Ratchet effect in an optical lattice with biharmonic driving: A numerical analysis

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We analyze numerically a rocking ratchet for cold atoms. The ratchet setup consists of a spatially symmetric dissipative optical lattice, and a biharmonic driving force. This setup corresponds to recent experimental realizations. We investigate the dependence of the features of the current generation on the scattering rate, which plays the role of the noise strength. Such a dependence is then used to investigate the origin of current reversals and of the rectification of fluctuations in a cold atom ratchet with broken Hamiltonian time symmetry.

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I. INTRODUCTION

The ratchet effect [1–20] is the rectification of fluctuations in a system out of thermodynamic equilibrium in the absence of net bias forces. This phenomenon is very general, and applies to many different systems. Correspondingly, a large variety of ratchet setups have been proposed, and partially experimentally demonstrated [21–36], as reviewed in Refs. [20].

All ratchet mechanisms can be traced back to two fundamental requirements: the system has to be out of equilibrium, so to overcome the limitations imposed by the second principle of thermodynamics, and the relevant symmetries of the system, which would otherwise inhibit current generation, have to be broken [15,17–19].

The typical ratchet setup corresponds to Brownian particles in a periodic potential. The system is driven out of equilibrium by either pulsing the potential ("flashing ratchet") [1,6] or by applying a time-dependent force with zero average ("rocking ratchet") [2–5]. The symmetry of the system can be broken by choosing a spatially asymmetric potential [2–6,10,12,14], or a temporally asymmetric driving force [7–9,11,13].

Quite recently, an extensive experimental investigation of rocking ratchets was carried out using cold atoms in near-resonant optical lattices [28–34]. In this system, the laser fields generate both a periodic potential for the atoms, and a damping force [37]. The system is driven out of equilibrium by applying a zero-mean driving force, which also controls the time symmetry of the system, and therefore directed motion.

In this work we present a detailed numerical analysis of the cold atom ratchet with biharmonic driving, thus extending previous numerical analysis done in parallel with the experimental work [28–33]. The analysis is carried out using semiclassical Monte Carlo simulations for the simple case of a one dimensional (1D) optical lattice, and a $J_g=1/2\rightarrow J_e=3/2$ atomic transition. The specific aim of the present work is to investigate the dependence of the features of the current generation on the scattering rate, which plays the role of the level of noise. Such a dependence will then be used to investigate the origin of current reversals and of the rectification of fluctuations in a cold atom ratchet with broken Hamiltonian time symmetry.

This paper is organized as follows. In Sec. II we describe the model. In Sec. III the results of our semiclassical Monte Carlo simulations are presented. In Sec. IV conclusions are drawn.

II. MODEL

A. Ratchet setup

In the present work we consider the simple case of a $J_g=1/2\rightarrow J_e=3/2$ atomic transition and a 1D optical lattice. The optical potential is generated by two counterpropagating laser fields, with orthogonal linear polarizations. This is the so-called lin $\perp$ lin configuration [37].

We notice that this configuration does not correspond exactly to the one considered in recent experimental realizations of cold atom ratchets [30–33], which involved atomic transitions with larger angular momentum and three-dimensional optical lattices. However, the case considered in this work, which has the advantage of being computationally easy to treat, corresponds to the simplest configuration in which Sisyphus cooling takes place, and therefore is expected to reproduce all essential features of the experimental realizations.

In the above described lin $\perp$ lin configuration, the interference of the laser fields creates a bipotential $U_{\pm}(z)$,

$$U_{\pm}=\frac{U_0}{2}[-2\pm\cos(2kz)] \tag{1}$$

for the atoms. More precisely, an atom in the ground state sublevel $|\pm\rangle=|J_g=1/2,M_g=\pm1/2\rangle$ will experience the potential $U_{\pm}$. Here U_0 is the potential depth, z is the coordinate along the light propagation direction, and k is the laser field wave vector.

The interaction with the light also produces dissipative couplings. The stochastic process of optical pumping between the two ground state sublevels leads to a friction force, the so-called Sisyphus cooling mechanism [37], and a fluctuating force. The strength of dissipation can be characterized by the scattering rate Γ'. This quantity can also be considered to quantify the level of noise in the system. We notice here that in experiments both U_0 and Γ' can be varied independently by changing appropriately the lattice beams detuning from atomic resonance and their intensity.

To investigate the ratchet effect we introduce a biharmonic drive of the form

$$F(t) = F_0[A_d \cos \omega_d t + B_d \cos(2\omega_d t + \phi)], \tag{2}$$

which plays a double role: it drives the system out of equilibrium, and controls the time symmetry of the system. This is in complete analogy with previous experimental work [30–33], where a driving of the form (2) was applied by phase modulating one of the lattice beams.

B. Symmetry analysis

We recall here the essence of the symmetry analysis which determines under which conditions directed motion in a spatially symmetric potential is possible [15,17]. This will be needed to interpret the numerical results. The two relevant symmetries are

$$\hat{S}_a: \quad (x,p,t) \rightarrow (-x,-p,t+T/2), \tag{3}$$

$$\hat{S}_b: \quad (x,p,t) \rightarrow (x,-p,-t). \tag{4}$$

Here T is the period of the driving. The invariance of the system under any of these two transformations prevents the generation of a current. In the Hamiltonian limit, the symmetries are controlled by the driving force $F(t)$. The driving is said to have F_s symmetry if, after some appropriate shift,

$$F(t) = F(-t+\tau). \tag{5}$$

We say that F has F_{sh} symmetry if

$$F(t) = -F(t+T/2). \tag{6}$$

In our case of biharmonic driving, as given by Eq. (2), the symmetry F_{sh} is broken independently of the value of the phase ϕ. This is because the driving consists of two harmonics with different parity. The transport is then controlled by the symmetry F_s, which is preserved for $\phi = n\pi$, with n integer, and broken otherwise.

III. NUMERICAL RESULTS

A. Atomic current vs relative phase of the driving field harmonics

The numerical work consists of studying the atomic current as a function of the phase difference ϕ between the two harmonics, for different choices of the scattering rate Γ', and the force amplitude F_0. This provides a clear insight on the key features of the ratchet, the role of the symmetries, and the current reversals. The present theoretical analysis corresponds closely to the experimental work presented in Refs. [28,31].

Our numerical analysis is based on semiclassical Monte Carlo simulations [37] for the atomic dynamics in the optical lattice in the presence of a driving of the form of Eq. (2). By averaging over the different atomic trajectories, it is then possible to determine the average atomic momentum for different choices of the parameters of the optical lattice and of the driving.

Figure 1(a) presents results for the average atomic velocity as a function of the phase ϕ for different values of the scattering rate Γ'. For the smallest scattering rate considered,

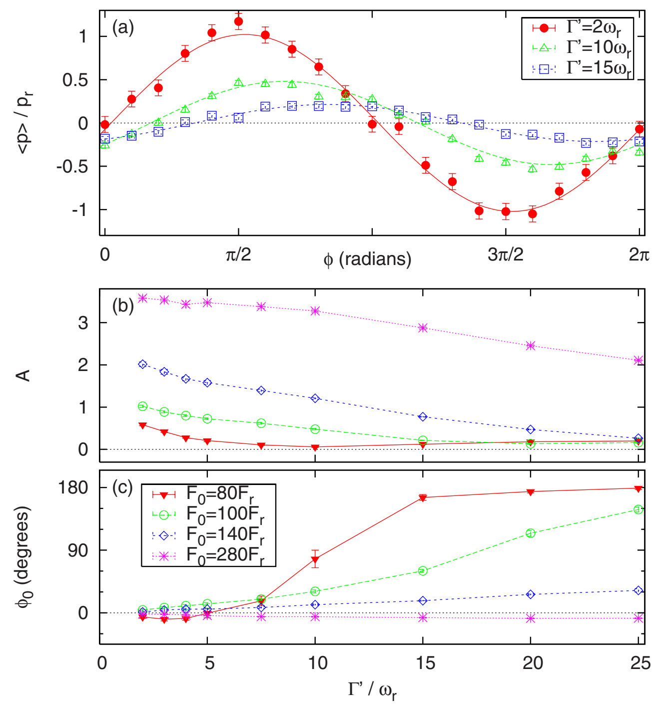

FIG. 1. (Color online) Results of semiclassical Monte Carlo simulations. In (a) we plot the average current versus ϕ for three different values of Γ' with $F_0 = 100F_r$, where $F_r = \hbar k \omega_r$. The curves represent the best fits of the function $\langle p \rangle / p_r = A \sin(\phi - \phi_0)$. In (b) and (c) we present the values of the fit parameters A and ϕ_0 for varying Γ' at three different F_0. The calculations were performed with the following parameters: $U_0 = 200\hbar\omega_r, \omega_d/\omega_v = 1, A_d = B_d = 1$ with a sample size of 10^4 atoms; ω_v is the vibrational frequency at the bottom of the potential wells.

the atomic current is well described by $I \sim \sin \phi$. This shows that, in the limit of small dissipation, the transport of atoms in the driven lattice is governed by the symmetries which hold in the Hamiltonian limit, with no current generated for $\phi = n\pi$ (n integer).

We now consider the current generation for larger values of the scattering rate. Figure 1(a) shows that at larger values of the scattering rate the current as a function of ϕ acquires a phase shift ϕ_0, i.e., the average atomic momentum shows a dependence on ϕ of the form

$$\frac{\langle p \rangle}{p_r} = A \sin(\phi - \phi_0), \tag{7}$$

with p_r the atomic momentum recoil.

By fitting data as those in Fig. 1(a) with the function Eq. (7) we derived the fitting amplitude A and phase lag ϕ_0 as a function of the scattering rate, for different values of the amplitude of the applied force, with results in Figs. 1(b) and 1(c), respectively. It appears that ϕ_0 is essentially zero at the lowest considered values of the scattering rate, and then increases to nonzero values for increasing scattering rate. This can be interpreted as dissipation-induced symmetry breaking. In fact, dissipation breaks the invariance under time-reversal transformation $\hat{S}_b$ [Eq. (4)]. Therefore, even for a symmetric

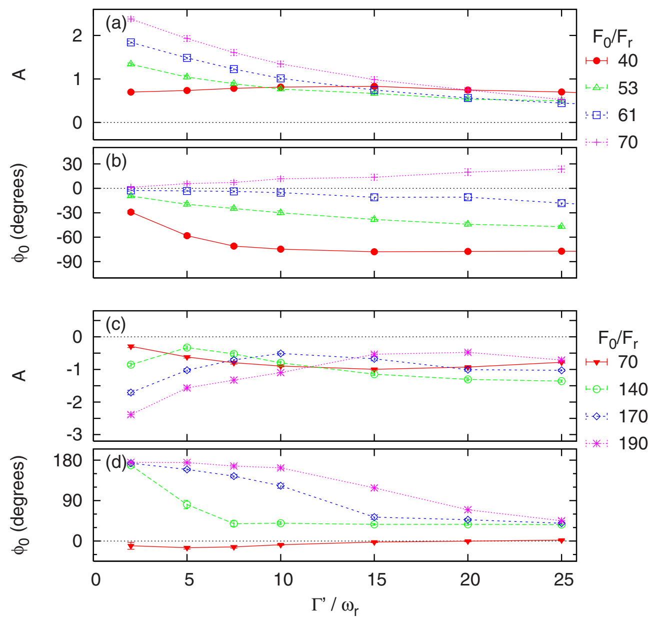

FIG. 2. (Color online) Values of the fit parameters A and ϕ_0 (see the caption of Fig. 1) for simulations with driving force parameters $A_d=1, B_d=4$. In (a) and (b) $U_0=100\hbar\omega_r, \omega_d/\omega_v=0.75$. In (c) and (d) $U_0=200\hbar\omega_r, \omega_d/\omega_v=1$, as for the results in Fig. 1. The lines are guides for the eye.

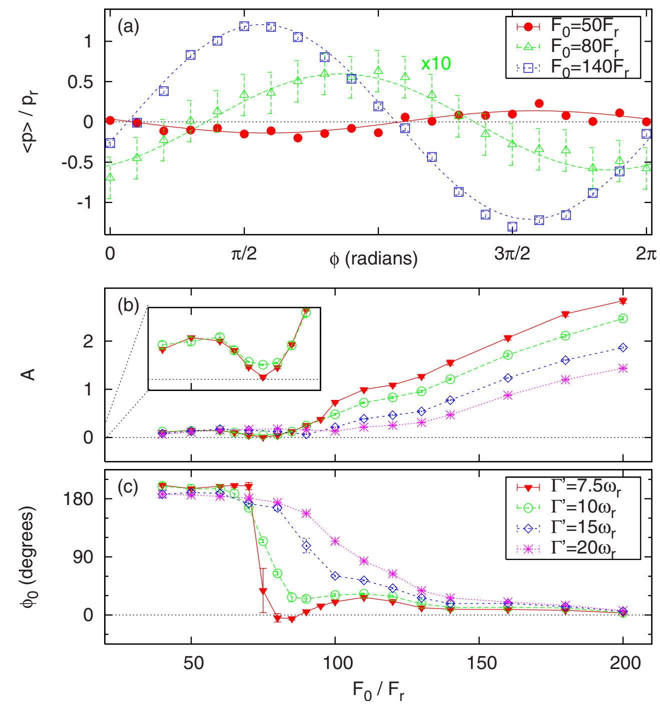

FIG. 3. (Color online) (a) Plot of the average current versus ϕ for three different values of the driving force amplitude F_0 with $\Gamma'=10\omega_r$. (b) and (c) Values of the fit parameters A and ϕ_0 for varying F_0 at three different Γ'; other parameters for the calculations as in Fig. 1. In (a) the lines are the best fits with the function Eq. (7); in (b) and (c) the lines are guides for the eye.

driving, i.e., for $\phi=n\pi$, a current can be generated. This phenomenon was experimentally observed in Ref. [31].

We now examine the dependence of the current amplitude A on the scattering rate Γ', as shown in Fig. 1(b). We also refer to Fig. 2, obtained for a different choice of the driving parameters A_d and B_d, and for two different values of the optical potential depth. It appears that the current amplitude A shows a complicated dependence on the scattering rate Γ', and such a dependence is sensitive to the driving strength. For strong drivings, the amplitude A is found to essentially decrease, in absolute value, for increasing scattering rate. Hence, in this case the noise does not appear to play any constructive role as the magnitude of the current decreases for increasing level of noise. Therefore the generation of a current can be traced back to dynamical harmonic mixing [38], with the noise acting as a disturbance and therefore decreasing the magnitude of the current. However, such a behavior is not completely general, and indeed for weak drivings a different dependence is observed. In fact, for the lowest value of the force amplitude considered, the current amplitude A, in absolute value, shows an initial increase for increasing scattering rate, i.e., the noise plays here a constructive role, as an increase in noise level leads to an increase of the current of atoms.

We also notice that the current amplitude exhibits in some cases a local minimum as a function of the scattering rate, as evidenced by the curve for $F_0=80F_r$ in Fig. 1(b), and the ones for $F_0=140$, 170, $190F_r$ in Fig. 2(c). Such a minimum occurs for a value of Γ' such that the phase lag ϕ_0 is equal to $\pi/2$. The existence of such a minimum can be understood as follows. For a phase lag ϕ_0 equal to $\pi/2$, the current maximum, which is also the amplitude of the sinlike curve representing the current as a function of ϕ, is obtained at $\phi=n\pi$, with n integer. But for this value of the phase ϕ the driving is symmetric under time reversal [see Eq. (5)]. Therefore the symmetry breaking, and the current generation, is entirely produced by the dissipation. In contrast, for phase lags ϕ_0 different from $\pi/2$ the driving breaks the time-reversal symmetry at the value $\phi=\phi_0+\pi/2+n\pi$ corresponding to maximum current. A larger current is expected in this case as both driving and dissipation contribute to the symmetry breaking. This explains the local minimum in the amplitude A observed for a scattering rate Γ' corresponding to a phase lag $\phi_0=\pi/2$. We notice that a local minimum in the current amplitude for a scattering rate Γ' giving rise to a phase shift $\phi_0\simeq\pi/2$ was observed experimentally [31,39].

The ratchet current is very sensitive not only to the value of the scattering rate, but also to the amplitude F_0 of the driving force. Figure 3(a) presents our numerical results for the average atomic momentum as a function of ϕ for different values of the amplitude of the driving force. Besides the aforementioned local minima in correspondence of a phase shift $\phi_0=\pi/2$, as evidenced in the inset, the amplitude A is found to be increasing with increasing strength of the driving. The phase shift ϕ is approximately zero, modulus π, for very weak and very strong drivings and it differs substantially from zero for intermediate forces, as shown in Fig. 3(b).

B. Current reversals in systems with broken Hamiltonian symmetry

In the previous section we discussed the dependence on the ratchet parameters of the amplitude A and phase lag ϕ_0 of the sinlike curve representing the current as a function of ϕ. Here we show that the obtained results allow us to give a clear insight into the origin of the current reversals obtained in ratchets with broken Hamiltonian symmetry, and we refer, in particular, to our experimental work of Ref. [29]. In that work, the phase ϕ was kept fixed at $\phi=\pi/2$, so to break the time-reversal symmetry of the Hamiltonian, and current reversals were observed as a function of the amplitude of the driving. We aim now to investigate the relationship between these current reversals, observed at $\phi=\pi/2$, and the dependence of the amplitude A and phase shift ϕ_0 of the sinlike curve of the current on the driving parameters. Two different scenarios are, in principle, possible. In the first one, the amplitude of the sinlike curve decreases to zero and then changes sign, i.e., the sinlike curve becomes completely flat and then changes sign. In the second scenario, the amplitude of the sinlike curve of the atomic current does not decrease to zero, and it is a variation in the phase shift ϕ_0 of the sinlike curve which determines the current reversal at $\phi=\pi/2$. Our numerical simulations will determine which scenario is actually behind the current reversals observed at $\phi=\pi/2$.

Figure 4 shows the dependence of the ratchet current as a function of ϕ for different choices of, in (a), the driving strength and, in (b), the scattering rate. These data will be used to clarify the relationship between the current reversals observed at $\phi=\pi/2$, and the dependence on the driving parameters of the amplitude and phase shift of the sinlike curve representing the ratchet current as a function of the phase shift ϕ.

Consider first the situation analyzed in Fig. 4(a) where the different curves correspond to different amplitudes of the driving, for a given scattering rate. The range of driving parameters considered there corresponds to the inset of Fig. 3(b). A current reversal as a function of the driving strength is observed for $\phi=\pi/2$. The current reversal results from both the dependence of the amplitude A and the phase shift ϕ_0 on the driving strength. In fact, the amplitude A is reduced for increasing F_0, then reaches a minimum, which can in some cases be essentially zero [e.g., curve in Fig. 3(b) for $\Gamma'=7.5\omega_r$], and increases again. In the same range of driving strengths, the phase shift ϕ_0 varies from an essentially zero value to π. In this way an increase in the force produces a global change of sign of the sinlike curve of the atomic current as a function of ϕ, as shown by the curves with $F_0=60F_r$ and $F_0=90F_r$ in Fig. 4(a).

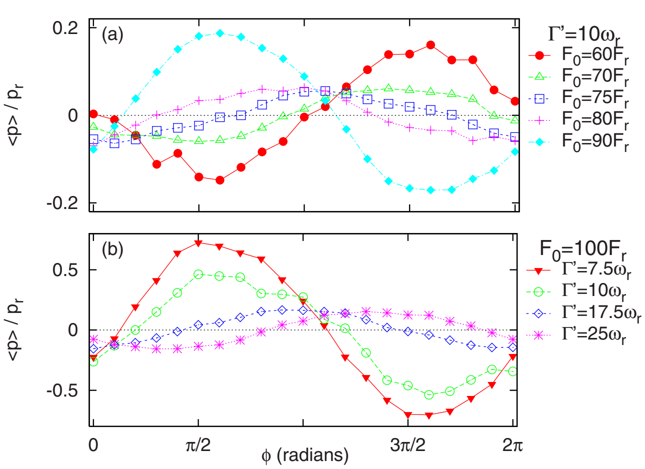

FIG. 4. (Color online) Current reversal by (a) varying F_0 for fixed Γ' (top graph), and (b) varying Γ' for fixed F_0 (lower graph). Other calculation parameters as in Fig. 3. The lines are guides for the eye.

Consider now the situation analyzed in Fig. 4(b), where the different curves correspond to different values of the scattering rate Γ', for a given strength of the driving. A current reversal is observed at $\phi=\pi/2$ as a function of the scattering rate. The phase shift of the sinlike curve is the key to explain the current reversal at $\phi=\pi/2$. In fact, Figs. 4(b) show that the amplitude A decreases for an increasing scattering rate, while the phase shift ϕ_0 varies from a small value ($\phi_0\approx 0.33$ rad for $\Gamma'=7.5\omega_r$) to a large value ($\phi_0=2.66$ rad for $\Gamma'=25\omega_r$). It is such a large variation in the phase shift which determines the current reversal observed at $\phi=\pi/2$.

In conclusion, we found that the examined current reversals at $\phi=\pi/2$ are determined by a significant variation, with respect to the parameters (driving strength or scattering rate) producing the current reversal, of the phase shift ϕ_0 of the ratchet current as a function of ϕ. The sinlike curve of the ratchet current does, in general, not flatten out, but shifts significantly, and this generates the current reversals at $\phi=\pi/2$.

C. Rectification of fluctuation in a system with broken Hamiltonian symmetry

We now address the issue of the rectification of fluctuations in a driven optical lattice with broken Hamiltonian symmetry, as introduced in Ref. [29]. In that work it was observed that in a setup with biharmonic driving and fixed phase offset $\phi=\pi/2$ the atomic velocity shows a nonmonotonic dependence on the scattering rate at low driving force amplitudes. We can now establish the link between these observations and our present results for the dependence on the scattering rate and driving strength of the amplitude A and phase shift ϕ_0 of the current as a function of the phase ϕ between the two driving field harmonics.

In Fig. 5 we report the results of our numerical simulations for the average atomic momentum as a function of the driving strength at different values of the scattering rate, for a fixed phase offset $\phi=\pi/2$ between the driving harmonics. These results agree qualitatively with the analysis of Ref. [29] and shows that the average momentum shows a nonmonotonic dependence on the scattering rate at low driving strength. More precisely, the average atomic velocity is an increasing function of the scattering rate at low values of Γ', then reaches a maximum and then decreases. This corresponds to the rectification of fluctuations, in the sense that an increase in the level of noise (scattering rate) leads to an increase in the current amplitude [29]. This shows that for

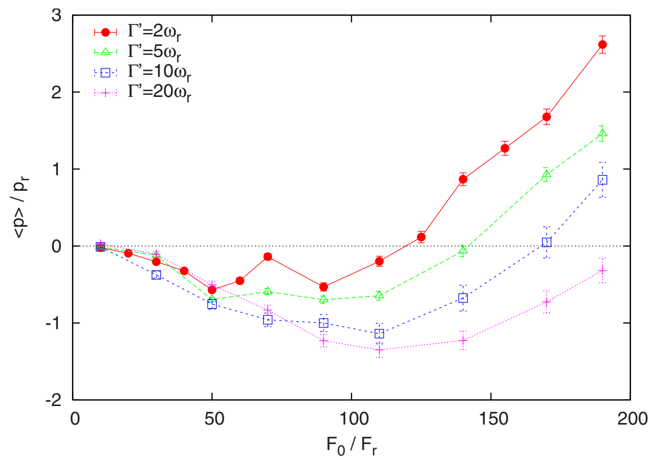

FIG. 5. (Color online) Average atomic momentum versus driving field amplitude F_0, at different values of the scattering rate Γ'. The simulations are for a biharmonic driving with fixed phase offset $\phi=\pi/2$. The other calculation parameters are as in Figs. 2(c) and 2(d).

small amplitudes of the driving the generation of a current can be traced back to the rectification of fluctuations. In contrast, at larger values of the driving force amplitude, the current magnitude is a monotonic and decreasing function of the scattering rate.

These results can now be linked with the results for the dependence on the scattering rate of the amplitude A and phase shift ϕ_0 of the current as a function of ϕ, as reported in Fig. 2. It is easy to show that the observed nonmonotonic dependence is determined by the nonmonotonic dependence of the amplitude A, and not by the change in the phase lag ϕ_0 with the scattering rate. In fact for small values of the scattering rate, ϕ_0 is essentially zero and then increases for increasing Γ'. This behavior alone, produces a decrease in the current generated for $\phi=\pi/2$. Therefore the initial increasing dependence on Γ' of the current for $\phi=\pi/2$ has to be attributed to the nonmonotonic dependence of the amplitude A of the current as a function of ϕ. As evidenced in Fig. 2, such a nonmonotonic dependence of A on Γ' is observed at small values of the driving force amplitude, consistent with the fact that (see Fig. 5) the rectification of fluctuations for $\phi=\pi/2$ is observed for weak drivings.

IV. CONCLUSIONS

In this work we analyzed numerically a rocking ratchet for cold atoms. The ratchet setup consists of a spatially symmetric optical potential, and a biharmonic driving force. This is the setup used in a recent series of experiments [28–33].

We studied the ratchet current as a function of the phase ϕ between the two harmonics. The average atomic velocity exhibits an approximately sinlike dependence on the phase ϕ, with the amplitude and phase shift of the sine strongly dependent on the driving force amplitude and scattering rate. We determined numerically the dependence of the ratchet current parameters (amplitude and phase shift) on the driving force amplitude and on the scattering rate.

We then considered the problem of the current reversals in rocking ratchets with broken time symmetry, and referred, in particular, to the experimental work of Ref. [29] where current reversals were observed as a function of the driving strength in a rocking ratchet with fixed phase $\phi=\pi/2$, i.e., for a value of the phase which breaks the time symmetry of the Hamiltonian. We determined numerically the link between these current reversals and the changes in amplitude and phase shift of the ratchet current as a function of ϕ. We showed that the current reversal at $\phi=\pi/2$ originates from a large variation in the phase shift of the sinlike ratchet current as a function of ϕ. This holds for both current reversals observed as a function of the driving strength, and as a function of the scattering rate. We also showed that the rectification of fluctuations in this rocking ratchet with fixed phase $\phi=\pi/2$ corresponds to a nonmonotonic dependence on the scattering rate of the amplitude of the ratchet current as a function of the phase offset ϕ.

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