Perfect teleportation, quantum-state sharing, and superdense coding through a genuinely entangled five-qubit state

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We investigate the usefulness of a recently introduced five-qubit state by Brown *et al.* [I. D. K. Brown, S. Stepney, A. Sudbery, and S. L. Braunstein, J. Phys. A **38**, 1119 (2005)] for quantum teleportation, quantum-state sharing, and superdense coding. It is shown that this state can be utilized for perfect teleportation of arbitrary single and two-qubit systems. We devise various schemes for quantum-state sharing of an arbitrary single- and two-particle state via cooperative teleportation. We later show that this state can be used for superdense coding as well. It is found that five classical bits can be sent by sending only three quantum bits.

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I. INTRODUCTION

Entanglement is central to all branches of quantum computation and information. This counterintuitive feature has been used to achieve what would be impossible in classical physics. Characterization and classification of multipartite entangled states is not yet firmly established [2]. Quantum teleportation is an important ingredient in distributed quantum networks, which exploit entanglement for transferring a quantum state between two or more parties. It also serves as an elementary operation in quantum computers and in a number of quantum communication protocols. It has been achieved experimentally in different quantum systems [3–5] and over long distances, inside [6] and outside [7] laboratory conditions.

Quantum teleportation is a technique for transfer of information between parties, using a distributed entangled state and a classical communication channel [8]. Existence of long range correlations assist in information transfer by a sender, unaware of the information to be sent, as well as the destination. Teleportation of an arbitrary single qubit, $|\psi\rangle_a=\alpha|0\rangle+\beta|1\rangle$, (with $|\alpha|^2+|\beta|^2=1$) through an entangled channel of Einstein-Podolsky-Rosen (EPR) pair between the sender and receiver was first demonstrated by Bennett *et al.* [9]. Superdense coding is another spectacular application of quantum information theory, receiving significant attention in recent times. It shares a close relationship with quantum teleportation [10]. Multipartite entangled states, namely the prototype–Greenberger-Horne-Zeilinger (GHZ) states [11], generalized W states [12,13], and the cluster states [14,15], have also been exploited for carrying out teleportation and superdense coding. With the increment in number of states, the complexity involved increases manifold due to scarce knowledge regarding characterization of multipartite entanglement.

The way in which a given shared multiparticle state is entangled plays a pivotal role in deciding the suitability of the state for teleportation. For instance, it is well known that the normal W states are not useful for perfect teleportation. However, assigning suitable weights and relative phases to individual terms of W states makes it suitable for perfect teleportation and superdense coding [12]. These modified W states are unitarily connected to GHZ states [16].

Though many types of states have been used for teleporting an arbitrary one-qubit state, very few known states are capable of teleporting an arbitrary two-qubit state. Even the multiqubit GHZ and the generalized W state cannot be used for this purpose. All the states that are known to be useful for this purpose are essentially four-qubit states [17,18]. Even higher dimensional generalizations have been explicated, which allow teleporting N-qubit systems using N Bell states [19]. In this paper, we describe a five-qubit state that can be used for perfect teleportation of both a one-qubit as well as an arbitrary two-qubit state and discuss its advantages over the previously known states. It is found suitable to carry out maximal teleportation and maximal superdense coding, satisfying the definition of task-oriented maximally entangled states (TMES) [20]. In addition to this, a new five-partite cluster state was introduced in [20], for perfect teleportation and superdense coding. Five-qubit entangled states play a key role in quantum information processing tasks and it is the threshold number of qubits required for quantum error correction [21,22]. Quantum mechanical entanglement of five particles was achieved using the spontaneous parametric down conversion as a source of entangled photons [23]. Teleportation was carried out experimentally using the five particle entangled GHZ state [23]. Recently, Brown *et al.* [1] arrived at a maximally entangled five-qubit state through an extensive numerical optimization procedure. This has the form:

$$|\psi_5\rangle=\frac{1}{2}(|001\rangle|\phi_-\rangle+|010\rangle|\psi_-\rangle+|100\rangle|\phi_+\rangle+|111\rangle|\psi_+\rangle), \tag{1}$$

where $|\psi_\pm\rangle=\frac{1}{\sqrt{2}}(|00\rangle\pm|11\rangle)$ and $|\phi_\pm\rangle=\frac{1}{\sqrt{2}}(|01\rangle\pm|10\rangle)$ are Bell states. This result was verified by yet another numerical

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search procedure carried out recently [24]. This state exhibits genuine multipartite entanglement according to both negative partial transpose measure, as well as von Neumann entropy measure. The von Neumann entropy between $(1234|5)$ is equal to 1 and between $(123|45)$ is 2. These are the maximum possible entanglement values between the respective subsets, thus satisfying the general condition for teleportation as shown in [16]. It is also to be noted that $\mathrm{Tr}(\rho_{i_1}^2)=\frac{1}{2}$ and $\mathrm{Tr}(\rho_{i_1,i_2}^2)\cdots=\frac{1}{4}$, where $i_1,i_2\ldots$ refer to the subsystems, respectively, thus satisfying the criteria for multiqubit entanglement as shown in [25]. The above state is also genuinely entangled according to the recently proposed multiple entropy measures (MEMS) [26]; it has MEMS of $S_1=1$ and $S_2=2$, respectively. This is more than the entanglement exhibited by the GHZ, W, and the cluster states. Even after tracing out one or two qubits from the state, entanglement sustains in the resulting subsystem and thus is highly "robust." Also, the state is maximally mixed, after we trace out any possible number of qubits, which is an indication of genuine multiparticle entanglement for the five-qubit state $|\psi_5\rangle$. Four-qubit states do not show such characteristic behavior and fail to attain maximal entropy [27]. Moreover, the above state assumes the same form for all ten splits as $(3+2)$. Thus Alice can have any pair of three qubits in the above state to teleport to Bob. This is not possible with the four-qubit states known before. Thus the five-qubit state can provide an edge over the four-qubit states for state transfer and coding. We shall now devise a suitable method to study the physical realization of this state and investigate its usefulness for quantum information tasks, namely teleportation state sharing and superdense coding.

II. PHYSICAL REALIZATION

Though the Brown state was initially obtained through an extensive numerical search procedure, it can be physically realized as follows. We start with two photons in the Bell state given by

$$|\phi_+\rangle = \frac{1}{\sqrt{2}}(|01\rangle + |10\rangle). \tag{2}$$

We need to prepare another photon in the state $\frac{1}{\sqrt{2}}(|0\rangle+|1\rangle)$. One can combine both these states and perform a Universal Controlled-NOT (UCNOT) operation on the last two qubits and get a W class of states as follows:

$$\frac{1}{2}(|01\rangle + |10\rangle)(|0\rangle + |1\rangle) \xrightarrow{\text{UCNOT}(3,2)} \frac{1}{2}(|100\rangle + |010\rangle + |001\rangle + |111\rangle). \tag{3}$$

We now take two photons in another Bell state,

$$|\psi_+\rangle = \frac{1}{\sqrt{2}}(|00\rangle + |11\rangle). \tag{4}$$

The Brown state can be obtained by applying a unitary transformation U_b to their combined state as follows:

$$|W\rangle|\psi_+\rangle \xrightarrow{U_b} |\psi_5\rangle. \tag{5}$$

The unitary transform U_b is given by a 32×32 matrix, with unity in the places $k_{1,1},k_{2,2},k_{5,6},k_{6,5},k_{9,9},k_{10,10},k_{13,14},k_{14,13}$, $k_{17,18}$, $k_{18,17}$, $k_{19,20}$, $k_{20,19}$, $k_{21,21},k_{23,23}$, $k_{24,24}$, $k_{25,26}$, $k_{26,25}$, $k_{27,28},k_{28,27},k_{29,29},k_{30,30},k_{31,31},k_{32,32}$ and -1 in the following places $k_{3,3},k_{4,4},k_{7,8},k_{8,7},k_{11,11},k_{12,12},k_{15,16},k_{16,15}$ and 0 in the other terms. Here $k_{i,j}$ represents the element in the ith row and jth column. This unitary operator can be further decomposed into known gates in quantum information. This procedure can lead to its possible experimental realization.

III. TELEPORTATION OF A SINGLE-QUBIT STATE

Let us first consider the situation in which Alice possesses qubits 1, 2, 3, 4 and particle 5 belongs to Bob. Alice wants to teleport $(\alpha|0\rangle+\beta|1\rangle)$ to Bob. So, Alice prepares the combined state,

$$(\alpha|0\rangle + \beta|1\rangle)|\psi_5\rangle = |\phi_1\rangle_{a_{1+}}(\alpha|0\rangle + \beta|1\rangle) + |\phi_2\rangle_{a_{1-}}(\alpha|0\rangle - \beta|1\rangle) + |\phi_3\rangle_{a_{2+}}(\beta|0\rangle + \alpha|1\rangle) + |\phi_4\rangle_{a_{2-}}(\beta|0\rangle - \alpha|1\rangle), \tag{6}$$

where the $|\phi_x\rangle_{a_i\pm}$ are mutually orthogonal states of the measurement basis. The states $|\phi_x\rangle_{a_i\pm}$ are given as

$$|\phi_x\rangle_{a_1\pm} = (-|00011\rangle + |00100\rangle + |01001\rangle + |01110\rangle) \pm (|10010\rangle - |10101\rangle + |11000\rangle + |11111\rangle),$$

$$|\phi_x\rangle_{a_2\pm} = (-|10011\rangle + |10100\rangle + |11001\rangle + |11110\rangle) \pm (|00010\rangle - |00101\rangle + |01000\rangle + |01111\rangle).$$

Alice can now make a five-particle measurement using $|\phi_x\rangle_{a_i\pm}$ and convey the outcome of her measurement to Bob via two classical bits. Bob can apply suitable unitary operations given by $(1,\sigma_1,i\sigma_2,\sigma_3)$ to recover the original state $(\alpha|0\rangle+\beta|1\rangle)$. This completes the teleportation protocol for the teleportation of a single-qubit state using the state $|\psi_5\rangle$. We now proceed to study the suitability of the Brown state for quantum-state sharing (QSTS) of a single-qubit state.

IV. QSTS OF A SINGLE-QUBIT STATE

A. Proposal I

Let us consider the situation in which Alice possesses qubit 1, Bob possesses qubits 2, 3, 4, and Charlie qubit 5. Alice has an unknown qubit $(\alpha|0\rangle+\beta|1\rangle)$ which she wants Bob and Charlie to share. Now, Alice combines the unknown qubit with the Brown state and performs a Bell measurement and conveys her outcome to Charlie by two classical bits (cbits). For instance, if Alice measures in the basis $|\psi_+\rangle$, then the Bob-Charlie system evolves into the entangled state:

TABLE I. The outcome of the measurement performed by Bob and the state obtained by Charlie.

Outcome of the measurement	State obtained
$\frac{1}{2}(\lvert 010\rangle-\lvert 101\rangle+\lvert 001\rangle+\lvert 110\rangle)$	$\alpha\lvert 1\rangle+\beta\lvert 0\rangle$
$\frac{1}{2}(\lvert 100\rangle-\lvert 011\rangle+\lvert 000\rangle+\lvert 111\rangle)$	$\alpha\lvert 0\rangle+\beta\lvert 1\rangle$
$\frac{1}{2}(\lvert 010\rangle-\lvert 101\rangle-\lvert 001\rangle-\lvert 110\rangle)$	$\alpha\lvert 1\rangle-\beta\lvert 0\rangle$
$\frac{1}{2}(\lvert 100\rangle-\lvert 000\rangle-\lvert 111\rangle-\lvert 011\rangle)$	$\alpha\lvert 0\rangle-\beta\lvert 1\rangle$

$$\alpha(\lvert 01\rangle\lvert\phi_-\rangle+\lvert 10\rangle\lvert\psi_-\rangle)+\beta(\lvert 00\rangle\lvert\phi_+\rangle+\lvert 11\rangle\lvert\psi_+\rangle). \tag{7}$$

Now Bob can perform a three-partite measurement and convey his outcome to Charlie by two cbits. Having known the outcome of both their measurements, Charlie can obtain the state by performing appropriate unitary transformations. The outcome of the measurement performed by Bob and the state obtained by Charlie are shown in Table I.

Here, Bob can also perform a single particle measurement followed by a two particle measurement instead of a three particle measurement. However, this would consume an extra cbit of information.

B. Proposal II

In this scenario we let Alice possess qubits 1, 2, Bob possess qubits 3, 4, and Charlie qubit 5. Alice combines the unknown qubit with her particles and makes a three-partite measurement. The outcome of the measurement performed by Alice and the entangled state obtained by Bob and Charlie are shown in Table II.

Alice can send the outcome of her measurement to Bob via three cbits of information. Now Bob and Charlie can meet and apply a joint unitary three particle transformation on their particles and convert it into the GHZ type of state as follows:

$$\alpha(\lvert 1\rangle\lvert\phi_-\rangle+\lvert 0\rangle\lvert\psi_-\rangle)+\beta(\lvert 0\rangle\lvert\phi_+\rangle+\lvert 1\rangle\lvert\psi_+\rangle)\rightarrow\alpha\lvert 000\rangle+\beta\lvert 111\rangle. \tag{8}$$

After performing the unitary transformation, Bob and Charlie can be spatially separated. Bob can perform a Bell measurement on his particles and Charlie can obtain the state by applying an appropriate unitary operator. This kind of strategy, might as well find applications other than state sharing in quantum information. As in the previous case, even in this scenario Alice can perform a Bell measurement followed by a single-partite measurement instead of a three particle measurement.

V. TELEPORTATION OF AN ARBITRARY TWO-QUBIT STATE

Alice has an arbitrary two-qubit state,

$$\lvert\psi\rangle=\alpha\lvert 00\rangle+\mu\lvert 10\rangle+\gamma\lvert 01\rangle+\beta\lvert 11\rangle, \tag{9}$$

which she has to teleport to Bob. Here α, β, γ, and μ are any set of complex numbers satisfying $\lvert\alpha\rvert^2+\lvert\beta\rvert^2+\lvert\gamma\rvert^2+\lvert\mu\rvert^2=1$. Qubits 1, 2, 3 and 4, 5, respectively, belong to Alice and Bob. Alice prepares the combined state,

$$\begin{aligned}\lvert\psi\rangle\lvert\psi_5\rangle=\frac{1}{4}[&\lvert\psi_5\rangle_1(\alpha\lvert 01\rangle+\gamma\lvert 00\rangle+\mu\lvert 11\rangle+\beta\lvert 10\rangle)+\lvert\psi_5\rangle_2(\alpha\lvert 01\rangle+\gamma\lvert 00\rangle-\mu\lvert 11\rangle-\beta\lvert 10\rangle)+\lvert\psi_5\rangle_3(\alpha\lvert 01\rangle-\gamma\lvert 00\rangle+\mu\lvert 11\rangle-\beta\lvert 10\rangle)\\&+\lvert\psi_5\rangle_4(\alpha\lvert 01\rangle-\gamma\lvert 00\rangle-\mu\lvert 11\rangle+\beta\lvert 10\rangle)+\lvert\psi_5\rangle_5(\alpha\lvert 11\rangle+\gamma\lvert 10\rangle+\mu\lvert 01\rangle+\beta\lvert 00\rangle)+\lvert\psi_5\rangle_6(\alpha\lvert 11\rangle-\gamma\lvert 10\rangle+\mu\lvert 01\rangle-\beta\lvert 00\rangle)\\&+\lvert\psi_5\rangle_7(\alpha\lvert 11\rangle+\gamma\lvert 10\rangle-\mu\lvert 01\rangle-\beta\lvert 00\rangle)+\lvert\psi_5\rangle_8(\alpha\lvert 11\rangle-\gamma\lvert 10\rangle-\mu\lvert 01\rangle+\beta\lvert 00\rangle)+\lvert\psi_5\rangle_9(\alpha\lvert 00\rangle+\gamma\lvert 01\rangle+\mu\lvert 10\rangle+\beta\lvert 11\rangle)\\&+\lvert\psi_5\rangle_{10}(\alpha\lvert 00\rangle-\gamma\lvert 01\rangle+\mu\lvert 10\rangle-\beta\lvert 11\rangle)+\lvert\psi_5\rangle_{11}(\alpha\lvert 00\rangle+\gamma\lvert 01\rangle-\mu\lvert 10\rangle-\beta\lvert 11\rangle)+\lvert\psi_5\rangle_{12}(\alpha\lvert 00\rangle-\gamma\lvert 01\rangle-\mu\lvert 10\rangle\\&+\beta\lvert 11\rangle)+\lvert\psi_5\rangle_{13}(\alpha\lvert 10\rangle+\gamma\lvert 11\rangle+\mu\lvert 00\rangle+\beta\lvert 01\rangle)+\lvert\psi_5\rangle_{14}(\alpha\lvert 10\rangle-\gamma\lvert 11\rangle+\mu\lvert 00\rangle-\beta\lvert 01\rangle)+\lvert\psi_5\rangle_{15}(\alpha\lvert 10\rangle+\gamma\lvert 11\rangle\\&-\mu\lvert 00\rangle-\beta\lvert 01\rangle)+\lvert\psi_5\rangle_{16}(\alpha\lvert 10\rangle-\gamma\lvert 11\rangle-\mu\lvert 00\rangle+\beta\lvert 01\rangle)].\end{aligned} \tag{10}$$

Here, $\lvert\psi_5\rangle_i$'s forming the mutual orthogonal basis of measurement are given by

$$\lvert\psi_5\rangle_3=\frac{1}{2}[\lvert\psi_-\rangle\lvert 001\rangle+\lvert\psi_+\rangle\lvert 100\rangle-\lvert\phi_+\rangle\lvert 010\rangle-\lvert\phi_-\rangle\lvert 111\rangle];$$

$$\lvert\psi_5\rangle_4=\frac{1}{2}[\lvert\psi_-\rangle\lvert 001\rangle+\lvert\psi_+\rangle\lvert 100\rangle-\lvert\phi_-\rangle\lvert 010\rangle-\lvert\phi_+\rangle\lvert 111\rangle];$$

$$\lvert\psi_5\rangle_5=\frac{1}{2}[\lvert\psi_+\rangle\lvert 111\rangle-\lvert\psi_-\rangle\lvert 010\rangle-\lvert\phi_-\rangle\lvert 001\rangle+\lvert\phi_+\rangle\lvert 100\rangle];$$

$$\lvert\psi_5\rangle_6=\frac{1}{2}[\lvert\psi_-\rangle\lvert 111\rangle-\lvert\psi_+\rangle\lvert 010\rangle+\lvert\phi_+\rangle\lvert 001\rangle-\lvert\phi_-\rangle\lvert 100\rangle];$$

$$\lvert\psi_5\rangle_7=\frac{1}{2}[\lvert\psi_-\rangle\lvert 111\rangle-\lvert\psi_+\rangle\lvert 010\rangle-\lvert\phi_+\rangle\lvert 001\rangle+\lvert\phi_-\rangle\lvert 100\rangle];$$

$$\lvert\psi_5\rangle_8=\frac{1}{2}[\lvert\psi_+\rangle\lvert 111\rangle-\lvert\psi_-\rangle\lvert 010\rangle+\lvert\phi_-\rangle\lvert 001\rangle-\lvert\phi_+\rangle\lvert 100\rangle];$$

TABLE II. The outcome of the measurement performed by Alice and the state obtained by Bob and Charlie.

Outcome of the measurement	State obtained
$\frac{1}{4}(\|000\rangle+\|001\rangle+\|110\rangle+\|111\rangle)$	$\alpha(\|1\rangle\|\phi_-\rangle+\|0\rangle\|\psi_-\rangle)+\beta(\|0\rangle\|\phi_+\rangle+\|1\rangle\|\psi_+\rangle)$
$\frac{1}{4}(\|000\rangle-\|001\rangle-\|110\rangle+\|111\rangle)$	$\alpha(\|1\rangle\|\phi_-\rangle-\|0\rangle\|\psi_-\rangle)-\beta(\|0\rangle\|\phi_+\rangle-\|1\rangle\|\psi_+\rangle)$
$\frac{1}{4}(\|000\rangle-\|001\rangle+\|110\rangle-\|111\rangle)$	$\alpha(\|1\rangle\|\phi_-\rangle-\|0\rangle\|\psi_-\rangle)+\beta(\|0\rangle\|\phi_+\rangle-\|1\rangle\|\psi_+\rangle)$
$\frac{1}{4}(\|000\rangle+\|001\rangle-\|110\rangle-\|111\rangle)$	$\alpha(\|1\rangle\|\phi_-\rangle+\|0\rangle\|\psi_-\rangle)-\beta(\|0\rangle\|\phi_+\rangle+\|1\rangle\|\psi_+\rangle)$
$\frac{1}{4}(\|100\rangle+\|101\rangle+\|010\rangle+\|011\rangle)$	$\beta(\|1\rangle\|\phi_-\rangle+\|0\rangle\|\psi_-\rangle)+\alpha(\|0\rangle\|\phi_+\rangle+\|1\rangle\|\psi_+\rangle)$
$\frac{1}{4}(\|100\rangle-\|101\rangle-\|010\rangle+\|011\rangle)$	$\beta(\|1\rangle\|\phi_-\rangle-\|0\rangle\|\psi_-\rangle)-\alpha(\|0\rangle\|\phi_+\rangle-\|1\rangle\|\psi_+\rangle)$
$\frac{1}{4}(\|100\rangle-\|101\rangle+\|010\rangle-\|011\rangle)$	$\beta(\|1\rangle\|\phi_-\rangle-\|0\rangle\|\psi_-\rangle)+\alpha(\|0\rangle\|\phi_+\rangle-\|1\rangle\|\psi_+\rangle)$
$\frac{1}{4}(\|100\rangle+\|101\rangle-\|010\rangle-\|011\rangle)$	$\beta(\|1\rangle\|\phi_-\rangle+\|0\rangle\|\psi_-\rangle)-\alpha(\|0\rangle\|\phi_+\rangle+\|1\rangle\|\psi_+\rangle)$

$$|\psi_5\rangle_9 = \frac{1}{2}[|\psi_+\rangle|111\rangle + |\psi_-\rangle|010\rangle + |\phi_-\rangle|001\rangle + |\phi_+\rangle|000\rangle];$$

$$|\psi_5\rangle_{10} = \frac{1}{2}[|\psi_-\rangle|111\rangle + |\psi_+\rangle|010\rangle - |\phi_-\rangle|001\rangle - |\psi_+\rangle|100\rangle];$$

$$|\psi_5\rangle_{11} = \frac{1}{2}[|\psi_-\rangle|111\rangle + |\psi_+\rangle|010\rangle + |\phi_+\rangle|001\rangle + |\phi_-\rangle|100\rangle];$$

$$|\psi_5\rangle_{12} = \frac{1}{2}[|\psi_+\rangle|111\rangle + |\psi_-\rangle|010\rangle - |\phi_+\rangle|100\rangle - |\phi_-\rangle|000\rangle];$$

$$|\psi_5\rangle_{13} = \frac{1}{2}[|\psi_+\rangle|100\rangle - |\psi_-\rangle|001\rangle - |\phi_-\rangle|010\rangle + |\phi_+\rangle|111\rangle];$$

$$|\psi_5\rangle_{14} = \frac{1}{2}[|\psi_-\rangle|100\rangle - |\psi_+\rangle|001\rangle + |\phi_+\rangle|010\rangle - |\phi_-\rangle|111\rangle];$$

$$|\psi_5\rangle_{15} = \frac{1}{2}[|\psi_-\rangle|100\rangle - |\psi_+\rangle|001\rangle - |\phi_+\rangle|010\rangle + |\phi_-\rangle|111\rangle];$$

$$|\psi_5\rangle_{16} = \frac{1}{2}[|\psi_+\rangle|100\rangle - |\psi_-\rangle|001\rangle + |\phi_-\rangle|010\rangle - |\phi_+\rangle|111\rangle]. \quad (11)$$

TABLE III. Set of unitary operators needed to obtain $|\psi\rangle_b$.

State	Unitary operation
$(\alpha\|01\rangle+\gamma\|00\rangle+\mu\|11\rangle+\beta\|10\rangle)$	$I\otimes\sigma_1$
$(\alpha\|01\rangle+\gamma\|00\rangle-\mu\|11\rangle-\beta\|10\rangle)$	$\sigma_3\otimes\sigma_1$
$(\alpha\|01\rangle-\gamma\|00\rangle+\mu\|11\rangle-\beta\|10\rangle)$	$I\otimes i\sigma_2$
$(\alpha\|01\rangle-\gamma\|00\rangle-\mu\|11\rangle+\beta\|10\rangle)$	$\sigma_3\otimes i\sigma_2$
$(\alpha\|11\rangle+\gamma\|10\rangle+\mu\|01\rangle+\beta\|00\rangle)$	$\sigma_1\otimes\sigma_1$
$(\alpha\|11\rangle-\gamma\|10\rangle+\mu\|01\rangle-\beta\|00\rangle)$	$\sigma_1\otimes i\sigma_2$
$(\alpha\|11\rangle+\gamma\|10\rangle-\mu\|01\rangle-\beta\|00\rangle)$	$i\sigma_2\otimes\sigma_1$
$(\alpha\|11\rangle-\gamma\|10\rangle-\mu\|01\rangle+\beta\|00\rangle)$	$i\sigma_2\otimes i\sigma_2$
$(\alpha\|00\rangle+\gamma\|01\rangle+\mu\|10\rangle+\beta\|11\rangle)$	$I\otimes I$
$(\alpha\|00\rangle-\gamma\|01\rangle+\mu\|10\rangle-\beta\|11\rangle)$	$I\otimes\sigma_3$
$(\alpha\|00\rangle+\gamma\|01\rangle-\mu\|10\rangle-\beta\|11\rangle)$	$\sigma_3\otimes I$
$(\alpha\|00\rangle-\gamma\|01\rangle-\mu\|10\rangle+\beta\|11\rangle)$	$\sigma_3\otimes\sigma_3$
$(\alpha\|10\rangle+\gamma\|11\rangle+\mu\|00\rangle+\beta\|01\rangle)$	$\sigma_1\otimes I$
$(\alpha\|10\rangle-\gamma\|11\rangle+\mu\|00\rangle-\beta\|01\rangle)$	$\sigma_1\otimes\sigma_3$
$(\alpha\|10\rangle+\gamma\|11\rangle-\mu\|00\rangle-\beta\|01\rangle)$	$i\sigma_2\otimes I$
$(\alpha\|10\rangle-\gamma\|11\rangle-\mu\|00\rangle+\beta\|01\rangle)$	$i\sigma_2\otimes\sigma_3$

Alice can make a five-particle measurement and then convey her results to Bob. Bob then retrieves the original state $|\psi\rangle_b$ by applying any one of the unitary transforms shown in Table III to the respective states. As is evident, each of the above states are obtained with equal probability. This successfully completes the teleportation protocol of a two-qubit state using $|\psi_5\rangle$.

VI. QSTS OF AN ARBITRARY TWO-QUBIT STATE

QSTS of an arbitrary two-particle state was previously carried out using four Bell pairs among two controllers and then generalized to N agents [28]. We now demonstrate the utility of the Brown state for the QSTS of an arbitrary two-qubit state. It is evident that this protocol requires a lesser number of particles than the previously known protocol and due to the properties of the Brown state, the protocol is also more robust against decoherence. We propose one possible protocol for the QSTS of an arbitrary two-qubit state.

We let Alice possess particles 1 ,2, Bob has particle 3, and Charlie has particles 4 and 5 in the Brown state, respectively. Alice first combines the state $|\psi\rangle$ with the Brown state and makes a four-particle measurement. The outcome of the measurement made by Alice and the entangled state obtained by Bob and Charlie are shown in Table IV, where

$$|\Omega_1\rangle = \frac{1}{2}(|101\rangle - |110\rangle), \quad (12)$$

$$|\Omega_2\rangle = \frac{1}{2}(|000\rangle - |011\rangle), \quad (13)$$

$$|\Omega_3\rangle = \frac{1}{2}(|001\rangle + |010\rangle), \quad (14)$$

TABLE IV. The outcome of the measurement performed by Alice and the state obtained by Bob and Charlie (the coefficient $\frac{1}{4}$ is removed for convenience).

Outcome of the measurement	State obtained
$(\|0000\rangle+\|1001\rangle+\|0110\rangle+\|1111\rangle)$	$\alpha\|\Omega_1\rangle+\mu\|\Omega_2\rangle+\gamma\|\Omega_3\rangle+\beta\|\Omega_4\rangle$
$(\|0000\rangle-\|1001\rangle+\|0110\rangle-\|1111\rangle)$	$\alpha\|\Omega_1\rangle-\mu\|\Omega_2\rangle+\gamma\|\Omega_3\rangle-\beta\|\Omega_4\rangle$
$(\|0000\rangle+\|1001\rangle-\|0110\rangle-\|1111\rangle)$	$\alpha\|\Omega_1\rangle+\mu\|\Omega_2\rangle-\gamma\|\Omega_3\rangle-\beta\|\Omega_4\rangle$
$(\|0000\rangle-\|1001\rangle-\|0110\rangle+\|1111\rangle)$	$\alpha\|\Omega_1\rangle-\mu\|\Omega_2\rangle-\gamma\|\Omega_3\rangle+\beta\|\Omega_4\rangle$
$(\|0010\rangle+\|0101\rangle+\|1000\rangle+\|1101\rangle)$	$\alpha\|\Omega_3\rangle+\gamma\|\Omega_4\rangle+\mu\|\Omega_1\rangle+\beta\|\Omega_2\rangle$
$(\|0010\rangle-\|0101\rangle+\|1000\rangle-\|1101\rangle)$	$\alpha\|\Omega_3\rangle-\gamma\|\Omega_4\rangle+\mu\|\Omega_1\rangle-\beta\|\Omega_2\rangle$
$(\|0010\rangle+\|0101\rangle-\|1000\rangle-\|1101\rangle)$	$\alpha\|\Omega_3\rangle+\gamma\|\Omega_4\rangle-\mu\|\Omega_1\rangle-\beta\|\Omega_2\rangle$
$(\|0010\rangle-\|0101\rangle-\|1000\rangle+\|1101\rangle)$	$\alpha\|\Omega_3\rangle-\gamma\|\Omega_4\rangle-\mu\|\Omega_1\rangle+\beta\|\Omega_2\rangle$
$(\|0001\rangle+\|0100\rangle+\|1011\rangle+\|1110\rangle)$	$\alpha\|\Omega_2\rangle+\gamma\|\Omega_1\rangle+\mu\|\Omega_4\rangle+\beta\|\Omega_3\rangle$
$(\|0001\rangle-\|0100\rangle+\|1011\rangle-\|1110\rangle)$	$\alpha\|\Omega_2\rangle-\gamma\|\Omega_1\rangle+\mu\|\Omega_4\rangle-\beta\|\Omega_3\rangle$
$(\|0001\rangle+\|0100\rangle-\|1011\rangle-\|1110\rangle)$	$\alpha\|\Omega_2\rangle+\gamma\|\Omega_1\rangle-\mu\|\Omega_4\rangle-\beta\|\Omega_3\rangle$
$(\|0001\rangle-\|0100\rangle-\|1011\rangle+\|1110\rangle)$	$\alpha\|\Omega_2\rangle-\gamma\|\Omega_1\rangle-\mu\|\Omega_4\rangle+\beta\|\Omega_3\rangle$
$(\|0011\rangle+\|1010\rangle+\|0101\rangle+\|1100\rangle)$	$\alpha\|\Omega_4\rangle+\mu\|\Omega_3\rangle+\gamma\|\Omega_2\rangle+\beta\|\Omega_1\rangle$
$(\|0011\rangle-\|1010\rangle+\|0101\rangle-\|1100\rangle)$	$\alpha\|\Omega_4\rangle-\mu\|\Omega_3\rangle+\gamma\|\Omega_2\rangle-\beta\|\Omega_1\rangle$
$(\|0011\rangle+\|1010\rangle-\|0101\rangle-\|1100\rangle)$	$\alpha\|\Omega_4\rangle+\mu\|\Omega_3\rangle-\gamma\|\Omega_2\rangle-\beta\|\Omega_1\rangle$
$(\|0011\rangle-\|1010\rangle-\|0101\rangle+\|1100\rangle)$	$\alpha\|\Omega_4\rangle-\mu\|\Omega_3\rangle-\gamma\|\Omega_2\rangle+\beta\|\Omega_1\rangle$

$$|\Omega_4\rangle = \frac{1}{2}(|100\rangle + |111\rangle). \tag{15}$$

Neither Bob nor Charlie can reconstruct the original state $|\psi\rangle$ from the above states by local operations. Now Bob performs a measurement on his particle in the basis $\frac{1}{\sqrt{2}}(|0\rangle \pm |1\rangle)$. For instance, if Bob gets the state $(\alpha|\Omega_1\rangle+\mu|\Omega_2\rangle+\gamma|\Omega_3\rangle+\beta|\Omega_4\rangle)$ then Charlie's particle evolves into any one of the following states: $(\pm\alpha|\phi_-\rangle+\mu|\psi_-\rangle+\gamma|\phi_+\rangle\pm\beta|\psi_+\rangle)$.

Alice sends the outcome of her measurement by four classical bits and Bob by one classical bit to Charlie. Having known the outcomes of both their measurements, Charlie can do appropriate unitary transformations to get back the state $|\psi\rangle$. For instance, if Charlie gets the above states, then he performs the unitary transformations $(\pm|11\rangle\langle\psi_+|+|10\rangle\times\langle\phi_+|)\pm(|00\rangle\langle\phi_-|+|01\rangle\langle\psi_-|)$ on his two particles to get back $|\psi\rangle$. Another possible scenario is that Alice can send the result of her measurement to Bob by four classical bits of information. Then Bob and Charlie can cooperate and apply joint unitary transformations on their particles and convert their state into $(\alpha|\Omega_1\rangle+\mu|\Omega_2\rangle+\gamma|\Omega_3\rangle+\beta|\Omega_4\rangle)$ and then be spatially separated. From now on the protocol follows the previous scenario.

Suppose we let Alice have particle 1, Bob have particles 2, 3 and Charlie have particles 4 and 5 in the Brown state. Alice can combine the state $|\psi\rangle$ with the Brown state and make a three-particle measurement. Thus the Bob-Charlie system is left with four qubits. But, it is not possible to obtain the state $|\psi\rangle$ from their combined state in a straightforward manner. The Brown state could also be used for QSTS of a three-partite GHZ type state given by $|\mathrm{GHZ}\rangle=\alpha|000\rangle+\beta|111\rangle$.

These results could be directly generalized to N agents using the following state:

$$|\psi_n\rangle = \frac{1}{2}(|\eta_1\rangle_n|001\rangle|\phi_-\rangle + |\eta_2\rangle_n|010\rangle|\psi_-\rangle + |\eta_3\rangle_n|100\,|\phi_+\rangle + |\eta_4\rangle_n|111\rangle|\psi_+\rangle), \tag{16}$$

where the $|\eta_i\rangle$'s form the computational basis of the nth order. For example, if $n=2$, $|\eta_i\rangle$ equals any combination from the set $(|00\rangle,|11\rangle,|10\rangle,|01\rangle)$. This we call the "generalized Brown state." We let Alice have the first two particles, Charlie have the last two particles, and the other agents in the network have the remaining particles.

VII. SUPERDENSE CODING

We now proceed to show the utility of $|\psi_5\rangle$ for superdense coding. Entanglement is quite handy in communicating information efficiently in a quantum channel. Suppose Alice and Bob share an entangled state, namely $|\psi\rangle_{AB}$. Then Alice can convert her state into different orthogonal states by applying suitable unitary transforms on her particle [29]. Bob then does appropriate Bell measurements on his qubits to retrieve the encoded information. It is known that two classical bits per qubit can be exchanged by sending information through a Bell state. In this section, we shall discuss the suitability of $|\psi_5\rangle$, as a resource for superdense coding. Let us assume that Alice has the first three qubits, and Bob has the last two qubits. Alice can apply the set of unitary transforms on her particle and generate 64 states out of which 32 are mutually orthogonal as shown below:

$$U_x^3 \otimes I \otimes I \to |\psi_5\rangle_{x_i}. \tag{17}$$

Bob can then perform a five-partite measurement in the basis of $|\psi_5\rangle_{x_i}$ and distinguish these states. The appropriate unitary transforms applied and the respective states obtained by Alice are shown in Table V.

The capacity of superdense coding is defined as [30],

$$X(\rho^{AB}) = \log_2 d_A + S(\rho^B) - S(\rho^{AB}), \tag{18}$$

where d_A is the dimension of Alice's system, $S(\rho)$ is the von Neumann entropy. For the state $|\psi_5\rangle$, $X(\rho^{AB})=3+2-0=5$. The Holevo bound of a multipartite quantum state gives the maximum amount of classical information that can be encoded [30]. It is equal to 5 for the five-qubit state ($\log_2 N$). Thus the superdense coding reaches the "Holevo bound" allowing five classical bits to be transmitted through three quantum bits consuming only two entangled bits (ebits). The Brown state could be used to send two classical bits by sending a qubit consuming one ebits. Hence the Brown state could be used instead of the Bell state considered in [29]. One can also send four classical bits by sending two qubits consuming two ebits. Thus the Brown state could also be used instead of the four-partite cluster state considered in [31]. It could be shown that, using the generalized Brown state, it is possible to send $(2N-1)$ qubits by sending N classical bits, if N is odd, or else send $2N$ qubits by sending N classical bits if N is even thus satisfying the definition of TMES for superdense coding [20].

TABLE V. States $|\psi_5\rangle_{x_i}$ obtained by Alice after performing unitary operations U_x^3.

Unitary operation	State
$I\otimes I\otimes I$	$\frac{1}{2}(\|001\rangle\|\phi_-\rangle+\|010\rangle\|\psi_-\rangle+\|100\rangle\|\phi_+\rangle+\|111\rangle\|\psi_+\rangle)$
$I\otimes \sigma_3\otimes I$	$\frac{1}{2}(\|001\rangle\|\phi_-\rangle-\|010\rangle\|\psi_-\rangle+\|100\rangle\|\phi_+\rangle-\|111\rangle\|\psi_+\rangle)$
$\sigma_3\otimes I\otimes I$	$\frac{1}{2}(\|001\rangle\|\phi_-\rangle+\|010\rangle\|\psi_-\rangle-\|100\rangle\|\phi_+\rangle-\|111\rangle\|\psi_+\rangle)$
$\sigma_3\otimes \sigma_3\otimes I$	$\frac{1}{2}(\|001\rangle\|\phi_-\rangle-\|010\rangle\|\psi_-\rangle-\|100\rangle\|\phi_+\rangle+\|111\rangle\|\psi_+\rangle)$
$\sigma_1\otimes \sigma_1\otimes I$	$\frac{1}{2}(\|111\rangle\|\phi_-\rangle+\|100\rangle\|\psi_-\rangle+\|010\rangle\|\phi_+\rangle+\|001\rangle\|\psi_+\rangle)$
$\sigma_1\otimes i\sigma_2\otimes I$	$\frac{1}{2}(\|111\rangle\|\phi_-\rangle-\|100\rangle\|\psi_-\rangle+\|010\rangle\|\phi_+\rangle-\|001\rangle\|\psi_+\rangle)$
$i\sigma_2\otimes \sigma_1\otimes I$	$\frac{1}{2}(\|111\rangle\|\phi_-\rangle+\|100\rangle\|\psi_-\rangle-\|010\rangle\|\phi_+\rangle-\|001\rangle\|\psi_+\rangle)$
$i\sigma_2\otimes i\sigma_2\otimes I$	$\frac{1}{2}(\|111\rangle\|\phi_-\rangle-\|100\rangle\|\psi_-\rangle-\|010\rangle\|\phi_+\rangle+\|001\rangle\|\psi_+\rangle)$
$I\otimes \sigma_1\otimes I$	$\frac{1}{2}(\|011\rangle\|\phi_-\rangle+\|000\rangle\|\psi_-\rangle+\|110\rangle\|\phi_+\rangle+\|101\rangle\|\psi_+\rangle)$
$I\otimes i\sigma_2\otimes I$	$\frac{1}{2}(\|011\rangle\|\phi_-\rangle-\|000\rangle\|\psi_-\rangle+\|110\rangle\|\phi_+\rangle-\|101\rangle\|\psi_+\rangle)$
$\sigma_3\otimes \sigma_1\otimes I$	$\frac{1}{2}(\|011\rangle\|\phi_-\rangle+\|000\rangle\|\psi_-\rangle-\|110\rangle\|\phi_+\rangle-\|101\rangle\|\psi_+\rangle)$
$\sigma_3\otimes i\sigma_2\otimes I$	$\frac{1}{2}(\|000\rangle\|\psi_-\rangle-\|011\rangle\|\phi_-\rangle-\|110\rangle\|\phi_+\rangle+\|101\rangle\|\psi_+\rangle)$
$\sigma_1\otimes I\otimes I$	$\frac{1}{2}(\|101\rangle\|\phi_-\rangle+\|110\rangle\|\psi_-\rangle+\|000\rangle\|\phi_+\rangle+\|011\rangle\|\psi_+\rangle)$
$\sigma_1\otimes \sigma_3\otimes I$	$\frac{1}{2}(\|100\rangle\|\phi_-\rangle-\|110\rangle\|\psi_-\rangle+\|000\rangle\|\phi_+\rangle-\|011\rangle\|\psi_+\rangle)$
$i\sigma_2\otimes I\otimes I$	$\frac{1}{2}(\|000\rangle\|\phi_+\rangle-\|101\rangle\|\phi_-\rangle-\|110\rangle\|\psi_-\rangle+\|011\rangle\|\psi_+\rangle)$
$i\sigma_2\otimes \sigma_3\otimes I$	$\frac{1}{2}(\|101\rangle\|\phi_-\rangle+\|110\rangle\|\psi_-\rangle-\|000\rangle\|\phi_+\rangle-\|011\rangle\|\psi_+\rangle)$
$I\otimes I\otimes \sigma_1$	$\frac{1}{2}(\|000\rangle\|\phi_-\rangle+\|011\rangle\|\psi_-\rangle+\|101\rangle\|\phi_+\rangle+\|110\rangle\|\psi_+\rangle)$
$I\otimes \sigma_3\otimes \sigma_1$	$\frac{1}{2}(\|000\rangle\|\phi_-\rangle-\|011\rangle\|\psi_-\rangle+\|101\rangle\|\phi_+\rangle-\|110\rangle\|\psi_+\rangle)$
$\sigma_3\otimes \sigma_1\otimes \sigma_1$	$\frac{1}{2}(\|000\rangle\|\phi_-\rangle+\|011\rangle\|\psi_-\rangle-\|101\rangle\|\phi_+\rangle-\|110\rangle\|\psi_+\rangle)$
$\sigma_3\otimes \sigma_3\otimes \sigma_1$	$\frac{1}{2}(\|000\rangle\|\phi_-\rangle-\|011\rangle\|\psi_-\rangle-\|101\rangle\|\phi_+\rangle+\|110\rangle\|\psi_+\rangle)$
$\sigma_1\otimes \sigma_1\otimes \sigma_1$	$\frac{1}{2}(\|110\rangle\|\phi_-\rangle+\|101\rangle\|\psi_-\rangle+\|011\rangle\|\phi_+\rangle+\|000\rangle\|\psi_+\rangle)$
$\sigma_1\otimes i\sigma_2\otimes \sigma_1$	$\frac{1}{2}(\|110\rangle\|\phi_-\rangle-\|100\rangle\|\psi_-\rangle+\|011\rangle\|\phi_+\rangle-\|000\rangle\|\psi_+\rangle)$
$\imath\sigma_2\otimes \sigma_1\otimes \sigma_1$	$\frac{1}{2}(\|110\rangle\|\phi_-\rangle+\|101\rangle\|\psi_-\rangle-\|011\rangle\|\phi_+\rangle-\|000\rangle\|\psi_+\rangle)$
$i\sigma_2\otimes i\sigma_2\otimes \sigma_1$	$\frac{1}{2}(\|110\rangle\|\phi_-\rangle-\|101\rangle\|\psi_-\rangle-\|011\rangle\|\phi_+\rangle+\|000\rangle\|\psi_+\rangle)$
$I\otimes \sigma_1\otimes \sigma_1$	$\frac{1}{2}(\|010\rangle\|\phi_-\rangle+\|001\rangle\|\psi_-\rangle+\|111\rangle\|\phi_+\rangle+\|100\rangle\|\psi_+\rangle)$
$I\otimes i\sigma_2\otimes \sigma_1$	$\frac{1}{2}(\|010\rangle\|\phi_-\rangle-\|001\rangle\|\psi_-\rangle+\|111\rangle\|\phi_+\rangle-\|100\rangle\|\psi_+\rangle)$
$\sigma_3\otimes \sigma_1\otimes \sigma_1$	$\frac{1}{2}(\|001\rangle\|\psi_-\rangle-\|010\rangle\|\phi_-\rangle+\|111\rangle\|\phi_+\rangle-\|100\rangle\|\psi_+\rangle)$
$\sigma_3\otimes i\sigma_2\otimes \sigma_1$	$\frac{1}{2}(\|001\rangle\|\psi_-\rangle-\|010\rangle\|\phi_-\rangle-\|111\rangle\|\phi_+\rangle+\|100\rangle\|\psi_+\rangle)$
$\sigma_1\otimes I\otimes \sigma_1$	$\frac{1}{2}(\|100\rangle\|\phi_-\rangle+\|111\rangle\|\psi_-\rangle+\|001\rangle\|\phi_+\rangle+\|010\rangle\|\psi_+\rangle)$
$\sigma_1\otimes \sigma_3\otimes \sigma_1$	$\frac{1}{2}(\|100\rangle\|\phi_-\rangle-\|111\rangle\|\psi_-\rangle+\|001\rangle\|\phi_+\rangle-\|010\rangle\|\psi_+\rangle)$
$i\sigma_2\otimes I\otimes \sigma_1$	$\frac{1}{2}(\|001\rangle\|\phi_+\rangle-\|100\rangle\|\phi_-\rangle-\|111\rangle\|\psi_-\rangle+\|010\rangle\|\psi_+\rangle)$
$i\sigma_2\otimes \sigma_3\otimes \sigma_1$	$\frac{1}{2}(\|100\rangle\|\phi_-\rangle+\|111\rangle\|\psi_-\rangle-\|001\rangle\|\phi_+\rangle-\|010\rangle\|\psi_+\rangle)$

It is worth mentioning that all the calculations in the paper with regard to teleportation, state sharing, and superdense coding could be carried out using the following state:

$$|\psi_5\rangle = \frac{1}{2}(|\Omega_1\rangle|\phi_-\rangle + |\Omega_2\rangle|\psi_-\rangle + |\Omega_3\rangle|\phi_+\rangle + |\Omega_4\rangle|\psi_+\rangle), \tag{19}$$

where $|\Omega_i\rangle$'s form a tripartite orthogonal basis. However, the Brown state makes it possible for Alice to have any three particles because it has the same form for all (3+2) splits [32]. All the applications considered in this paper could also be carried out using a state of the type

$$|\psi_5\rangle = A_1|001\rangle|\phi_-\rangle + A_2|010\rangle|\psi_-\rangle + A_3|100\rangle|\phi_+\rangle + A_4|111\rangle|\psi_+\rangle, \tag{20}$$

where A_i is an integer, if the following relations are satisfied:

$$-\sum_{n=1}^{4} A_i^2(1 + \log_2 A_i^2) = 1, \tag{21}$$

$$-(A_3^2 + A_4^2)\log_2(A_3^2 + A_4^2) = \frac{1}{2}. \tag{22}$$

This is a necessary but not sufficient condition.

VIII. CONCLUSION

We have shown that the new five-partite state obtained by Brown *et al.* [1] has many useful applications in quantum information. We show that this state can be physically realized by a pair of Bell states and a single-qubit state. We use this state for perfect teleportation and quantum-state sharing of arbitrary one-qubit and two-qubit states under different scenarios. This state is also a very useful resource for superdense coding. The superdense capacity for the state reaches the Holevo bound of five classical bits. The state under consideration helps one to carry out teleportation and superdense coding maximally. In the future, we wish to generalize these protocols for other states having an odd number of qubits and qudits. The decoherence property of this state also needs careful investigation in the case of any practical applications. The comparison between the cost function and decoherence properties of different classes of states in Eq. (19) and the Brown state can also be an interesting future work. A series of papers will follow.

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[1] I. D. K. Brown, S. Stepney, A. Sudbery, and S. L. Braunstein, J. Phys. A **38**, 1119 (2005).
[2] M. B. Plenio and S. Virmani, Quantum Inf. Comput. **7**, 1 (2007).
[3] D. Bouwmeester, J. W. Pan, K. Mattle, M. Eibl, H. Weinfurter, and A. Zeilinger, Nature (London) **390**, 575 (1997).
[4] M. Riebe, H. Hffner, C. F. Roos, W. Hnsel, J. Benhelm, G. P. T. Lancaster, T. W. Krber, C. Becher, F. S. Kaler, D. F. V. James, and R. Blatt, Nature (London) **429**, 734 (2004).
[5] M. D. Barrett, J. Chiaverini, T. Schaetz, J. Britton, W. M. Itano, J. D. Jost, E. Knill, C. Langer, D. Leibfried, R. Ozeri, and D. J. Wineland, Nature (London) **429**, 737 (2004).
[6] I. Marcikic, H. de Riedmatten, W. Tittel, H. Zbinden, and N. Gisin, Nature (London) **421**, 509 (2003).
[7] R. Ursin, T. Jennewein, M. Aspelmeyer, R. Kaltenbaek, M. Lindenthal, P. Walther, and A. Zeilinger, Nature (London) **430**, 849 (2004).
[8] M. A. Nielsen and I. L. Chuang, *Quantum Computation and Quantum Information* (Cambridge University Press, Cambridge, England, 2002).
[9] C. H. Bennett, G. Brassard, C. Crepeau, R. Jozsa, A. Peres, and W. K. Wootters, Phys. Rev. Lett. **70**, 1895 (1993).
[10] R. F. Werner, J. Phys. A **34**, 7081 (2001).
[11] A. Karlsson and M. Bourennane, Phys. Rev. A **58**, 4394 (1998).
[12] P. Agrawal and A. Pati, Phys. Rev. A **74**, 062320 (2006).
[13] V. N. Gorbachev, A. I. Trubilko, and A. A. Rodichkina, Phys. Lett. A **314**, 267 (2003).
[14] H. J. Briegel and R. Raussendorf, Phys. Rev. Lett. **86**, 910 (2001).
[15] R. Raussendorf and Hans J. Briegel, Phys. Rev. Lett. **86**, 5188 (2001).
[16] L. Li and D. Qiu, J. Phys. A **40**, 10871 (2007).
[17] G. Rigolin, Phys. Rev. A **71**, 032303 (2005).
[18] Y. Yeo and W. K. Chua, Phys. Rev. Lett. **96**, 060502 (2006).
[19] G. Rigolin, e-print arXiv:quant-ph/0407193v2.
[20] P. Agrawal and B. Pradhan, e-print arXiv:quant-ph/0707.4295v2.
[21] C. H. Bennett, D. P. DiVincenzo, J. A. Smolin, and W. K. Wootters, Phys. Rev. A **54**, 3824 (1996).
[22] R. Laflamme, C. Miquel, J. P. Paz, and W. H. Zurek, Phys. Rev. Lett. **77**, 198 (1996).
[23] Z. Zhao, Y. A. Chen, T. Yang, and H. J. Briegel, Nature (London) **430**, 54 (2004).
[24] A. Borras, A. R. Plastino, J. Batle, C. Zander, M. Casas, and A. Plastino, J. Phys. A **40**, 13407 (2007).
[25] G. Rigolin, T. R. de Oliveira, and M. C. de Oliveira, Phys. Rev. A **74**, 022314 (2006).
[26] D. Liu, X. Zhao, G. L. Long, e-print arXiv:quant-ph/0705.3904v4.
[27] A. Higuchi and A. Sudbery, Phys. Lett. A **273**, 213 (2000).
[28] F. G. Deng, X. H. Li, C. Y. Li, P. Zhou, and H. Y. Zhou, Phys. Rev. A **72**, 044301 (2005).
[29] C. H. Bennett and S. J. Wiesner, Phys. Rev. Lett. **69**, 2881 (1992).
[30] D. Bruss, G. M. D'Ariano, M. Lewenstein, C. Macchiavello, and A. Sen(De), and U. Sen, Phys. Rev. Lett. **93**, 210501 (2004).
[31] B. Pradhan, P. Agrawal, and A. K. Pati, e-print arXiv:quant-ph/0705.1917v1.
[32] A. Sudbery (private communication).