavity modes are often assumed to remain those of a closed cavity provided output coupling loss is small. The interaction with an outside reservoir is phenomenologically introduced by adding a decay term in the amplitude rate equation. This is an approximation limited only to low loss cavities. The use of close-cavity eigenmodes for an open cavity will falsely lead to a complete suppression of atom-cavity-field interaction at the modal node position. Another approach is to use the normal modes defined by an infinitely large cavity that represents the reservoir surrounding the cavity of interest [11]. The atomic dynamics is obtained by summing over the interaction between the atom and all normal modes. This approach typically focuses only on the atomic dynamics. The field dynamics is lost after summation. To better describe the dynamics of the atom-cavity-field interaction while retaining the cavity-field dynamics, the ideal modes to use are the leaky cavity eigenmodes.

The concept of leaky cavity eigenmodes, the so-called Fox-Li eigenmodes or quasimodes, is commonly used in semiclassical laser physics but less popular in quantum optics. These eigenmodes are nonorthogonal in general. In a semiclassical calculation using Fox-Li modes, it was shown that there is an enhancement factor in the quantum limited laser linewidth. This excess noise factor was later generalized as a result of eigenmode nonorthogonality [12–15] and has been experimentally confirmed [16–20]. This factor can be significant and it shows that the eigenmode nonorthogonality does have significant effects in quantum optics. It raised the interests in how to analyze the problem in a fully quantum approach [21–25] and what the quantization of lossy cavity eigenmodes should be [26–32]. Several quantization approaches have been proposed and applied to spontaneous decay analyses. However, the analyses have all used traveling field to describe the atom-field interaction even when a standing wave cavity is considered. The atom interacts with both forward and backward propagating waves. To accurately describe the dynamics of the atom-cavity-field interaction, the interference between the counterpropagating fields needs to be considered along with the eigenmode nonorthogonality.

Let us first briefly review the notation and the quantization of lossy cavity eigenmodes. For a lossy cavity, the eigenmodes $\{u_n\}$ are in general not orthogonal but instead biorthogonal to a set of adjoint modes $\{\phi_n\}$, i.e., $(\phi_n|u_m)=\delta_{nm}$. The inner product [14,16] is defined as

$$(\phi_n|u_m = \int_{\mathbf{x}} \phi_n^\dagger \cdot u_m, \tag{1}$$

where the eigenmode vector u_m is defined as

$$u_m = \begin{pmatrix} u_{m+} \\ u_{m-} \end{pmatrix}, \tag{2}$$

and is power normalized $(u_n|u_n)=1$. The subscripts + and −, respectively, denote the components that travel in the positive and negative optical axis directions. Similar definition of forward and backward propagation components is also applied to adjoint mode ϕ_n. This inner dot product definition can be rigorously derived from a Sturm-Liouville problem [26]. There have been several different quantization proposals for the lossy cavity eigenmodes. Here, the author's previous result [31] will be used, which was also proposed by other researchers [27] from a different approach. In the conventional method, E field is referenced to an orthonormal mode basis $\{e_k\}$ defined by a closed boundary condition,

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where each mode is quantized as a simple harmonic oscillator. Following the above inner dot product definition, the cavity mode e_k also has forward and backward propagation components. For example, when a one-dimensional (1D) cavity is considered,

$$e_k = \frac{1}{\sqrt{2L}}\begin{pmatrix} e^{ikz} \\ -e^{-ikz} \end{pmatrix}, \tag{3}$$

where L is the cavity length and e_k is power normalized $(e_k|e_k)=1$. The quantized E field is $\hat{E}=\Sigma_k\sqrt{\hbar\omega_k/(2\epsilon_0)}\{\hat{a}_{e_k}e_k+\hat{a}^\dagger_{e_k}e^*_k\}$, where the amplitude operator $\hat{a}_{e_k}$ commutation relation is $[\hat{a}_{e_k},\hat{a}^\dagger_{e'_k}]=\delta_{kk'}$. This is the mode of the universe representation of $\hat{E}$ when L is sufficiently large. Now, assume there is a lossy cavity inside the universe defined by length L and we can express the E field inside the lossy cavity in terms of its true cavity eigenmodes $\{u_n\}$ in a similar form, i.e., $\hat{E}=\Sigma_n\sqrt{\hbar\omega_n/(2\epsilon_0)}\{\hat{a}_n u_n+\hat{a}^\dagger_n u^*_n\}$, where $\hat{a}_n$ and $\hat{a}^\dagger_n$ are the new creation and annihilation operators assigned to mode u_n, and u_n is power normalized. Both of these two $\hat{E}$ expressions can be used to describe the E field inside a lossy cavity. Projecting the annihilation part of the two $\hat{E}$ expressions by $(\phi_n|$ and using the biorthogonality $(\phi_n|u_m)=\delta_{nm}$, we obtain the transformation

$$\hat{a}_n = \sum_k (\phi_n|e_k \hat{a}_{e_k}. \tag{4}$$

The commutation relation for $\hat{a}_n$ is then

$$[\hat{a}_n,\hat{a}^\dagger_m] = \sum_{i,j} (\phi_n|e_i(e_j|\phi_m[\hat{a}_{e_i},\hat{a}^\dagger_{e_j}] = (\phi_n|\phi_m, \tag{5}$$

where the closure relation $\Sigma_i\, |e_i)(e_i| = 1$ is used.

We now use the lossy cavity eigenmodes to describe the dynamics of atom-cavity-field interaction. The atomic variables of interest are the upper and lower population operators $\hat{\sigma}_e \equiv |e\rangle\langle e|$, $\hat{\sigma}_g \equiv |g\rangle\langle g|$, and dipole operator $\hat{\sigma}\equiv|g\rangle\langle e|$. The electric-dipole interaction Hamiltonian is

$$H_I = \sum_n g_n\hbar[\hat{a}^\dagger_{u_n}(u^*_{n+}+u^*_{n-})\hat{\sigma}(\mathbf{x}) + \hat{\sigma}^\dagger\hat{a}_{u_n}(u_{n+}+u_{n-})], \tag{6}$$

where $g_n=\sqrt{\omega_n/(2\epsilon_0\hbar)}\vec{\epsilon}_n\cdot e\vec{d}$. $\vec{\epsilon}_n$ is the mode polarization vector and $e\vec{d}=e\langle e|\vec{r}|g\rangle$ is the atomic dipole moment. The atom interacts with both forward and backward propagating waves. The interference creates a standing wave-dependent atom-field interaction. This spatial dependence of interaction is explicitly described by the product $(u^*_{n+}+u^*_{n-})\hat{\sigma}(\mathbf{x})$, which were over looked in all of the previous analyses using lossy cavity eigenmodes.

From the above interaction Hamiltonian, we have the coupled quantum Langevin operator equations

$$\frac{d}{dt}\hat{\sigma}_e(\mathbf{x}) = i\sum_n g_n[\hat{a}^\dagger_n(u^*_{n+}+u^*_{n-})\hat{\sigma}(\mathbf{x}) - \hat{\sigma}^\dagger(\mathbf{x})\hat{a}_n(u_{n+}+u_{n-})], \tag{7}$$

$$\frac{d}{dt}\hat{\sigma}_g(\mathbf{x}) = -i\sum_n g_n[\hat{a}^\dagger_n(u^*_{n+}+u^*_{n-})\hat{\sigma}(\mathbf{x}) - \hat{\sigma}^\dagger(\mathbf{x})\hat{a}_n(u_{n+}+u_{n-})], \tag{8}$$

$$\frac{d}{dt}\hat{\sigma}(\mathbf{x}) = -i\omega_a\hat{\sigma}(\mathbf{x}) + i\sum_n g_n[\hat{\sigma}_e(\mathbf{x}) - \hat{\sigma}_g(\mathbf{x})]\hat{a}_n(u_{n+}+u_{n-}), \tag{9}$$

$$\frac{d}{dt}\hat{a}_n = -(i\omega_n+\gamma_n)\hat{a}_n - ig_n(\phi^*_{n+}+\phi^*_{n-})\hat{\sigma}(\mathbf{x}) + F_n, \tag{10}$$

where γ_n is the cavity eigenmode decay rate. F_n is the noise operator associated with the cavity dissipation, where $[\hat{F}_n(t),\hat{F}^\dagger_n(t')]=2\gamma_n[\hat{a}_n,\hat{a}^\dagger_n]\delta(t-t')$ to conserve the commutation relations of $\hat{a}_n$. The presence of adjoint mode ϕ_n in the amplitude operator rate (10) is derived from the interaction Hamiltonian

$$\frac{i}{\hbar}[H_I,\hat{a}_n] = i\sum_m g_m[\hat{a}^\dagger_m(u^*_{m+}+u^*_{m-})\hat{\sigma}(\mathbf{x}),\hat{a}_n] \tag{11}$$

$$= -i\sum_m g_m(\phi_n|\phi_m)(u_m|\delta)\hat{\sigma}(\mathbf{x}) \tag{12}$$

$$= -ig_n(\phi_n|\delta)\hat{\sigma}(\mathbf{x}) \tag{13}$$

$$= -ig_n(\phi^*_{n+}+\phi^*_{n-})\hat{\sigma}(\mathbf{x}), \tag{14}$$

where the closure relation $\Sigma_n\, |\phi_m)(u_m|=1$ is used [26] and the inner product $(u_m|\delta)$ is defined as

$$(u_m|\delta \equiv \int_{\mathbf{x}'} (u^*_{m+}(\mathbf{x}')\ \ u^*_{m-}(\mathbf{x}'))\begin{pmatrix}\delta(\mathbf{x}-\mathbf{x}') \\ \delta(\mathbf{x}-\mathbf{x}')\end{pmatrix}. \tag{15}$$

Equation (10) shows that the eigenmode amplitude operator is driven by the adjoint mode projection of dipole, $(\phi^*_{n+}+\phi^*_{n-})\hat{\sigma}(\mathbf{x})$. If one just considers a traveling wave interaction, it will be driven by $\phi^*\hat{\sigma}(\mathbf{x})$. This is the same expression that one will have from a semiclassical derivation [13]. The ability to lead to a quantum Langevin equation similar in form to its classical counterpart physically justifies the commutator relation $[\hat{a}_n,\hat{a}^\dagger_m]=(\phi_n|\phi_m)$. The above coupled rate equations describe the dynamics of atom-cavity-field interaction, where cavity is damped by the reservoir. In practice, the atom may also directly interact with part of the reservoir field, e.g., those propagate perpendicular to the cavity axis. Here, we consider a case where the atom mainly interacts with the cavity field, therefore neglecting the latter contribution.

We first use these equations to analyze the atom-field interaction and pay particular attention to its spatial dependence. We consider the case where the atom-field interaction is in the weak coupling regime. The atom will irreversibly decay without Rabi oscillations. We assume that the cavity decay rate is much faster than the atomic decay rate. The field amplitude $\hat{a}_n$ then adiabatically follows dipole moment

$$\hat{a}_n = \frac{-ig_n(\phi^*_{n+}+\phi^*_{n-})\hat{\sigma}(\mathbf{x})+F_n}{\gamma_n+i(\omega_n-\omega_a)}. \tag{16}$$

Substituting this expression into Eq. (7), we have

$$\frac{d}{dt}\hat{\sigma}_e = -\hat{\sigma}_e\sum_n g_n^2\frac{(\phi^*_{n+}+\phi^*_{n-})(u_{n+}+u_{n-})}{\gamma_n+i(\omega_n-\omega_a)}+\text{c.c.} + \sum_n g_n\frac{-i\hat{\sigma}^\dagger F_n(u_{n+}+u_{n-})}{\gamma_n+i(\omega_n-\omega_a)}+\text{H.c.} \tag{17}$$

The first term on the right-hand side of the above equation is the cavity modified spontaneous decay. The term including F_n is the stimulated decay due to nonzero cavity field. When the cavity field is at vacuum state, the reservoir average of the second term is zero [31]. The derived modified spontaneous decay depends on the sum of each eigenmode and adjoint mode product weighted by a frequency detuning factor. If the cavity decay rate γ_n is much smaller than the eigenmode frequency spacing and one of the modes u_n is close to atomic transition frequency, one can reduce the summation to just one mode and the decay rate can be approximated by $\propto|u_{n+}+u_{n-}|^2/[\gamma_n+i(\omega_n-\omega_a)]$, where $\phi\simeq u$ for small cavity decay rate is used. This is the single mode low loss approximation often quoted in literals [9]. To be more precise, however, the mode nonorthogonality and multimode contributions need to be considered. The derived expression provides an exact expression and the correction can be significant when cavity loss is not small.

We consider next the strong atom-field interaction regime. When the coupling constant g_n is large and cavity loss is small, the energy can cycle between atom and photons before it is eventually lost to the reservoir. Since the dimension of the cavity is usually small in order to have Rabi oscillations, the cavity mode spacing is often much larger than the atomic linewidth. The atom primarily interacts with only one cavity mode. Assuming a low excitation linear approximation $\hat{\sigma}_e-\hat{\sigma}_g\approx-1$ [33], one can calculate the Rabi frequency from Eqs. (9) and (10) using spectral analysis,

$$\hat{\sigma}(\omega) = \frac{-ig_0\hat{a}_0(\omega)(u_{0+}+u_{0-})}{-i(\omega-\omega_a)}, \tag{18}$$

$$\hat{a}_0(\omega) = \frac{-ig_0(\phi^*_{0+}+\phi^*_{0-})\hat{\sigma}(\omega)+F_0}{\gamma_0-i(\omega-\omega_0)}. \tag{19}$$

Substituting Eq. (18) into (19), we have

$$\hat{a}_0(\omega) = \frac{i(\omega-\omega_a)F_0}{(\omega-\omega_-)(\omega-\omega_+)}, \tag{20}$$

where $\omega_\pm=(\omega_a+\omega_0-i\gamma_0\pm\Delta)/2$ and the splitting frequency is defined as $\Delta=\sqrt{(\omega_a-\omega_0+i\gamma_0)^2+4g_0^2(\phi^*_{0+}+\phi^*_{0-})(u_{0+}+u_{0-})}$. The spatial dependence is similar to the decay rate dependence on the eigenmode and adjoint mode product.

To gain a physical picture of the above results, let us consider a simple 1D problem, where a cavity is formed by two symmetric mirrors located at $z=0$ and $-L$ with amplitude reflective coefficient r. The eigenmode and adjoint mode are

$$u_n = \frac{1}{p}\begin{pmatrix} e^{(ik_n+\gamma_z)z} \\ re^{(-ik_n-\gamma_z)z}\end{pmatrix} \quad \text{and} \quad \phi_n = \frac{p}{2L}\begin{pmatrix} e^{(ik_n-\gamma_z)z} \\ \frac{1}{r}e^{(-ik_n+\gamma_z)z}\end{pmatrix}, \tag{21}$$

where $r=-e^{-\gamma_z L}$, $k_n=n\pi/L$, and $p=\sqrt{(1-r^2)L/\ln|r|}$ is the power normalization constant that makes $(u_n|u_n)=1$. The spontaneous decay rate for an atom at z is

$$\gamma_a = \sum_n g_n^2\frac{(\phi^*_{n+}+\phi^*_{n-})(u_{n+}+u_{n-})+\text{c.c.}}{\gamma_n+i(\omega_n-\omega_a)} \tag{22}$$

$$=\sum_n g_n^2\frac{2-e^{-i2k_nz-2\gamma_z(z+L/2)}-e^{i2k_nz+2\gamma_z(z+L/2)}}{2L[\gamma_n+i(\omega_n-\omega_a)]}+\text{c.c.} \tag{23}$$

For simplicity, assume that the atomic transition frequency ω_a matches to one of the mode frequency ω_0. The interaction factor g_n is approximated by the center mode value g_0 and moved out of the summation. This is a reasonable approximation when atomic transition frequency is significantly larger than the axial mode frequency spacing. We first carry out the summation for the second term in the numerator,

$$\sum_n \frac{e^{-i2k_nz-2\gamma_z(z+L/2)}}{2L[\gamma_n+i(\omega_n-\omega_a)]} = \sum_n\frac{e^{-in\Delta k2z}}{2Lc(\gamma_z+in\Delta k)}e^{-ik_02z-2\gamma_z(z+L/2)}, \tag{24}$$

where $k_n=k_0+n\Delta k$, $\omega_n-\omega_a=nc\Delta k$, $\gamma_z=c\gamma_n$, and the mode spacing $\Delta k=\pi/l$. The summation can be carried out,

$$\sum_n\frac{e^{-in\Delta k2z}}{(\gamma_z+in\Delta k)} = \int_{-\infty}^{\infty}\frac{1}{\gamma_z+ik}\sum_n\delta(k-n\Delta k)e^{-ik2z}dk \tag{25}$$

$$=\int_{\infty}^{0} e^{\gamma_z 2z'}\sum_n\frac{2\pi}{\Delta k}\delta\left(2z-2z'-\frac{2\pi n}{\Delta k}\right)d2z' \tag{26}$$

$$=2L\frac{e^{\gamma_z 2z}}{1-e^{\gamma_z 2L}}, \tag{27}$$

where the convolution theorem of Fourier transform is used in the second equality. Similar calculations can be applied to other terms in the numerator, and the decay rate becomes

$$\gamma_a = \frac{2g_0^2}{c}\frac{1+e^{-2\gamma_zL}-e^{-\gamma_zL}2\cos 2k_0z}{1-e^{-\gamma_z 2L}} \tag{28}$$

$$=\frac{2g_0^2}{c}\frac{1+e^{-2\gamma_zL}-2e^{-\gamma_zL}+4e^{-\gamma_zL}\sin^2 k_0z}{1-e^{-\gamma_z2L}} \tag{29}$$

$$=\frac{2g_0^2}{c}\frac{(1+r)^2-4r\sin^2 k_0z}{1-r^2}. \tag{30}$$

When reflective coefficient $r\to 0$, it reduces to the free space 1D spontaneous decay $2g_0^2/c$. The second term on the right-hand side of the equation is due to the cavity modification. The general behavior is illustrated in Fig. 1 for a one wavelength cavity. When $r\to -1$, it reduces to conventional 1D

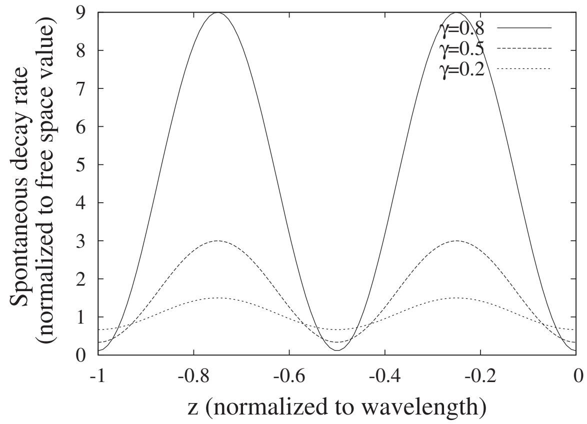

FIG. 1. The position dependence of spontaneous decay rate for three amplitude reflective coefficients. The decay rate is normalized to 1D free space value. The spontaneous decay rate is suppressed (enhanced) at the node (antinode) position as expected. Note that the decay rate is not completely suppressed at the node position.

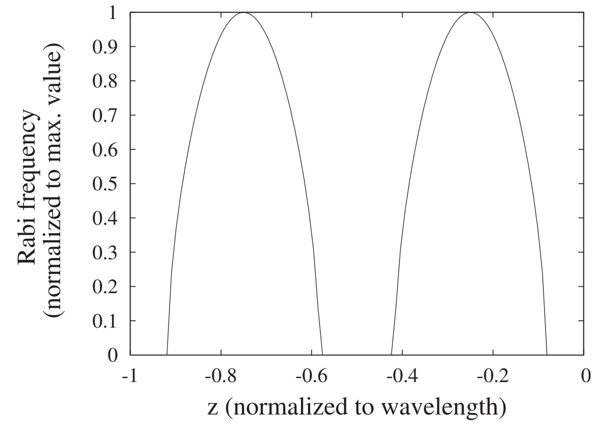

FIG. 2. The position dependence of Rabi oscillation frequency. The Rabi frequency is normalized to its value at the antinode position. There is no Rabi oscillation around the node position, where the spatially dependent atom-field interaction is smaller than cavity decay rate.

modification factor $\propto Q(\lambda/L)\sin^2 k_0 z$, where quality factor $Q=\omega_0/2\gamma_0$. For arbitrary r, the modified decay rate has a sinusoidal spatial modulation with amplitude $2|r|$ on top of a constant $(1+|r|^2)$.

In the Rabi oscillation regime, assuming again $\omega_a=\omega_0$, the splitting frequency Δ defined previously becomes

$$\Delta = \sqrt{-\gamma_o^2 + \frac{4g_0^2}{2L}(2 - e^{-i2k_n z - 2\gamma_z(z+L/2)} - e^{i2k_n z + 2\gamma_z(z+L/2)})} \quad (31)$$

$$\simeq \sqrt{-\gamma_0^2 + \frac{4g_0^2}{L}[1-\cos(2k_0 z) + i\gamma_z(z+L/2)\sin(2k_0 z)]} \quad (32)$$

$$\simeq \sqrt{-\gamma_0^2 + \frac{4g_0^2}{L}[2\sin^2(k_0 z) + i\gamma_z(z+L/2)\sin(2k_0 z)]}, \quad (33)$$

where approximation $e^x \simeq 1+x$ for $x \ll 1$ is used in the second equality. This is justified because γ_z is typically very small in the Rabi oscillation regime and the imaginary term can be dropped. To have Rabi frequency splitting, $8g_n^2\sin^2(k_0 z)/L$ must be greater than γ_0. This can be achieved by designing a cavity with $L \ll 1$ and $\gamma_0 \ll 1$ and positioning the atom close to $|\sin(k_0 z)|=1$. The spatial dependence of the Rabi frequency is then reduced to $\sqrt{8g_n^2\sin^2(k_0 z)/L - \gamma_0^2}$. When $8g_n^2\sin^2(k_0 z)/L - \gamma_0^2 < 0$, there is no Rabi oscillation. The spatial dependence is illustrated in Fig. 2 for a one wavelength cavity.

In summary, the atom-cavity-field interaction is analyzed based on the quantized quasimodes. The spatial dependence of the spontaneous decay and Rabi frequency in the weak and strong coupling regimes are derived, respectively. The spatial dependence of spontaneous decay is proportional to the sum of each eigenmode and adjoint mode product weighted by a detuning factor. The Rabi frequency also depends on the eigenmode and adjoint mode product. A 1D example shows that the decay rate has a sinusoidal spatial modulation on top of a dc value, where interaction with all cavity eigenmodes are considered. The Rabi frequency also has a sinusoidal spatial dependence. The 1D cavity problem considered above shows the general longitudinal spatial dependence property. For a more realistic cavity, it will require numerical eigenmode calculation. The transverse dependence will be cavity geometry specific. The longitudinal dependence, however, should still have similar physical features.

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