Magneto-optical force in a resonant field of elliptically polarized light waves

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In a one-dimensional model we consider the dependence of the magneto-optical force on the elliptical polarization of laser beams for different types of closed atomic transitions $J_g \to J_e$. The linear dependencies on atomic velocity and magnetic field of the magneto-optical force are found analytically. These results establish the possibility (for $J_g > 0$) to capture atoms in a magneto-optical trap even at zero detuning (i.e., at the exact resonance of atoms with laser field) in certain $(\epsilon-\theta-\bar{\epsilon})$ field configurations as well as in an anomalous blue-detuned region. The nonlinear dependencies of the force on atomic velocity and magnetic field are studied numerically. The capture velocity and the number of trapped atoms are estimated.

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I. INTRODUCTION

The mechanical action of resonant laser field on atoms is one of the most important subjects in modern atomic and laser physics. Nowadays magneto-optical traps (MOTs) of various design are the major source of cold atoms with temperatures in a range of hundreds to a few μK. A fortunate combination of the effective laser cooling with the deep magneto-optical potential (of the order of a few K), originating from the action of the resonant light pressure in the presence of a spatially nonuniform magnetic field, leads to the stable MOT operation with requirements of the setup parameters that are not too stringent (vacuum, magnetic field gradient, intensity, size of laser beams, etc.). Cold atomic samples prepared in a MOT are widely used in various branches of physical research, for example, in nonlinear high-resolution spectroscopy, in studies on cooling and dynamics of atoms in optical lattices, in Bose-Einstein condensation of atoms, and in the field of quantum metrology (atomic frequency standards).

MOT theory, as well as laser cooling and trapping theory as a whole [1–3], was basically developed for the circularly or linearly polarized laser beams commonly used in experiments. Recently, using a one-dimensional optical lattice as an example, we have shown that the atomic kinetics in a resonant field of elliptically polarized light waves has several qualitative differences from the cases of circular and linear polarizations [4,5]. For instance, the use of waves with elliptical polarization leads to unexpected features in the dependence of the light-induced force, acting on atoms, on the laser frequency (detuning). As a result, the laser cooling becomes possible at exact resonance and, moreover, in some anomalous blue-detuned regions [6]. Another interesting effect arising in elliptically polarized fields is the dipole force rectification in the absence of magnetic field [7].

In the present paper we investigate the polarization peculiarities of the magneto-optical force acting on atoms in a resonant field of elliptically polarized waves in the presence of a static magnetic field. The consideration is carried out in the standard one-dimensional model of MOT [8], but beyond the usual limitations on the light wave polarizations, which are elliptical in the general case. In the linear approximation with respect to atomic velocity and magnetic field we find the friction coefficient and the string constant of the linear trapping force (the curvature of the magneto-optical potential). The main attention is focused on conditions, at which the stable MOT operation is possible at the exact resonance of radiation with atomic transition. In this case the qualitative distinction of atomic kinetics related to the elliptical polarization is most pronounced. Apart from this, using numerical methods, we investigate the nonlinear dependence of the magneto-optical force on velocity and magnetic field. The number of trapped atoms is estimated in various regimes.

II. FORMULATION OF THE PROBLEM

We consider a one-dimensional model of the magneto-optical trap. An atom with optical transition $J_g \to J_e$ (where J_g and J_e are the total angular momenta in the ground and excited states, respectively) moves in a light field of elliptically polarized counterpropagating waves in the presence of a magnetic field. The wave vectors and magnetic field are parallel to the z axis. The Hamiltonian of the system under consideration can be written in the form

$$\hat{H} = \hat{H}_0 - \hat{\boldsymbol{\mu}} \cdot \mathbf{B} - \hat{\mathbf{d}} \cdot \mathbf{E}, \tag{1}$$

where $\hat{H}_0$ is the Hamiltonian of a free atom in the rotating frame (in the energy pseudospin space):

$$\hat{H}_0 = \frac{\hat{p}^2}{2M} - \hbar\delta\hat{\Pi}_e, \tag{2}$$

where $\delta=(\omega-\omega_0)$ is the detuning of the laser frequency from the atomic transition frequency ω_0, M is the mass of atom, and the projection operator

$$\hat{\Pi}_e = \sum_{\mu_e} |J_e,\mu_e\rangle\langle J_e,\mu_e| \tag{3}$$

is constructed from the Zeeman substates $|J_e,\mu_e\rangle$ of the excited level. The last two terms in Eq. (1) describe the interaction of atom with an external magnetic field $\mathbf{B}$ and with a resonant monochromatic laser field $\mathbf{E}(z,t)=\mathbf{E}(z)e^{-i\omega t}+\text{c.c.}$

The spatially inhomogeneous vector amplitude of the laser field is written as

$$\mathbf{E}(z) = E(z)\mathbf{e}(z)e^{i\Phi(z)}, \tag{4}$$

where $E(z)$ is the real-valued scalar amplitude, $\Phi(z)$ is the phase, and $\mathbf{e}(z)$ is the complex polarization vector. The vector $\mathbf{e}(z)$ is normalized, so that $[\mathbf{e}^*(z)\cdot\mathbf{e}(z)]=1$. In the spherical basis $[\mathbf{e}_0=\mathbf{e}_z;\mathbf{e}_{\pm1}=\mp(\mathbf{e}_x\pm i\mathbf{e}_y)/\sqrt{2}]$ the polarization vector is decomposed as

$$\mathbf{e}(z) = \mathbf{e}_{+1}\sin\left[\epsilon(z)+\frac{\pi}{4}\right]e^{-i\phi(z)} - \mathbf{e}_{-1}\cos\left[\epsilon(z)+\frac{\pi}{4}\right]e^{i\phi(z)},$$

where the angle $\phi(z)$ determines the orientation of the polarization ellipse in the xy plane; and the ellipticity angle $\epsilon(z)$ is defined so that $|\tan\epsilon|$ is equal to the ratio of the minor semiaxis to the major semiaxis and the sign of ϵ determines the helicity. In the dipole and rotating-wave approximations the operator of the resonant atom-field interaction has the form

$$\hat{V}(z) = \hbar\Omega(z)\sum_{q=0,\pm1}\hat{D}_q e^q(z) + \text{H.c.} \tag{5}$$

Here $\Omega=-dE/\hbar$ is the Rabi frequency, d is the reduced dipole matrix element, and $e^q(z)$ are the contravariant spherical components of the polarization vector. The operators $\hat{D}_q$ are expressed in terms of the Clebsch-Gordan coefficients

$$\hat{D}_q = \sum_{\mu_e,\mu_g}|J_e,\mu_e\rangle C^{J_e\mu_e}_{J_g\mu_g,1q}\langle J_g,\mu_g|. \tag{6}$$

The magnetic Hamiltonian, describing the linear Zeeman splitting of the magnetic sublevels, can be written in the following form:

$$\hat{H}_B = \hbar\Omega_Z\left(\hat{\mathbf{J}}_g + \frac{g^{(e)}}{g^{(g)}}\hat{\mathbf{J}}_e\right)\cdot\mathbf{b}, \tag{7}$$

where $\hat{\mathbf{J}}_g$ and $\hat{\mathbf{J}}_e$ are the total angular momentum operators of the ground and exited states, Ω_Z is the Zeeman splitting of the ground state, $g^{(e,g)}$ are the Lande factors of the ground and excited states, and $\mathbf{b}=\mathbf{B}/B$.

In the steady-state conditions the force acting on an atom is the quantum-mechanical average of the force operator (see, e.g., Ref. [1]):

$$F(z,v) = \text{Tr}[\hat{F}(z)\hat{\rho}(z,v)], \quad \hat{F}(z) = -\partial_z\hat{V}(z). \tag{8}$$

In the semiclassical approximation [1,2] the atomic density matrix in the Wigner representation $\hat{\rho}(z,v)$ obeys the generalized optical Bloch equations [6]

$$v\partial_z\hat{\rho}(z,v) = -\frac{i}{\hbar}[\hat{V}(z)+\hat{H}_B,\hat{\rho}(z,v)] - \left[\left(\frac{\gamma}{2}-i\delta\right)\hat{\Pi}_e\hat{\rho}(z,v) + \left(\frac{\gamma}{2}+i\delta\right)\hat{\rho}(z,v)\hat{\Pi}_e\right] + \gamma\sum_q\hat{D}_q^\dagger\hat{\rho}(z,v)\hat{D}_q, \tag{9}$$

where γ is the radiation relaxation rate.

III. LINEAR THEORY OF MOT

The solution of the Eq. (9) can be analytically found for a field with intensity and polarization gradients only in various limiting situations. As applied to the laser cooling and trapping in MOT, of certain interest is an analysis of slow atoms and weak magnetic fields, i.e., the case when the corresponding Doppler shift and Zeeman splitting are much less than the relaxation rate on the internal degrees of freedom. To first order in these terms, the force reduces to

$$F(z,v) \approx F^{(0)}(z) + v\xi(z) + \Omega_Z f(z), \tag{10}$$

where $F^{(0)}(z)$ is the light pressure force on an atom at rest and at zero magnetic field, the second term is the friction force, and the third term is the linear trapping magneto-optical force. Usually the zero-order force vanishes in the average over the spatial period. The friction and linear magneto-optical forces have nonzero averages, leading to the formation of the MOT potential and to the cooling and trapping of atoms. Further, in the usual way, a weak (with respect to the wavelength) linear spatial dependence of the Zeeman splitting is assumed: $\Omega_Z=\alpha z$. Then $\kappa=\alpha\langle f(z)\rangle_z$ is the string constant or the curvature of the potential near the minimum point $z=0$.

In the general case the coefficients f and ξ can be decomposed in the spatial gradients of the local field parameters [5,6]

$$f = \hbar\sum_k\psi_k g^{(k)}, \quad \xi = \hbar\sum_{kk'}\chi_{kk'}g^{(k)}g^{(k')},$$

where $g^{(k)}$ are defined as follows:

$$g^{(1)} = \partial_z\ln E, \quad g^{(2)} = \partial_z\Phi, \quad g^{(3)} = \partial_z\epsilon, \quad g^{(4)} = \partial_z\phi.$$

Thus, the problem reduces to an analysis of the coefficients $\chi_{kk'}$ and ψ_k, which depend on the intensity and polarization of the local field, and on the type of optical transition.

IV. ANALYTICAL RESULTS

For several optical transitions explicit analytical expressions for the coefficients $\chi_{kk'}$ have been obtained in Refs. [5,6]. The same method can be applied to find the coefficients ψ_k. As an example, omitting cumbersome intermediate calculations, we write the final result for $J_g=1/2\to J_e=1/2$ optical transition

$$\psi_1 = -\left(1+\frac{g^{(e)}}{g^{(g)}}\right)\frac{S_\epsilon\sin(2\epsilon)}{(1+S_\epsilon)^2}\frac{\delta^2}{(\gamma^2/4+\delta^2)},$$

$$\psi_2 = \frac{\left(1+\frac{g^{(e)}}{g^{(g)}}\right)}{2}\frac{S_\epsilon\sin(2\epsilon)}{(1+S_\epsilon)^2}\frac{\delta\gamma}{(\gamma^2/4+\delta^2)},$$

$$\psi_3 = -\frac{\left(1+\frac{g^{(e)}}{g^{(g)}}\right)}{2\cos(2\epsilon)}\frac{S_\epsilon}{(1+S_\epsilon)^2}\times\frac{[(1+S_\epsilon)\gamma^2/4+\delta^2(S_\epsilon-1)+2\delta^2\cos^2(2\epsilon)]}{(\gamma^2/4+\delta^2)},$$

$$\psi_4 = 0, \tag{11}$$

where

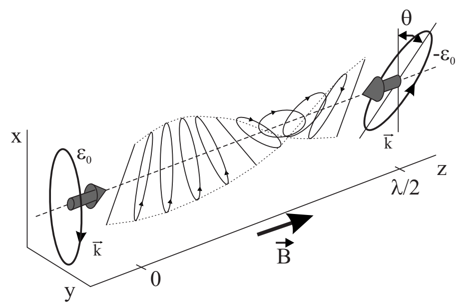

FIG. 1. The $\epsilon-\theta-\bar{\epsilon}$ configuration of light field formed by two counterpropagating waves with elliptical polarizations. Their ellipticity angles are ϵ_0 and $-\epsilon_0$. The magnetic field direction is parallel to the z axis.

$$S_\epsilon = \frac{2}{3}\frac{\cos^2(2\epsilon)\Omega^2}{(\gamma^2/4+\delta^2)}$$

is the effective saturation parameter.

In a one-dimensional field of counterpropagating waves with elliptical polarizations all the field gradients $g^{(k)}$ differ from zero. A simple but nontrivial example of such a field is the $\epsilon-\theta-\bar{\epsilon}$ configuration (see Fig. 1) produced by counterpropagating waves with equal intensities and opposite ellipticity angles ϵ_0 and $-\epsilon_0$. The major axes of the polarization ellipses are oriented at an angle θ to each other. As is seen, the terms ψ_1 and ψ_3 in Eq. (11) are even in the detuning. In particular, at the exact resonance $\delta=0$ the coefficient ψ_3 originating from the ellipticity gradient does not vanish.

In the general case the friction and trapping magneto-optical forces contain terms with both odd and even dependencies on the detuning. These contributions do not vanish in the average over the spatial period. This circumstance is principal for the cooling and trapping of atoms in MOT even in the case of exact resonance of atoms with radiation.

Here we consider, as an example, atoms with optical transition $J_g=1/2 \to J_e=1/2$ in a field of the $\epsilon-\theta-\bar{\epsilon}$ configuration. In the low saturation limit $S \ll 1$ it is possible to average the magneto-optical force over the spatial coordinate z in an analytical form. In the case of exact resonance the result reads

$$\langle f \rangle_z = \hbar k \frac{2S_0}{3}\left(1+\frac{g^{(e)}}{g^{(g)}}\right)\tan(\theta)\left[\sqrt{1-\cos^2(2\epsilon_0)\cos^2(\theta)}-1\right], \tag{12}$$

where S_0 is the saturation parameter per single light beam. The expression (12) shows that the magneto-optical force at the exact resonance is an even function of the ellipticity angle ϵ_0 and an odd function of the angle θ. The corresponding friction coefficient has been found in Ref. [5]:

$$\langle \xi \rangle_z = \hbar k^2 \frac{3}{8}\frac{\sin(2\theta)\cos(2\epsilon_0)\sin(4\epsilon_0)}{[1-\cos^2(2\epsilon_0)\cos^2(\theta)]^{3/2}}. \tag{13}$$

It has an odd dependence on the ellipticity angle ϵ_0 as well as on the angle θ. Let the Lande factors of the excited and ground states and the magnetic field gradient be positive $\partial_z\Omega_Z > 0$. Then at the exact resonance the stable trapping of atoms takes place for $-\pi/2 < \theta < 0$ and $0 < \epsilon_0 < \pi/4$.

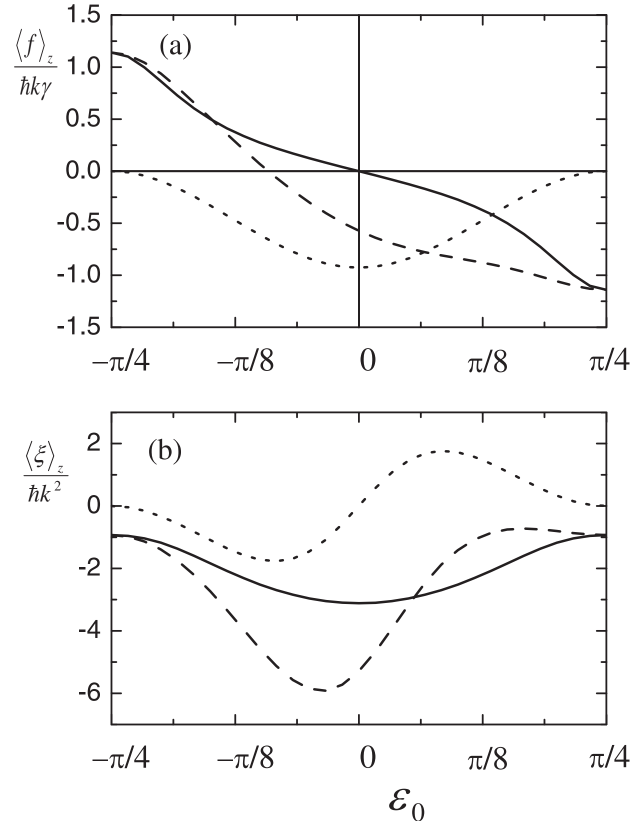

FIG. 2. The linear magneto-optical force (a) and friction coefficient (b) as functions of the ellipticity angle ϵ_0 in the $\epsilon-\theta-\bar{\epsilon}$ field configuration for atoms with $J_g=1 \to J_e=2$ optical transition. Solid lines correspond to $\delta=-0.5\gamma$ and $\theta=\pi/2$, dashed lines to $\delta=-0.5\gamma$ and $\theta=-\pi/4$, and dotted lines to $\delta=0$ and $\theta=-\pi/4$. The single-beam Rabi frequency is $\Omega=\gamma$.

V. NUMERICAL RESULTS

For optical transitions with the total angular momenta $J_{e,g} > 1/2$ similar results can be obtained numerically by solving a set of linear algebraic equations for elements of the first-order correction matrix introduced in Ref. [5]. For atoms with optical transition $J_g=1 \to J_e=2$ the linear magneto-optical force and friction coefficient as functions of the ellipticity angle ϵ_0 in the $\epsilon-\theta-\bar{\epsilon}$ field configuration are shown in Fig. 2 for negative and zero detunings, and for some particular values of the angle θ. The Lande factors are taken to be equal $g^{(g)}=1/2$ and $g^{(e)}=2/3$. Note that in the case of orthogonal ellipses $\theta=\pi/2$ the magneto-optical force is odd, while the friction coefficient is even functions of the ellipticity angle ϵ_0. Here the point $\epsilon_0=0$ corresponds to the so-called lin$\perp$lin field configuration, where the magneto-optical force vanishes. If the angle θ differs from zero and $\pi/2$, additional contributions to the force and friction even in the detuning appear. For the friction coefficient these terms are odd functions of the angle θ and ellipticity ϵ_0, and they have been discussed in Refs. [4,5]. Additional terms in the magneto-

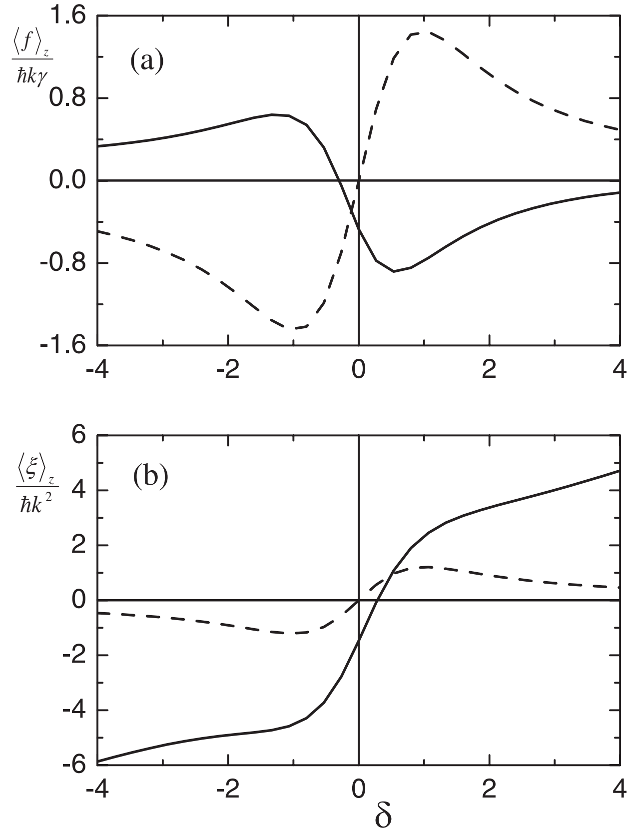

FIG. 3. The linear magneto-optical force (a) and friction coefficient (b) as functions of the detuning for atoms with $J_g=1\rightarrow J_e=2$ optical transition. Dashed lines correspond to the standard $\sigma_+-\sigma_-$ field configuration, and solid lines correspond to the $\epsilon-\theta-\bar{\epsilon}$ field configuration with $\epsilon_0=-\pi/8$ and $\theta=-\pi/4$. The single-beam Rabi frequency $\Omega=\gamma$.

optical force have the same nature, i.e., they originate from the imbalance of the light pressure forces caused by optical pumping of atoms in an elliptically polarized field. These terms are even in the ellipticity and odd in the angle θ [see, for example, Eq. (12)].

The linear magneto-optical force and friction coefficient are shown in Fig. 3 as functions of the detuning for the two particular $\epsilon-\theta-\bar{\epsilon}$ configurations: $\{\epsilon_0=-\pi/8,\theta=-\pi/4\}$ and $\{\epsilon_0=\pi/4,\theta=0\}$ (i.e., $\sigma_+-\sigma_-$ field configuration). As is seen, for both the magneto-optical force and friction coefficient are negative at $\delta=0$, what is necessary for the stable MOT operation at positive magnetic field gradients $\partial_z\Omega_Z>0$. For reference, the results for the standard $\sigma_+-\sigma_-$ configuration are shown in the same figure. From this comparison one can see that the string and friction coefficients for the both particular configurations are of the same order. It is important to note that for the stable MOT operation in the two different regimes (at $\delta=0$ and at $\delta<0$) we need in the opposite helicities (i.e., opposite signs of ϵ_0).

VI. NONLINEAR THEORY OF MOT

In order to estimate the number N_c of atoms trapped in a MOT we have to know the character capture velocity v_c (v_c is the maximum velocity an atom can have and be captured). As is well known, $N_c\propto v_c^4$ [9]. To find v_c one has to go beyond the framework of linear MOT theory. We develop a numerical method of calculation of the spatially average force $\langle F(v,\Omega_Z)\rangle_z$ on atoms with a full nonlinear dependence on velocity and magnetic field. Our approach is based on the numerical solution of the Eq. (9) by expanding the Wigner density matrix into Fourier series on the spatial coordinate. Then the continued fraction method [10] is used to find the density matrix spatial harmonics.

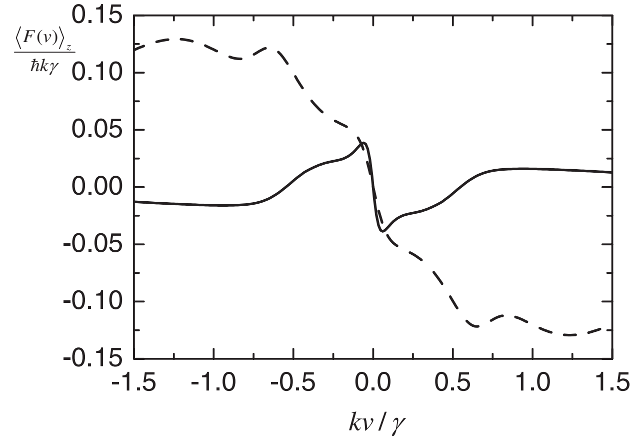

FIG. 4. The magneto-optical force as a function of velocity for atoms with closed optical transition $J_g=1\rightarrow J_e=2$ in a field of the $\epsilon-\theta-\bar{\epsilon}$ configuration. Solid line corresponds to $\epsilon_0=-\pi/8$, $\theta=-\pi/4$ and zero detuning $\delta=0$. Dashed line corresponds to the standard $\sigma_+-\sigma_-$ field configuration and $\delta=-0.5\gamma$. The single-beam Rabi frequency $\Omega=\gamma$.

At zero detuning $\delta=0$ typical nonlinear dependencies of $\langle F(v,\Omega_Z=0)\rangle_z$ and $\langle F(v=0,\Omega_Z)\rangle_z$ are shown in Figs. 4 and 5 for the closed atomic transition $J_g=1\rightarrow J_e=2$ and the Lande factors $g^{(g)}=1/2$ and $g^{(e)}=3/2$ in the $\epsilon-\theta-\bar{\epsilon}$ configuration

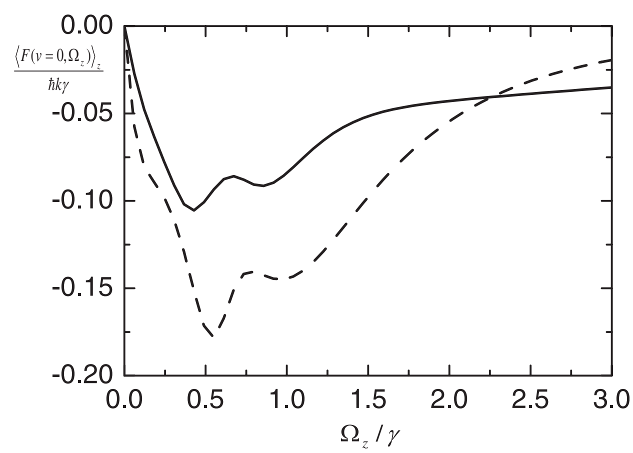

FIG. 5. The magneto-optical force as a function of magnetic field for atoms with closed optical transition $J_g=1\rightarrow J_e=2$ in a field of the $\epsilon-\theta-\bar{\epsilon}$ configuration. Solid line corresponds to $\epsilon_0=-\pi/8$, $\theta=-\pi/4$ and zero detuning $\delta=0$. Dashed line corresponds to the standard $\sigma_+-\sigma_-$ field configuration and $\delta=-0.5\gamma$. The single-beam Rabi frequency $\Omega=\gamma$.

with $\epsilon_0=-\pi/8$, $\theta=-\pi/4$ and the single-beam Rabi frequency $\Omega=\gamma$. For reference, the results for the standard $\sigma_+-\sigma_-$ configuration and $\delta=-0.5\gamma$ are shown in the same figure. As is seen from these figures, the velocity range where the friction is effective in the case $\delta=0$ is much less than in the standard $\sigma_+-\sigma_-$ configuration at $\delta=-0.5\gamma$, while the depths of magneto-optical potential are comparable in the two cases under consideration. This should lead to the significantly lower values of v_c and N_c in the case of our primary interest $\delta=0$.

Indeed, the numerical solution of the equation of motion with the full nonlinear dependence $\langle F(v,\Omega_Z)\rangle_z$ shows that for $\delta=0$ the capture velocity v_c is well determined by the point v, where the force equals to zero at the entrance into the trap. In the other words, v_c is the solution of the algebraic equation $\langle F(v_c,\Omega_Z)\rangle_z=0$, where $\Omega_Z=-\alpha L$ is the Zeeman splitting at the entrance and $2L$ is the trap size determined by the laser beam diameter. In intense laser fields, for example, at $\Omega \geq \gamma$ that corresponds to the light field intensity $\mathcal{I} \geq 12$ mW/cm^2 (hereafter we assume the atomic parameters of the D_2 line of ^{87}Rb), the capture velocity v_c at $\delta=0$ can be as large as $\sim\gamma/k\simeq 4.5$ m/s. The number of trapped atoms in a MOT can be estimated by Eq. (1) from Ref. [9]. For the obtained value of v_c, assuming $L=1$ cm, we get $N_c\simeq 2\times 10^5$. Thus, the number of trapped atoms at $\delta=0$ is in $2-3$ orders less than for a standard MOT. However, even such a small number of trapped atoms can be easily detected by the standard resonance fluorescence techniques. At the same time, the optimum Zeeman splitting at the entrance into the trap $\alpha L\simeq 0.01\gamma$ corresponds to the magnetic field gradient $\partial_z B\simeq 0.1$ G/cm, which is in $1-2$ orders less than the usually used gradients (~ 10 G/cm) [11].

VII. CONCLUSION

Concluding, we briefly summarize the main results. In a 1D model of MOT we have analyzed the dependence of the magneto-optical force on the elliptical polarization of light waves forming the MOT field. In the linear approximation on atomic velocity and magnetic field the analytical expressions for the string and friction coefficients have been obtained. An analysis of these analytical expression allows us to make a conclusion about a principal possibility (for $J_g>0$) of the stable MOT operation at the exact resonance of atoms with radiation ($\delta=0$) in the case of special $\boldsymbol{\epsilon}-\theta-\bar{\boldsymbol{\epsilon}}$ light field configuration. The nonlinear dependence of magneto-optical force on velocity and magnetic field has been studied by numerical methods. The estimates of the capture velocity and the number of trapped atoms have been obtained. The comparison of the two regimes of the MOT operation (in the $\boldsymbol{\epsilon}-\theta-\bar{\boldsymbol{\epsilon}}$ field configuration at $\delta=0$ versus the standard $\sigma_+-\sigma_-$ field at $\delta<0$) has been done. The following differences have been observed: (1) for the stable MOT operation in the case $\delta=0$ the opposite (with respect to the $\sigma_+-\sigma_-$ case) helicity of the polarization field configuration is required, (2) the maximum number of trapped atoms in a standard MOT is in $2-3$ orders greater than for the $\boldsymbol{\epsilon}-\theta-\bar{\boldsymbol{\epsilon}}$ case at $\delta=0$, (3) the optimum magnetic field gradient, corresponding to the maximum of trapped atom, is in $1-2$ orders greater in a standard MOT.

ACKNOWLEDGMENTS

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