Pulse-length dependence of multiphoton ionization by phase-controlled ultrashort x-ray pulses

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Adapting earlier results that are appropriate in x-ray and perturbative regimes, the double-differential probability of multiphoton ionization by a phase-controlled ultrashort pulse is given. The infinite-pulse-length limit is also discussed. The numerical calculation was carried out in the case of two-photon processes and for a hydrogen-type 1*s* initial state that describes well most inner (*K*) shells. The numerical results indicate that the efficiency of multiphoton ionization strongly increases with the shortening of the length of a few-cycle pulse. A moderate carrier-envelope phase dependence of the ionization probability has been found in the one-cycle case only.

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I. INTRODUCTION

The phase ϕ of few-cycle light pulses determines the relative location of the pulse envelope $E_0(t)$ and carrier harmonics $\cos(\omega_0 t+\phi)$ in the electric field strength given by $E(t)=E_0(t)\cos(\omega_0 t+\phi)$ [1]. Shortly after the first experimental evidence of phase control [2] it was theoretically pointed out [3] that the carrier frequency and pulse envelope are relevant down to the one-cycle regime. A recent observation shows that the carrier-envelope phase (CEP) ϕ of the isolated, single-cycle, attosecond (135-as) xuv pulse is a relevant parameter [4]. Therefore, it is reasonable to suppose that these features of ultrashort light pulses remain valid in the xuv and soft-x-ray domain. Motivated by the continuous development of attosecond metrology [5] we investigate here the double-differential, soft x-ray ionization probability for a bound-free transition in one pulse.

The scale parameter for bound-free transitions is $\alpha_{bf}=eE_0a_B/(\hbar\omega_0)=\gamma^{-1}$, where γ is the so-called Keldysh parameter [1]. In this form E_0 is the amplitude of a time-dependent electric field of angular frequency ω_0, $\hbar$ is the Planck constant over 2π, and e is the elementary charge. A convenient Bohr radius of the bound state with energy $E_b<0$ is introduced via $a_B=\hbar/\sqrt{2m|E_b|}$, with m the rest mass of the electron. In the $\alpha_{bf}\ll 1$ regime—i.e., in the so-called multiphoton ionization regime—the perturbative approach remains valid.

It was found [6] that the harmonic conversion efficiency η in Ar has a maximum at 41 eV and has a value $\eta\sim 3\times 10^{-6}$, producing a beam of about 6×10^9 W/cm^2 with the aid of a driven field of intensity 2×10^{15} W/cm^2 and of pulse length 7 fs of a Ti: sapphire laser. Coherent radiation of much lower intensity up to 1.3 keV photon energy was produced by intense femtosecond laser pulses [7]. Consequently, taking $a_B=a_0$, where a_0 is the usual Bohr radius, and considering that few-cycle soft-x-ray and x-ray pulses are mainly created in high-harmonic generation from also few cycle optical pulses, their intensity is expected to lie in the perturbative regime.

Carrying out time-dependent perturbation calculations, the strength of the coupling of the energy eigenstates i, k, in a monochromatic field of angular frequency ω and of amplitude of the electric field strength $\vec{E}_\omega$ is characterized by the quantity $c_{ki}=e\vec{r}_{ki}\cdot\vec{E}_\omega/(E_k-E_i\pm\hbar\omega)$. For atoms such as Be up to P, the *K*-shell binding energies (from 112.1 eV up to 2145.6 eV, respectively) are much larger than the *L*-shell binding eneries (from 8 eV up to 189.3 eV, respectively). There are no other states between them, and consequently resonances do not occur if the carrier angular frequency $\omega_0\sim|E_b|/(2\hbar)$. Let us denote i the initial bound state and k the final one, which is a free state near the threshold; then, the leading term $c_{ki}\simeq 2e\vec{r}_{ki}\cdot\vec{E}_\omega/\hbar\omega_0\ll 1$ for the above-mentioned atoms, in the photon energy range from $\hbar\omega_0=56$ eV up to $\hbar\omega_0=1073$ eV, respectively, and for the intensities discussed above. In bound-free transitions from the most inner (*K*) shell the initial (bound) state is modeled by a hydrogenlike 1*s* state with effective nuclear charge Ze, and Z is defined as $Z=\sqrt{|E_b|/R_y}$, where R_y is the Rydberg-energy.

The above facts together allow us to go back to earlier stages of the intense-field–matter interaction [8]. Thus, the present work rests on an analytical expression of the transition (ionization) probability for a bound-free transition in an intense laser field that can be applied for one pulse. Our model becomes better and better with increasing photon energy and decreasing intensity—i.e., in the x-ray and weak-field regimes. Therefore, the results obtained for this probability may be successful to explain forthcoming experiments on the attosecond time scale and in the xuv and soft-x-ray regimes. The problem of the interaction of a strong laser field with a correlated multielectron system can be, of course, solved by the multiconfiguration time-dependent Hartree-Fock approach [9]. However, this excellent method is only necessary if the radiation field couples strongly the electrons, but it is not the case in the weakfield and x-ray

regimes. In this regime the interaction of the field with the outer electrons is much weaker than the interaction with the most inner (K) shell.

II. DIFFERENTIAL IONIZATION PROBABILITY IN ONE PULSE

We deal with multiphoton, mainly with two-photon, ionization of few-cycle pulses. Our model is defined as follows [8]. We start with a one-electron problem—i.e., with a bound electron of wave function

$$\psi_i = \exp[ie\vec{r}\cdot\vec{A}(t)/\hbar c]u_i(\vec{r})\exp(-iE_b t/\hbar), \tag{1}$$

where $u_i(\vec{r})$ (initial state) is an eigenstate of the instantaneous energy operator

$$H_E = -\frac{\hbar^2}{2m}\Delta + V(\vec{r}) \tag{2}$$

in the electric-field gauge with energy eigenvalue $E_b<0$, t is the time, $\vec{r}$ is the position vector, and the factor $\exp[ie\vec{r}\cdot\vec{A}(t)/\hbar c]$ appears because of gauge invariance. Here c is the velocity of light and $\vec{A}(t)$ is the vector potential that describes the total radiation field. The electric field strength $\vec{E}(t)$ of radiation origin is determined as $\vec{E}(t)=-\partial/c\partial t\vec{A}(t)$. We use the dipole approximation. $V(\vec{r})$ is the attractive Coulomb potential. The wave function of the free electron (final state) is a nonrelativistic Volkov-type solution

$$\psi_{\vec{k}} = V^{-1/2}\exp\left\{i\left(\vec{K}\cdot\vec{r} - \int_{t_0}^{t}\frac{\hbar}{2m}\vec{K}(t)^2dt'\right)\right\}, \tag{3}$$

where t_0 is the initial time, $\vec{K}(t)=\vec{k}-\frac{e}{\hbar c}\vec{A}(t)$, $\vec{k}$ is the wave vector of the outgoing electron, and V is the volume of normalization. According to our discussion, we consider the perturbative regime; therefore, the $\vec{A}^2(t)$ term can be neglected in the exponent of (3). Furthermore, it is supposed that $2\omega_0 \gtrsim \omega_b$, where $\omega_b=|E_b|/\hbar$, so the energy of one photon is not enough for ionization.

The first-order, gauge-invariant, time-dependent transition amplitude $\tilde{c}_{\vec{k}}(t,t_0)$ is given [8] as $\tilde{c}_{\vec{k}}(t,t_0) = (2\pi)^{3/2}V^{-1/2}c_{\vec{k}}(t,t_0)$ with

$$c_{\vec{k}}(t,t_0) = -\int_{t_0}^{t}\exp[i(\omega+\omega_b)t']f^*(\vec{K},t')\frac{\partial}{\partial t'}U_i[\vec{K}(t')]dt', \tag{4}$$

where $\omega=E/\hbar$, $E=\hbar^2\vec{K}^2/(2m)$, which is the kinetic energy of the free electron,

$$f = \exp[i\beta\vec{k}\cdot\vec{F}(t)], \tag{5}$$

with $\vec{F}(t)=\int^t\vec{A}(t')dt'$ and $\beta=e/(mc)$, and

$$U_i[\vec{K}(t)] = \int\exp[-i\vec{K}(t)\cdot\vec{r}]u_i(\vec{r})d^3r. \tag{6}$$

If the electron initially (at time t_0) is in the bound state, then $|\tilde{c}_{\vec{k}}(t,t_0)|^2$ gives the probability of finding a free electron of energy E (of wave vector $\vec{K}$) at time t. Consequently, the ionization probability induced by one short pulse is defined as $P_{\vec{k}}=\lim_{t\to\infty,\ t_0\to-\infty}\Sigma_{\vec{k}}|\tilde{c}_{\vec{K}}(t,t_0)|^2$, which gives after the $\Sigma_{\vec{k}} \to Vd^3k/(2\pi)^3$ substitution the differential ionization probability in one pulse as $dP_{\vec{k}}/(d^3k)=|c_{\vec{K}}(\infty,-\infty)|^2$.

The effect of the Coulomb potential on the final-state wave function is taken into account in an approximate way with the aid of the enhancement factor

$$\eta_e = \frac{|\psi_Z(0)|^2}{|\psi_{\vec{k}}(0)|^2}, \tag{7}$$

where $\psi_Z(\vec{r})$ and $\psi_{\vec{k}}(\vec{r})$ are the continuum solutions of the stationary Schrödinger-equation with Coulomb-potential $-Ze^2/r$ and without it; both are without the laser field,

$$\eta_e = \xi/[1-\exp(-\xi)], \tag{8}$$

with $\xi=2\pi Z/(ka_0)$ [10]. Here a_0 is the Bohr radius. We deal with slow electrons, $E<|E_b|$—i.e., $k<Z/a_0$; therefore, the $\eta_e=2\pi Z/(ka_0)$ approximation is valid. Thus the differential ionization probability in one pulse is modified as

$$\frac{dP_{\vec{k}}}{d^3k} = \eta_e\left|\int_{-\infty}^{\infty}\exp[-i(\omega+\omega_b)t]f(\vec{K},t)\frac{\partial}{\partial t}U_i^*[\vec{K}(t)]dt\right|^2. \tag{9}$$

As the calculation is general, our result may be applied in the case of long pulses too. It will be shown below that if the pulse length τ is much longer than $1/\omega_0$, then Eq. (9) results in the usual N-photon ionization rate multiplied by τ.

III. K-SHELL IONIZATION: DOUBLE-DIFFERENTIAL PROBABILITY OF MULTIPHOTON IONIZATION

Next, we introduce the dimensionless variables $z=\omega_0 t$ and $T=\omega_0\tau$, where T is the parameter describing the pulse length. The electric field strength of a few-cycle pulse can be well described [1] with

$$\vec{E}(z) = E_0\vec{\varepsilon}\cosh^{-1}(z/T)\cos(\alpha), \tag{10}$$

where $\vec{\varepsilon}$ is the unit vector of polarization, $\alpha(\phi)=z+\phi$. We have found that pulses of similar shape can be obtained if we take

$$\vec{F}(z) = F_0\vec{\varepsilon}f_c(z,T)\cos(\alpha), \tag{11}$$

with $f_c(z,T)=\cosh^{-1}(z/T)$, and deduce the vector potential and the electric field strength from it. This recognition makes the numerical work easier. So we obtain

$$\vec{E}(z,T) = E_0\vec{\varepsilon}e(z,T), \tag{12}$$

with

$$e(z,T) = e_s(z,T)\sin[\alpha(\phi)] + e_c(z,T)\cos[\alpha(\phi)], \tag{13}$$

where

$$e_s(z,T) = -2\tanh(z/T)/[T\cosh(z/T)], \tag{14}$$

$$e_c(z,T) = \frac{1 + T^2 + (T^2 - 1)\sinh^2(z/T)}{T^2 \cosh^3(z/T)}, \tag{15}$$

and $E_0 = \omega_0^2 F_0 / c$.

We treat the case of multiphoton ionization of a hydrogenlike $1s$ state, which is appropriate for modeling transitions from the most inner (K) shells. If the initial state has spherical symmetry, then $U_i[\vec{K}(t)] = U_i[K(t)]$ with $K(t) = |\vec{K}(t)|$. Using $\vec{E}(t) = -\partial/c\partial t \vec{A}(t)$ and the relations $\vec{K} \gg e/\hbar c \vec{A}(t)$, $\partial_K U_i(K)/K \approx [\partial_K U_i(K)/K]_{K=0}$, valid in the case of slow ($E < |E_b|$) electrons, we obtain

$$\partial_t U_i[\vec{K}(t)] = \{[\partial_K U_i(K)]/K\}_{K=0} \vec{K} \cdot e\vec{E}(t)/\hbar. \tag{16}$$

Here the notation $\partial_t = \partial/\partial t$ and $\partial_K = \partial/\partial K$ is applied. Thus the K dependence originated from $U_i[\vec{K}(t)]$ has disappeared and the energy dependence of the slow electron spectrum is affected by the laser field alone. It is an advantage of the low-energy limit. For a hydrogenlike $1s$ state of effective nuclear charge Ze this approximation gives $\partial_t U_i[\vec{K}(t)] = -32\sqrt{\pi}(a_0/Z)^{7/2} e\vec{k} \cdot \vec{E}(t)/\hbar$.

Using it and introducing dimensionless variables $x = E/(\hbar\omega_0)$, $dx = \hbar k dk/(m\omega_0)$, and $x_b = |E_b|/(\hbar\omega_0)$ in (9) we have obtained for the double-differential probability of multiphoton ionization by a phase-controlled ultrashort pulse in the direction parallel with $\vec{\varepsilon}$ (i.e., for $\vec{k} \| \vec{\varepsilon}$)

$$\frac{dP_E}{dx\, d\Omega_E} = D_0 I_0 G(x,\phi), \tag{17}$$

with $D_0 = 2^{12}\pi^3 \alpha_f \hbar^3 c^2 / [(R_y Z^2)^3 mc^2] = 1.152 \times 10^{-12} Z^{-6}$ cm^2/W, where α_f is the fine structure constant, $R_y = \alpha_f^2 mc^2/2$ is the Rydberg energy; $I_0 = cE_0^2/(8\pi)$, and

$$G(x,\phi) = x|I(x,\phi)|^2. \tag{18}$$

Here

$$I(x,\phi) = \int_{-\infty}^{\infty} \exp[Q(x,\alpha(\phi))] e(z,T) dz, \tag{19}$$

with

$$Q(x,\alpha(\phi)) = -i(x + x_b)z + iL\sqrt{x} f_c(z,T)\cos[\alpha(\phi)], \tag{20}$$

where $L = (eF_0/\hbar)\sqrt{2/(mc^2)}$. The physical meaning of (17) is the following: $dP_E = D_0 I_0 G(x,\phi) dx\, d\Omega_E$ gives the probability of a bound-free transition (the $t_0 \to -\infty$ and $t \to \infty$ means that) in one pulse producing an electron of energy E between $E = \hbar\omega_0 x = \hbar^2 k^2/(2m)$ and $E + dE$, with $dE = \hbar\omega_0 dx = \hbar^2 k dk/m$, and having $\vec{k}$ of direction in a solid angle $d\Omega_E$ around the direction $\vec{\varepsilon}$. It can be seen from (17) that the quantity $G(x,\phi)$ gives the energy ($E \sim x$) and CEP (ϕ) difference of the double-differential transition probability.

The x, ϕ dependence of $G(x,\phi)$, which is proportional to the double-differential probability of multiphoton ionization by a phase-controlled ultrashort pulse, is plotted in the $-2\pi \le \phi \le 0$ interval of the phase ϕ and the $0 \le x \le 2.05$ interval of the kinetic energy $[x = E/(\hbar\omega_0)]$ for different lengths of the pulses ($T = 1$, 1.5, and 10 in Figs. 1–3, respectively) at binding energy $x_b = 1.95$ ("two-photon" process).

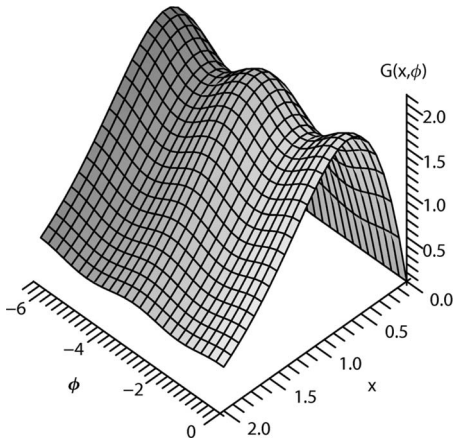

FIG. 1. Energy $[x = E/(\hbar\omega_0) < 2]$ and carrier-envelope phase (CEP) $(-2\pi \le \phi \le 0)$ dependence of $G(x,\phi) = x|I(x,\phi)|^2$ in the "two-photon" process $[x_b = |E_b|/(\hbar\omega_0) = 1.95]$ at pulse length $T = 1$ ($T = \omega_0\tau$, where ω_0 is the carrier angular frequency and τ is the pulse length) and with $L = 0.01$, $L = (eF_0/\hbar)\sqrt{2/(mc^2)} = \sqrt{2\alpha_f \lambda_0^3 I_0/(\pi^2 mc^3)}$, where $I_0 = cE_0^2/(8\pi)$ and λ_0 is the carrier wavelength. For other notation, see the text.

For these pulses the pulse length is short and therefore the width of the frequency distribution of the pulse is large enough to spread the peak expected around $x = 0.05$. For one-cycle pulses the bandwidth $\Delta\omega$ of the pulse has comparable magnitude with ω_0 [1]; therefore, the energy distribution of the outgoing electrons, which is proportional to $G(x,\phi)$, has an accountable magnitude in the $0 < E < \hbar\omega_0$ energy interval. A significant ϕ dependence has been found in the one-cycle case only. Qualitatively similar functions but with gradually decreasing values were obtained (not shown) in the three-, four-, five-, and six-photon cases. The seemingly high values of $G(x,\phi)$ compared to the infinitely long pulse-length limit (see below) motivated us to investigate the pulse-length dependence in more detail.

IV. PLANE-WAVE (INFINITE-PULSE-LENGTH) LIMIT

In order to be able to compare correctly the pulses of different length we take the infinitely long pulse as a point of

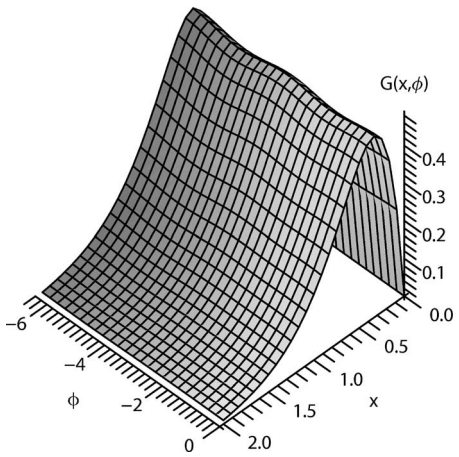

FIG. 2. The same as in Fig. 1 with $T = 1.5$.

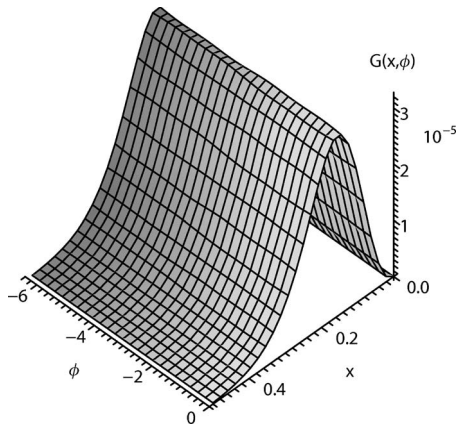

FIG. 3. The same as in Fig. 1 with $T=10$.

reference. In such a pulse the multiphoton ionization is well characterized by the transition rate W, which determines the number of ionization events as $N_0 W \tau_\infty$ in the case of an N_0 number of irradiated atoms. Therefore, for further clarification we plot $\log_{10}[G(x=0.05,\phi=0)/T]$ versus the dimensionless quantity T in Fig. 4. The sharp increase of $\log_{10}[G(x=0.05,\ \phi=0)/T]$ with decreasing T below $T=5$ indicates that short pulses will be much more efficient for ionization.

The transition to the infinite-pulse-length limit, $\tau\to\infty$, is straightforward; we have to take the $T\to\infty$ limit in (19). It results in $\lim_{T\to\infty} f_c(z,T)=\lim_{T\to\infty} e_c(z,T)=1$ and $\lim_{T\to\infty} e_s(z,T)=0$ that produces

$$I_\infty(x,\varphi)=\int_{-\infty}^{\infty}\exp[-i(x+x_b)z+iL\sqrt{x}\cos(\alpha)]\cos(\alpha)dz. \tag{21}$$

Using $\exp[iL\sqrt{x}\cos(z+\varphi)]=\Sigma_{N=-\infty}^{N=\infty} i^N \exp[iN(z+\varphi)]J_N(L\sqrt{x})$ [11], where J_N is the Bessel function of the first kind and of order N, and the identity $\cos(\alpha)=(e^{i\alpha}+e^{-i\alpha})/2$ with the definition of the Dirac δ one gets

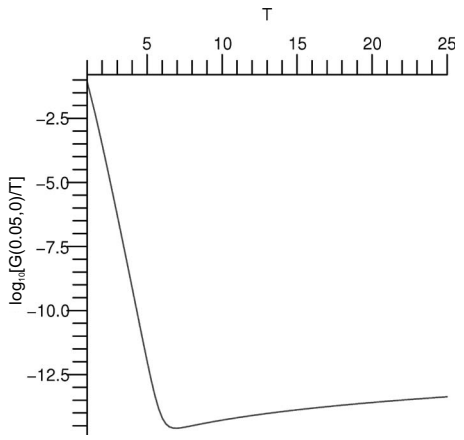

FIG. 4. The $\log_{10}[G(x=0.05,\ \phi=0)/T]$ versus the dimensionless quantity T of the "two-photon" ionization process at $x_b=1.95$. For other notation, see Fig. 1.

$$I_\infty(x,\varphi)=I_{\infty,-}(x,\varphi)+I_{\infty,+}(x,\varphi), \tag{22}$$

where

$$I_{\infty,\pm}(x,\varphi)=\sum_{N=-\infty}^{N=\infty} i^N e^{i(N\pm1)\varphi}J_N(L\sqrt{x})\pi\delta(N\pm1-x_b-x). \tag{23}$$

Now applying the identity $(2\pi)^2\delta(N-x-x_b)\delta(M-x-x_b)=\lim_{T\to\infty} T(2\pi)\delta(N-x-x_b)\delta_{N,M}$ and the definition of the transition probability per unit time [12],

$$W_{\vec{k}}=\lim_{t_0\to-\infty,\ t\to\infty}\frac{\sum_{\vec{k}}|\tilde{c}_{\vec{k}}(t,t_0)|^2}{t-t_0}, \tag{24}$$

with $t-t_0=T/\omega_0$, the double-differential transition probability per unit time in an infinitely long pulse can be introduced as

$$\frac{dW_E^\infty}{dx\,d\Omega_E}=\omega_0\lim_{T\to\infty}\frac{1}{T}\frac{dP_E}{dx\,d\Omega_E}. \tag{25}$$

The leading term is given by

$$\frac{dW_{n,E}^\infty}{dx\,d\Omega_E}=\omega_0 D_0 I_0 G_{\infty,n}(x), \tag{26}$$

with $n=N+1$ and

$$G_{\infty,n}(x)=\frac{x}{2}|J_N(L\sqrt{x})|^2\pi\delta(N+1-x_b-x). \tag{27}$$

Here N is the smallest integer that fulfills the $N+1-x_b>0$ inequality. J_N is the Bessel function of the first kind and of order N. The ϕ dependence, as was expected, disappeared.

Finally integration over x—i.e., over the energy—up to x_m [$x_m(n)=N+2-x_b-\varepsilon$, where $\varepsilon\ll n-x_b$ is any small number, in our case $n=2$, $x_b=1.95$, and $\varepsilon\ll 0.05$] gives the usual differential transition probability per unit time (the usual differential ionization rate), $dW_n^\infty/d\Omega_E$, for an $n=N+1$ photon process as

$$\frac{dW_n^\infty}{d\Omega_E}=\omega_0 D_0 I_0 \tilde{G}_{\infty,n}. \tag{28}$$

Here

$$\tilde{G}_{\infty,n}=\int_0^{x_m(n)} G_{\infty,n}(x)dx, \tag{29}$$

which gives

$$\tilde{G}_{\infty,n}=\pi\left(\frac{x}{2}|J_N(L\sqrt{x})|^2\right)_{x=N+1-x_b}. \tag{30}$$

Using the approximation $J_{|N|}(\xi)=(\xi/2)^{|N|}/(|N|!)$ valid for small arguments and the identity $J_{-m}(\xi)=(-1)^m J_m(\xi)$ for $m=0,1,2,\ldots$ in (30) (i.e., in $\tilde{G}_{\infty,N+1}$), it can be written as

$$\tilde{G}_{\infty,n} = \pi \left(\frac{L^{2N} x^{N+1}}{2^{2N+1} N!^2} \right)_{x=N+1-x_b}. \tag{31}$$

Thus $dW_n^\infty = \omega_0 D_0 I_0 \tilde{G}_{\infty,n} d\Omega_E$ is the rate of ionization events accompanied by electrons of energy $E=\hbar\omega_0(n-x_b)$ and of $\vec{k}$ lying in the solid angle $d\Omega_E$ around $\vec{\varepsilon}$. The differential ionization probability caused by an irradiation time of τ_{ir} is

$$\frac{dP_n^\infty}{d\Omega_E} = \tau_{ir} \frac{dW_n^\infty}{d\Omega_E} = \omega_0 \tau_{ir} D_0 I_0 \tilde{G}_{\infty,n}. \tag{32}$$

Thus irradiating an atomic electron on the K shell by a plane wave of τ_{ir} duration, $dP_n^\infty = \omega_0 \tau_{ir} D_0 I_0 \tilde{G}_{\infty,n} d\Omega_E$, is the probability of its ionization transition having the characteristics defined above.

V. RESULTS AND SUMMARY

A formula for the differential ionization probability of a short pulse in the "two-photon" energy interval, which is comparable with (32), can be obtained as

$$\frac{dP_n}{d\Omega_E} = D_0 I_0 \tilde{G}_n, \tag{33}$$

if we introduce the integrated quantity

$$\tilde{G}_n = \int_0^{x_m(n)} G(x, \phi = 0) dx. \tag{34}$$

Thus $dP_n = D_0 I_0 \tilde{G}_n d\Omega_E$ gives the ionization probability of ejecting electrons (with $\vec{k}$ as defined above) in the energy interval $0 < E < (n+1-x_b)\hbar\omega_0$, the upper limit being the threshold energy of the $(n+1)$-photon process in the infinite-pulse-length limit.

The ratio

$$r_n = \frac{dP_n}{dP_n^\infty} = \frac{\tilde{G}_n}{\omega_0 \tau_{ir} \tilde{G}_{\infty,n}} \tag{35}$$

characterizes well the essentially different effects of short and infinitely long pulses if we take $\omega_0 \tau_{ir} = T$. By fitting the results of numerical integration carried out at $T=1,2,3,4$ we have obtained $r_2 = 10^{\gamma - \delta T}$ with $\gamma = 8.83$ and $\delta = 1.56$ in the $1 \le T \le 4$ region while with $T \to \infty$ values of $r_2 \to$ const. It means that r_2 increases more than four orders of magnitude with the decrease of T from 4 to 1, while taking E_0 constant. The large increase of r_2 with decreasing pulse length can be qualitatively explained in the following manner.

$\tilde{G}_2$ and $\tilde{G}_{\infty,2}$ are integrated in the energy interval that is below the threshold energy of the three-photon process in the case of the infinite-pulse-length limit. As was mentioned above, for pulse lengths near the one-cycle case the bandwidth of the pulse $\Delta\omega \sim \omega_0$ and therefore events with energy $E > 2\hbar\omega_0 - |E_b|$ are also allowed by energy conservation. The narrowing of the bandwidth with increasing pulse length is manifested in the fact that increasing the length of the pulse from $T=1$, the spreading of $G(x, \phi=0)$ becomes smaller and smaller (see the $\phi=0$ cut of Figs. 1–3), and a peak arises around $x = 2 - x_b = 0.05$ with increasing T as is expected from the results of the $T \to \infty$ limit. In order to show how this peak is formed around $x=0.05$ we plot the electron energy distribution in the case of pulse length $T=300$ in Fig. 5.

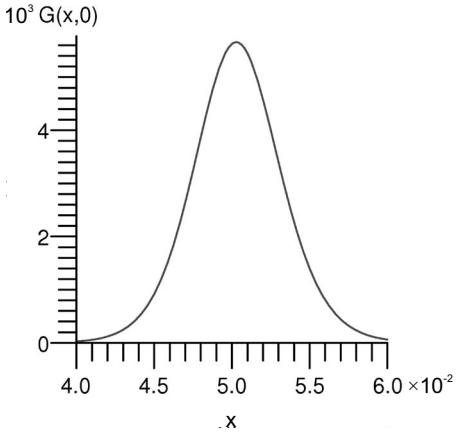

FIG. 5. The energy dependence of $G(x, \phi=0)$ in the "two-photon" ionization process ($x_b = 1.95$) in the $0.04 < x \le 0.06$ interval at pulse length $T=300$. For other notation, see Fig. 1.

In the plane-wave limit the differential ionization probability and the differential cross section $d\sigma_n^\infty / d\Omega_E$ of the n-photon process are related as

$$\frac{dP_n^\infty}{d\Omega_E} = \frac{d\sigma_n^\infty}{d\Omega_E} \Phi_0 \tau_{ir}, \tag{36}$$

where $\Phi_0 = I_0/(\hbar\omega_0)$ is the flux of photons of energy $\hbar\omega_0$ and I_0 is the intensity. From (32), (31), and (36) one can obtain the differential cross section of the n-photon channel in the infinite-pulse-length limit as

$$\frac{d\sigma_n^\infty}{d\Omega_E} = \sigma_0 \pi \left(\frac{L^{2N} x^{N+1}}{2^{2N+1} N!^2} \right)_{x=N+1-x_b}, \tag{37}$$

where $\sigma_0 = D_0 \hbar \omega_0^2$ and $n = N+1$.

As a numerical example, we discuss here the two-photon ionization of Be with $\hbar\omega_0 = 57.49$ eV, which corresponds to $x=0.05$. The K-shell binding energy is $|E_b| = 112.1$ eV, resulting in effective nuclear charge number $Z=2.87$. In this case $\sigma_0 = 1.65 \times 10^{-15}$ cm^2. Taking $L=0.01$, $x=0.05$, $x_b = 1.95$, $n=2$, and $N=1$ used in the numerical work we obtain $d\sigma_n^\infty / d\Omega_E = 1.63 \times 10^{-22}$ cm^2. Applying $L = 2.46 \times 10^{-11} \sqrt{I_0(\mathrm{W/cm^2})} \lambda_0^3(\mathrm{nm})$ and $\lambda_0(\mathrm{nm}) = 1.24/[\hbar\omega_0(\mathrm{keV})]$ where λ_0 is the carrier wavelength we get $I_0 = 1.65 \times 10^{13}$ W/cm^2 and $\Phi_0 = 1.79 \times 10^{30}$ cm^{-2} s^{-1}, which are much larger than the intensity and flux of the nowadays available uv and xuv sources of long pulses. The rate $W = (d\sigma_{n,E}^\infty / d\Omega_E) \Phi_0 (d\Omega_E) = 1.4 \times 10^6$ s^{-1}, taking $d\Omega_E = 4.8 \times 10^{-3}$, which corresponds to a $5^\circ \times 5^\circ$ solid angle. This rate produces $N_i = 1.4 \times 10^6 N_0 \tau_{ir}$ ionization events from a sample of N_0 number of Be atoms irradiated by the source of above characteristics and of τ_∞ time long.

In summary, the quantitative behavior of the double-differential probability of multiphoton ionization by a phase-controlled ultrashort pulse has been investigated in the per-

turbative regime. It is found that in two-photon ionization processes the pulse length plays a crucial, determining role; i.e., strongly increasing ionization efficiency has been found with decreasing pulse length. A moderate carrier-envelope phase dependence has been found in the one-cycle case only that seems to be in accordance with expectations based on a recent theoretical investigation [13]. Our results can be applied in different cases of xuv and soft x-ray two-photon ionization; e.g., in the case of Be the K-shell binding energy is 112.1 eV, which needs photon energy $\hbar\omega_0 \geq 56.05$ eV. The next (L_1) shell has a much lower binding energy (8 eV); therefore, its one-photon signal can be well separated in an experiment. Moreover, this situation is similar up to P when the K-shell binding energy is 2145.6 eV, which needs photon energy $\hbar\omega_0 \geq 1072.8$ eV, and the L_1 shell has binding energy 189.3 eV. As any advance in optical technology repeatedly opens new subfields of atomic physics, the results obtained in the present work may have applications in forthcoming experiments.

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