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Observation of optomechanical multistability in a high-Q torsion balance oscillator

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We observe the optomechanical multistability of a macroscopic torsion balance oscillator. The torsion oscillator forms the moving mirror of a hemispherical laser light cavity. When a laser beam is coupled into this cavity, the radiation pressure force of the intracavity beam adds to the torsion wire's restoring force, forming an optomechanical potential. In the absence of optical damping, up to 23 stable trapping regions were observed due to local light potential minima over a range of 4 μm oscillator displacement. Each of these trapping positions exhibits optical spring properties. Hysteresis behavior between neighboring trapping positions is also observed. We discuss the prospect of observing optomechanical stochastic resonance, aiming at enhancing the signal-to-noise ratio (SNR) in gravity experiments.

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The coupling of mechanical oscillator systems to the resonant radiation pressure field of a light cavity has been studied for various systems [1–13]. The applications under investigation range from radiation pressure dynamics and noise reduction in gravitational wave detectors [1,10,12], to cooling of micro-oscillators and nano-oscillators to their quantum-mechanical ground state [9,11]. A common feature of these systems is that the mechanical oscillator forms a part of a resonant optical cavity, and radiation pressure modifies the potential which the oscillator sees, locally changing its shape [3]. In most cases, this leads to the creation of additional potential minima, which serve as trapping positions for the oscillator, and due to a different potential gradient, this change can also tremendously change its intrinsic oscillation properties [3,13].

In a seminal work, Dorsel *et al.* [2] observed optical bistability of a cavity in which one mirror is suspended on a long pendulum. Furthermore, multistability was predicted [3] for pendular systems in two or three mirror configurations. However, due to the large restoring force associated with a pendulum, only a minute displacement was obtained with large laser powers (>100 mW) used in the experiment. Subsequently, only a single optomechanically coupled cavity resonance contributed experimentally. Another important set of pendular experiments are the suspended Fabry-Perot cavities in gravitational wave detectors [12], where optomechanical effects are observed in the presence of intracavity light power levels in the W range. Avoiding the disadvantages of pendular suspensions, the coupling of nano-oscillators and micro-oscillators to optical cavities has been successfully demonstrated. However, the results are mostly restricted to a higher frequency regime >1 kHz up to the MHz range, equaling those oscillators' mechanical eigenfrequencies.

In this paper, we report the observation of optomechanical multistability in the lowest swinging mode of the coupled oscillator using a torsion balance oscillator as the moving mirror. Moreover, an electronic feedback system is implemented to further reduce the torsion oscillator's restoring force. The system is sensitive for radiation pressure forces down to the femto-Newton (fN) range [22], and its equivalent noise temperature can be reduced electronically to as low as 300 mK [14], giving a temperature reduction of almost 10^6 with respect to the dominant microseismic noise. With applied cooling, the noise on the oscillator is greatly reduced. Thus, the detection system can operate in a very high sensitivity mode, enabling precise control of the oscillator's dynamic behavior. Applying a slowly changing force to the oscillator, we observe "hopping" between neighboring trapping states, either unidirectional in a staircaselike manner, or bidirectional between two or more states, also showing hysteresis behavior. Finally, with such a multistable system, it becomes possible to investigate another interesting effect, namely the stochastic resonance between neighboring trapping states [15–17]. Exploiting this technique, it appears feasible to further enhance the signal-to-noise ratio (SNR) in bistable optomechanical systems [17].

The experimental system is shown schematically in Fig. 1. It consists mainly of a torsional oscillator [18] made of a gold-coated glass plate, 50 mm × 10 mm × 0.15 mm in size, doubly suspended on a 15 cm long, 25 μm thick tungsten wire. The gold-coated glass plate serves as the moving flat mirror of a hemispherical optical cavity. The oscillator body has a mass of ~0.2 g and a moment of inertia $I=4.4\times10^{-8}$ kg m^2. The torsion constant is measured to be $\tau=2.2\times10^{-7}$ Nm rad^{-1}. The torsion pendulum has a natural frequency of $f_0=0.35$ Hz with a quality factor $Q\sim2600$.

A laser beam is reflected from the center of the oscillator and then detected by a high-sensitivity quadrant diode detector followed by a lock-in detector [19]. The voltage signal proportional to the oscillator's angular position is digitized at a sampling rate of 5 kHz. This scheme allows for measuring the oscillator's angular position θ with an accuracy of 2 nrad Hz$^{-1/2}$. The signal is then used as the input of a computerized, digital proportional (P) and differential (D) digital control loop. The generated digital computer control signal is then converted to an analog output control signal. The end of the torsion balance opposite to the cavity side is placed between two electronic feedback electrodes. Since the torsion balance is electrically grounded, varying the voltages applied

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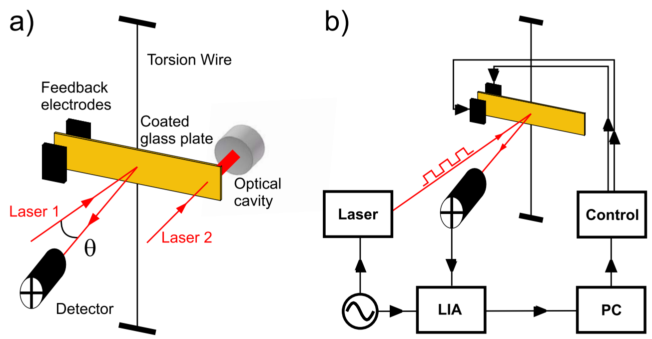

FIG. 1. (Color online) Simplified schematic of the experiment. The torsion balance oscillator (a) is a well-known precision force measurement device, sensitive down to the fN range. In addition, there exist simple linear control techniques for the applied electrostatic feedback (b). By choosing the axis of interaction to be horizontally aligned, we avoid disturbing effects of seismic surrounding which appear in standard low-frequency pendula. Therefore, the optical coupling into the hemispherical cavity is stable enough to observe low order TEM cavity modes. LIA: Lock-in amplifier.

to the feedback electrodes enables efficient control of the balance's angular position [20,21]. This gives the possibility of active damping (or heating) of the torsion oscillator, as well as controlling its torsion constant. In the experiment, the effective eigenfrequency can be changed easily within a range from 20 mHz to 3 Hz [14]. This means that the effective torsion constant can be reduced to $\tau_{\mathrm{eff}} \approx 7 \times 10^{-10}$ Nm rad^{-1}. For the tungsten wire used in the experiment (diameter 25 μm), a length of ~9 m is required to give such a "soft" restoring potential. The apparatus is placed in a high vacuum (10^{-7} mbar) environment, giving the high quality factor $Q \sim 2600$. The entire setup is mounted on top of an active vibration isolation system.

The hemispherical optical cavity is formed by the gold-coated glass surface and a second, rigidly mounted spherical mirror with a radius of curvature 25 mm, at a distance of 12.5 mm. Experimentally, we observe Laguerre-Gaussian TEM_{00} and TEM_{20} modes with a free spectral range (FSR) of the fundamental mode at ~13.5 GHz. This optical cavity has a finesse of $F=11$, giving a mirror reflectivity of $R=0.87$.

For the free torsion oscillator, the mechanical restoring force is proportional to the linear displacement, resulting in a quadratic mechanical potential. Now, the position-dependent radiation pressure inside an optical cavity is added. This cavity is approximated to be of a Fabry-Perot type. The position-dependent intracavity light power $P(x)$ produces a radiation pressure force $F(x) \propto 2P(x)/c$. Adding the potential of this radiation pressure force for a cavity of length d to that of the torsion wire, we obtain an analytic form for the total potential

$$U(x) = \frac{\tau}{2L^2}x^2 - \frac{2P_0}{c(1-R^2)} \frac{\lambda}{2\pi\sqrt{1+\left(\frac{2F}{\pi}\right)^2}} \times \tan^{-1}\left[\sqrt{1+\left(\frac{2F}{\pi}\right)^2}\tan\left[\frac{2\pi(d-x)}{\lambda}\right]\right]. \quad (1)$$

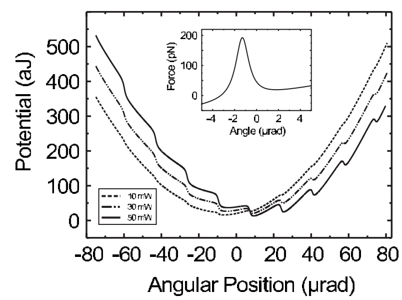

FIG. 2. Analytic calculation of the optomechanical potential for an amplitude range of 160 μrad. By increasing the optical input power into the cavity successively from 10 to 50 mW, stable position minima are formed. This is caused by a constant torsion wire restoring force added to the repulsive cavity radiation pressure force. Due to the cavity's low finesse, a small force caused by light coupled into an off-resonance cavity leads to a shift of the overall potential minima. The inset shows the optomechanical force for an optical input power of 10 mW. The oscillator's angular zero position coincides with a falling edge of the cavity resonance. Thus, an additional radiation pressure force offset is seen, even for very high cavity finesse.

Here, x is the moving mirror's linear displacement, L is the balance arm's half length, P_0 is the light power incident to the cavity, c is the speed of light, and λ is the optical wavelength. Here, we explicitly exclude a time dependence in the optomechanical potential. This assumption is justified because the measured resonant linewidth of our cavity $\gamma \approx 1.2$ GHz is far greater than the mechanical eigenfrequency. In other words, the oscillator's restoring force $F(x) = -\nabla U(x) = [k_0 + k_{\mathrm{cav}}(x)]x \approx k_{\mathrm{cav}}(x)x$ does not contain imaginary parts in the optical "spring" constant k_{cav}, which otherwise will lead to damping or heating. Here, the torsion balance's mechanical restoring constant $\propto k_0$ can be experimentally adjusted to be negligibly small [14]. Therefore, at any given time t, the intracavity intensity is determined only by the cavity length given by the linear moving mirror displacement x. Figure 2 shows a plot of the optomechanical potential given in Eq. (1) for three different optical input powers to the cavity. In this case, possible higher order TEM modes are neglected. This analytical result is very similar to that in Ref. [2]. For a pure quadratic potential $U(x)$, the conservative force $F(x)$ due to the relation $F(x) = -\nabla U(x)$ should be linear over the oscillator's position. It is important to note that the region of maximum cavity transmission is mechanically extremely unstable. The trapping positions are located in the area of maximum force gradients.

In order to investigate the effect of multistability of the oscillator potential, we first experimentally lower its natural frequency down to ~70 mHz. The reason is simply that by flattening the overall mechanical potential, the potential contribution by the light field becomes dominant. This is achieved by applying a well-chosen inverse proportional feedback signal using the digital feedback control. Effectively, this creates a "softer spring." In addition, we supply a weak velocity-dependent damping force which cools the oscillator by removing fluctuation but does not influence the torsion constant. This scheme is applied in all subsequent measurements.

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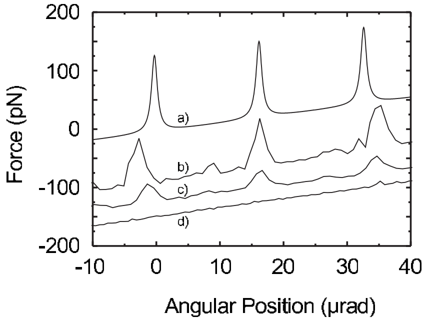

FIG. 3. Theoretical and measured optomechanical forces as functions of angular position. (a) Theoretical calculation for an incident light power of 5 mW. (b), (c) Measurements for light powers of 15 and 3 mW, respectively, entering the cavity. Close to resonance, instabilities prohibit the oscillator from moving to maximum intracavity light intensity regions, causing the amplitude to decrease. (d) Torsion oscillator's intrinsic restoring force, measured by blocking the incident light. All curves are shifted for better view. The experimental results in (b) and (c) are in good agreement with theoretical prediction (a).

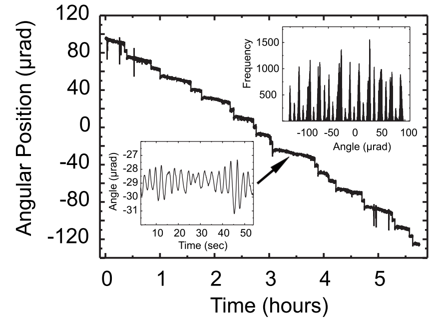

FIG. 4. The torsion wire's linear drift shows 23 stable trapping positions which are mainly caused by the TEM_{00} and TEM_{20} modes of the cavity. The upper inset shows a histogram plot over the full drift range, indicating that some of the main modes are split. This fact can be explained by trapping in higher order cavity modes (with lower probability). The lower inset shows a zoomed-in time trace of the central trapping position. Here, the oscillator clearly follows an optical spring behavior. Its oscillation period has changed from the soft spring period of 15 to 2.5 s.

We first map the potential of the moving mirror by measuring the force which is necessary to hold the oscillator arm at a well-defined position. In the presence of the cavity light field, we use the electronic feedback loop for scanning the oscillator's angular position and simultaneously measure the equivalent electrostatic force for stabilizing the holding position. Figure 3 shows measurements for cavity light input powers of 15 and 3 mW, respectively. We find distinct force maxima at equidistant positions, if the additional cavity force field is present. The spacing of such maxima equals the calculated and measured FSR of our cavity. In comparison to the theoretical calculation [shown in Fig. 3(a)], we find good experimental agreement for the curve's shape. As mentioned earlier, we expect a very unstable mechanical position exactly at the cavity resonances. This is seen in comparison of the amplitudes of the optomechanical force peaks. Measured peaks are only ~1/5 in amplitude compared to the calculated ones. In other words, in the region of maximum cavity intensity, the moving mirror does not remain stable long enough for experimental averaging.

If the oscillator is allowed to drift slowly after applying a constant offset voltage, it seeks a new overall equilibrium position given by the wire potential. Instead of a linear drift which is expected for the free torsion oscillator, the cavity-coupled system should seek locally stable positions. Figure 4 shows the time trace of the torsion balance's angular position in this experimental configuration. We observe "hopping" of the moving mirror between neighboring trapping positions. Plotting a position histogram (upper inset of Fig. 4), we find as many as 23 locally stable positions over a range of ~220 μrad in angular position, or equivalent to ~4 μm of linear displacement. The trapping potential minima are mainly formed by TEM_{20} and TEM_{20} cavity modes, equally spaced in position. The local curvature of the optomechanical potential is expected to be steeper inside a trapping region. This effectively changes the local torsion constant of the torsion oscillator. Such a behavior can be seen in the lower inset of Fig. 4. It shows a magnified view of the time trace for the central trapping position. We find that the adjusted (~70 mHz) oscillation frequency in the absence of light force now changes to a local oscillation frequency of ~0.4 Hz, indicating the well-known optical spring effect [7,12,13]. We note that the oscillator shows a simultaneous amplitude reduction due to "confinement" in the presence of the optical spring [13].

Finally, it is interesting to investigate whether the torsion oscillator will display positional hysteresis when we allow only a limited displacement range. To do this, the oscillator is moved to a region containing two stable trapping positions by applying an offset voltage to the control electrodes. In addition, a small modulation voltage of 20 mV *p-p* in amplitude at a frequency of 10 mHz is applied to the feedback electrodes. This gives a force modulation of 52 pN *p-p*. The adjustment and modulation procedure induces a discrete change between two neighboring trapping positions. Figure 5 shows the time trace of this bistable oscillator. Instead of following the applied modulation sinusoidally, the moving

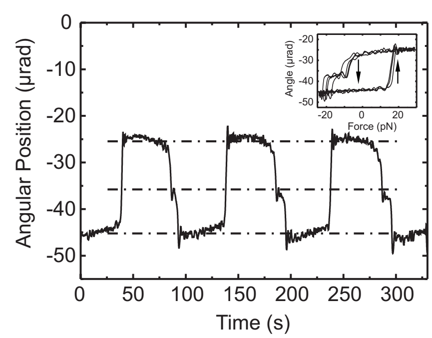

FIG. 5. The oscillator's angular position changes in discrete steps when a sinusoidal electrostatic modulation is applied. The left slope is steeper than the right one due to the asymmetric shape of the optomechanical potential. In addition, a higher order unstable trapping state is found on the right slope. The inset shows a plot of the angular position over the modulation force applied. The behavior of hysteresis is clearly seen.

mirror "jumps" between two distinct positions. When we plot its angular position over the applied modulation force (inset of Fig. 5), we find a clear hysteresis in position. The effect of optomechanical position hysteresis is thus verified experimentally for this optomechanical system.

In conclusion, we have demonstrated a macroscopic, controlled, optomechanical oscillator system. Here, the oscillator is efficiently coupled to the resonant modes of a hemispherical light cavity. We are able to exclude the effect of velocity-dependent cooling and/or heating radiation pressure forces. In combination, a quasistatic multistable system is formed which exhibits a wealth of optomechanical effects such as optical spring effects, hopping, and position hysteresis. We believe that the presented experimental system is a useful test environment for further exploration of multistability effects in the low-frequency regime. The extent of the system's optomechanical coupling is linearly tunable with high stability, opening the way for additional related experiments. For example, the hopping behavior reported here will be further examined in the presence of added noise, similar to that of the stochastic resonance. The dynamics of the multistable system in a noisy environment will also be studied, particularly in light of its claimed applications [15–17] on enhancing SNR for sensitive force detection systems.

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