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Elementary optical gate for expanding an entanglement web

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We introduce an elementary optical gate for expanding polarization entangled W states in which every pair of photons is entangled alike. The gate is composed of a pair of 50:50 beamsplitters and ancillary photons in the two-photon Fock state. By seeding one of the photons in an n-photon W state into this gate, we obtain an $(n+2)$-photon W state after postselection. This gate gives a better efficiency and a simpler implementation than previous proposals for W-state preparation.

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I. INTRODUCTION

Quantum entanglement lies at the heart of most of the quantum-information processing tasks, e.g., teleportation [1], key distribution (QKD) [2], and computation [3]. Entanglement between a pair of systems is fairly simple in the sense that there is a maximally entangled state from which any entangled state can be generated under local operations and classical communication (LOCC). In contrast, multipartite entanglement among three or more systems exhibits more variety. For example, three qubits can be entangled in two inequivalent ways, namely, the Greenberger-Horne-Zeilinger (GHZ) state $|\mathrm{GHZ}\rangle=(|000\rangle+|111\rangle)/\sqrt{2}$ and the W state $|W\rangle=(|001\rangle+|010\rangle+|100\rangle)/\sqrt{3}$ can never be converted to each other under LOCC, even probabilistically [4].

The distinction between these two types of entanglement becomes clearer if we consider their generalizations to the N-qubit case: $|W_N\rangle=|N-1,1\rangle/\sqrt{N}$ and $|\mathrm{GHZ}_N\rangle=(|N,0\rangle+|0,N\rangle)/\sqrt{2}$, where $|N-k,k\rangle$ is the sum over all the terms with $N-k$ modes in $|0\rangle$ and k modes in $|1\rangle$. In $|W_N\rangle$, every pair of qubits is entangled with each other directly; namely, the pairwise entanglement survives even after the rest of the qubits are discarded [5–7]. In fact, it was shown that the state $|W_N\rangle$ has the maximum amount of such pairwise entanglement shared by every pair [5]. It looks as if it is forming a weblike structure in which every qubit has a bond with every other qubit [see Fig. 1(a)]. On the other hand, the entanglement in $|\mathrm{GHZ}_N\rangle$ is sustained by all of the N qubits, and loss of only one particle destroys the entanglement completely. But if access to every qubit is allowed, it shows a maximal violation of local realism [8]. These distinct properties make the W and GHZ states interesting resources for multiparty tasks and fundamental studies of quantum mechanics. Thus, there have been many proposals and experimental implementations in photons [9–22], trapped ions [23,24], and NMR systems [25].

The distinction also shows up when we consider how one can increase the number of qubits forming W or GHZ states. In the case of GHZ states, there is a systematic way to extend its size without accessing all of the qubits: One can pick the Nth qubit of $|\mathrm{GHZ}_N\rangle$ and let it interact with a new qubit to produce $|\mathrm{GHZ}_{N+1}\rangle$. This is not surprising since (i) the marginal state of the remaining untouched $N-1$ qubits is the same for $|\mathrm{GHZ}_N\rangle$ and $|\mathrm{GHZ}_{N+1}\rangle$, and (ii) the Nth qubit is pivotal such that if we remove and discard it, the rest of the qubits will be disentangled. On the other hand, it is not so trivial whether such a local extension of W states is possible

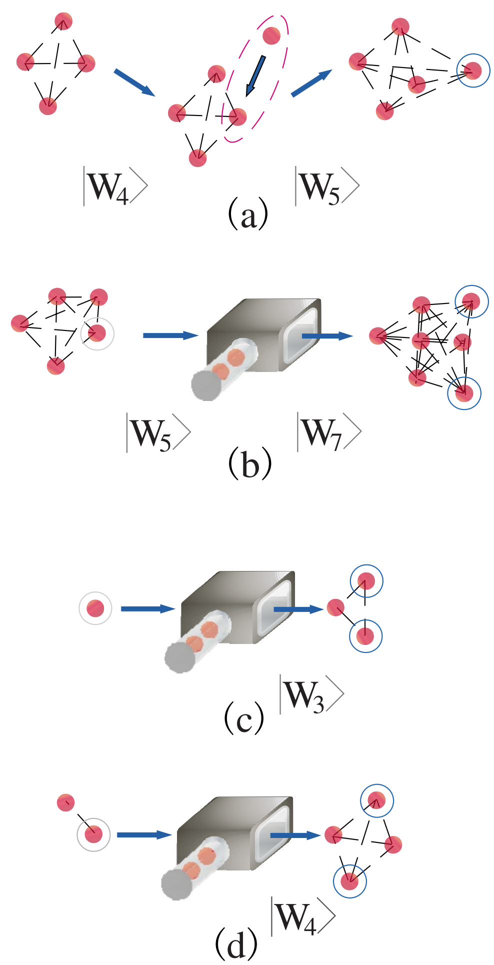

FIG. 1. (Color online) (a) Local extension of W states. (b) The proposed optical gate (T^W_{+2}) converts $|W_N\rangle$ to $|W_{N+2}\rangle$. (c) If a photon in state $|1\rangle$ is seeded, the gate produces $|W_3\rangle$. (d) If we start with two photons in state $(|01\rangle+|10\rangle)/\sqrt{2}$, we obtain $|W_4\rangle$.

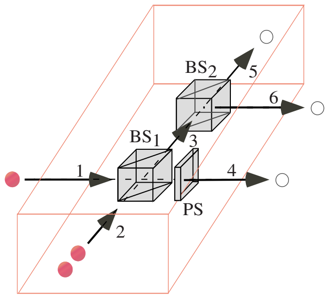

FIG. 2. (Color online) The schematic diagram of the setup for T^W_{+2} gate.

or not. For one thing, the marginal states of $N-1$ qubits are different for $|W_N\rangle$ and $|W_{N+1}\rangle$. Hence no unitary operation on the Nth qubit and a new qubit makes $|W_{N+1}\rangle$. In addition, newly added qubits must form the pairwise entanglement with each of the uninteracted $N-1$ qubits [see Fig. 1(a)].

In this paper, we show that such a local extension of polarization-entangled photonic W states is possible using a surprisingly simple probabilistic gate composed of a two-photon Fock state, two 50:50 beamsplitters (BSs), and a phase shifter (PS), based on postselection. Interestingly, the same gate can be used for the expansion $|W_N\rangle \to |W_{N+2}\rangle$ of any size N [see Fig. 1(b)]. The gate also works for $N=1,2$ if we extrapolate the definition of W states naturally: seeding $|W_1\rangle = |1\rangle$ results in $|W_3\rangle$ [see Fig. 1(c)], and seeding an Einstein-Podolsky-Rosen (EPR) pair $|W_2\rangle = (|01\rangle + |10\rangle)/\sqrt{2}$ results in $|W_4\rangle$ [see Fig. 1(d)]. In particular, the latter case is experimentally easier and more efficient than any other linear optical scheme of generating $|W_4\rangle$ proposed so far. Starting with Fig. 1(c) and applying Fig. 1(b) successively $N-1$ times, we can prepare W states with an odd number of photons, $|W_{2N+1}\rangle$. In the same way, states with an even number of photons $|W_{2N}\rangle$ can be prepared starting with Fig. 1(d). Thus, in principle it is possible to prepare any $|W_N\rangle$ using this gate.

II. WORKING PRINCIPLE OF THE ELEMENTARY GATE T^W_{+2}

In Fig. 2 we show the schematic of the proposed gate. The gate receives one photon from mode 1 as the input, and mixes it by a 50:50 beamsplitter (BS1) with two ancilla photons in horizontal (H) polarization (we denote it by $|2_H\rangle_2$, where the subscript number signifies the spatial mode). One of the output modes of BS1 is further divided into two modes by another 50:50 beamsplitter (BS2). The gate operation is successful when each of the output modes 4, 5, and 6 has a photon. The phase shifter (PS), which is a half-wave plate introducing a π-phase shift between H and V (vertical) polarizations, is in place just in order to keep the final W state in the standard symmetric form.

First we analyze how the gate works when the input photon in mode 1 is H polarized ($|1_H\rangle_1$) or V polarized ($|1_V\rangle_1$). The action of BS1 on H polarization is represented by the transformation $\hat{a}^\dagger_{1H} = (\hat{a}^\dagger_{3H} - \hat{a}^\dagger_{4H})/\sqrt{2}$ and $\hat{a}^\dagger_{2H} = (\hat{a}^\dagger_{3H} + \hat{a}^\dagger_{4H})/\sqrt{2}$, where $\hat{a}^\dagger_{jH}$ is the photon creation operator for mode j in H polarization. We assume that the BS1 is polarization independent, namely, the transformation for V polarization has the same form. Using these relations, we see that the initial states $|1_{H(V)}\rangle_1 \otimes |2_H\rangle_2 = 2^{-1/2}\hat{a}^\dagger_{1H(V)}(\hat{a}^\dagger_{2H})^2|0\rangle$ evolve as

$$|1_H\rangle_1|2_H\rangle_2 \to \frac{\sqrt{3}}{2\sqrt{2}}|3_H\rangle_3|0\rangle_4 + \underline{\frac{1}{2\sqrt{2}}|2_H\rangle_3|1_H\rangle_4} - \frac{1}{2\sqrt{2}}|1_H\rangle_3|2_H\rangle_4 - \frac{\sqrt{3}}{2\sqrt{2}}|0\rangle_3|3_H\rangle_4,$$

$$\begin{aligned}|1_V\rangle_1|2_H\rangle_2 \to{}& \frac{1}{2\sqrt{2}}|1_V 2_H\rangle_3|0\rangle_4 + \underline{\frac{1}{2}|1_H 1_V\rangle_3|1_H\rangle_4} \\ &+ \frac{1}{2\sqrt{2}}|1_V\rangle_3|2_H\rangle_4 - \underline{\frac{1}{2\sqrt{2}}|2_H\rangle_3|1_V\rangle_4} \\ &- \frac{1}{2}|1_H\rangle_3|1_H 1_V\rangle_4 - \frac{1}{2\sqrt{2}}|0\rangle_3|1_V 2_H\rangle_4. \end{aligned} \quad (1)$$

For the gate operation to be successful, there must be two photons in mode 3 and one photon in mode 4. Hence we are interested only in the underlined terms. The states $|2_H\rangle_3$ and $|1_H 1_V\rangle_3$ appearing in the underlined terms are transformed at BS$_2$ as

$$|2_H\rangle_3 \to \frac{1}{2}|2_H\rangle_5|0\rangle_6 + \underline{\frac{1}{\sqrt{2}}|1_H\rangle_5|1_H\rangle_6} + \frac{1}{2}|0\rangle_5|2_H\rangle_6,$$

$$\begin{aligned}|1_H 1_V\rangle_3 \to{}& \frac{1}{2}|1_H 1_V\rangle_5|0\rangle_6 + \underline{\frac{1}{2}|1_H\rangle_5|1_V\rangle_6} + \underline{\frac{1}{2}|1_V\rangle_5|1_H\rangle_6} \\ &+ \frac{1}{2}|0\rangle_5|1_H 1_V\rangle_6. \end{aligned} \quad (2)$$

Clearly, only the underlined terms in Eq. (2) contributes to the successful operation. Therefore, if we postselect the successful events, the action of the gate is given by the following state transformations:

$$|1_H\rangle_1|2_H\rangle_2 \to \frac{1}{4}|1_H\rangle_4|1_H\rangle_5|1_H\rangle_6, \quad (3)$$

$$\begin{aligned}|1_V\rangle_1|2_H\rangle_2 \to{}& \frac{1}{4}|1_H\rangle_4|1_H\rangle_5|1_V\rangle_6 + \frac{1}{4}|1_H\rangle_4|1_V\rangle_5|1_H\rangle_6 \\ &+ \frac{1}{4}|1_V\rangle_4|1_H\rangle_5|1_H\rangle_6, \end{aligned} \quad (4)$$

where we have included the effect of PS. There are two essential features in this gate operation: One is the symmetrization among the input photon and the ancilla photons, and the other is that the success probability (1/16) for the $|1_H\rangle_1$ input is one third of the probability (3/16) for the $|1_V\rangle_1$ input. In other words, all the four terms appearing in Eq. (3) and Eq. (4) have the same amplitude.

The above calculation may be physically understood as follows. When the input is a V-polarized photon, we can always determine the origin of an output photon, namely, distinguish whether it has come from the mode 1 or the mode 2 by looking at its polarization. Hence the result is the same as in the case of classical distinguishable particles, and the symmetrization in Eq. (4) can be understood classically. On the other hand, when the input is an H-polarized photon, quantum interference comes into play. In this case, there are three indistinguishable paths leading to the final state $|1_H\rangle_4|1_H\rangle_5|1_H\rangle_6$, depending on which one of the three photons originates from the input mode 1. One of the paths (the input photon going to mode 4) bears the opposite sign from the other two, and hence the probability becomes one third due to destructive interference.

III. SEEDING AND EXPANDING POLARIZATION ENTANGLED W STATES

Since the right-hand side of Eq. (4) is $|W_3\rangle$, we can prepare state $|W_3\rangle$ with a success probability of $3/16$ by applying the gate T^W_{+2} on the input photon in state $|1_V\rangle_1$. If we prepare an EPR pair $(|1_H\rangle_0|1_V\rangle_1+|1_V\rangle_0|1_H\rangle_1)/\sqrt{2}$ and feed the spatial mode 1 into the T^W_{+2} gate, the two terms $|1_V\rangle_1$ and $|1_H\rangle_1$ should evolve coherently as in Eq. (3) and Eq. (4), leading to the $|W_4\rangle$ state

$$|W_4\rangle = \frac{1}{2}[|1_H\rangle_0|1_H\rangle_4|1_H\rangle_5|1_V\rangle_6 + |1_H\rangle_0|1_H\rangle_4|1_V\rangle_5|1_H\rangle_6 + |1_H\rangle_0|1_V\rangle_4|1_H\rangle_5|1_H\rangle_6 + |1_V\rangle_0|1_H\rangle_4|1_H\rangle_5|1_H\rangle_6] \quad (5)$$

with success probability $1/8$. These values of success probability are significant improvements over other linear optics-based schemes. For instance, the most efficient schemes so far are those in [12,14], respectively, for $|W_4\rangle$ and $|W_3\rangle$ with the corresponding success probabilities of $2/27$ and $1/9$, which are lower than those of our proposal.

Next we discuss how this gate can be used to expand a general W state $|W_N\rangle=|N-1,1\rangle/\sqrt{N}$, where $|N-1,1\rangle$ is the sum over all the terms with $N-1$ modes in $|1_H\rangle$ and one mode in $|1_V\rangle$. This state may be rewritten as $[|N-2,1\rangle \otimes |1_H\rangle_1+|N-1,0\rangle\otimes|1_V\rangle_1]/\sqrt{N}$. If we apply the gate T^W_{+2} on mode 1, we obtain $[|N-2,1\rangle\otimes|3,0\rangle+|N-1,0\rangle \otimes|2,1\rangle]/4\sqrt{N}=|N+1,1\rangle/4\sqrt{N}$, implying that the gate produces $|W_{N+2}\rangle$ with success probability $(N+2)/(16N)$, which approaches a constant $1/16$ when N becomes large. Note that while this probability partly comes from the inefficiency associated with linear optics schemes, the probabilistic nature itself plays an essential role of updating the marginal state of each of the untouched photons from $\rho_N \equiv N^{-1}[(N-1)|1_H\rangle\langle 1_H|+|1_V\rangle\langle 1_V|]$ to that of ρ_{N+2}. By cascading T^W_{+2} gates, we can prepare W states over 5 or more photons. Starting with $|1_V\rangle$ as an input and cascading the gate k times, one can prepare $(2k+1)$-photon W state, $|W_{2k+1}\rangle$, provided that coincidence detection is observed at $2k+1$ output spatial modes. The success probability of such an event is given by $p_{\text{success}}=(2k+1)2^{-4k}$. Similarly, starting with an EPR pair and cascading k gates, one can prepare $2(k+1)$-photon W state, $|W_{2(k+1)}\rangle$, with a success probability of $p_{\text{success}}=(k+1)2^{-4k}$. Besides our current proposal, the scheme based on $N\times N$ multiport interferometers [14,15] is so far the only proposal encompassing generation of $|W_N\rangle$ with arbitrary N. This scheme requires a different multiport device for each N. In addition, numerical calculation up to $N=7$ shows that our proposal has better efficiency, e.g., for $N=5$ our proposal succeeds with a probability 12 times higher than that of the multiport interferometer. Note also that $N\times N$ interferometers cannot generate the $|W_6\rangle$ state because of the zero probability of coincidence detection due to destructive interference.

IV. FEASIBILITY ANALYSIS FOR $|W_4\rangle$

So far, several linear optical schemes for preparing $|W_4\rangle$ have been proposed [12–14], but no experiments have been done yet. It is thus interesting to consider the feasibility of our scheme with practical photon sources, namely, parametric down conversion (PDC) and/or weak coherent pulses (WCP) obtained by attenuating laser pulses. First, suppose that an EPR pair and ancillary two photons are both generated from PDC with rates γ and g, respectively, which are $\sim 10^{-4}$ in typical experiments. In this case, the errors are mainly caused by generation of three pairs of photons in total. Such events occur with rate $O(\gamma^2 g)$ or $O(\gamma g^2)$, which is small compared with the rate $O(\gamma g)$ of the desired events. Alternatively, we may also use WCP instead of PDC for the ancillary photons in mode 2. If the mean photon number of WCP is ν, the desired events occur with rate $O(\gamma\nu^2)$. With the requirement $\nu \ll 1$ as usual, the main source of errors in this case is two-pair production at PDC, resulting in two photons in the input mode 1. Then, one photon in the WCP leads to triple coincidence at modes 4, 5, and 6, which occurs with rate $O(\gamma^2\nu)$. Thus we need to satisfy $\gamma \ll \nu \ll 1$to obtain a high fidelity. In both cases, the contribution of the dark counts of detectors are negligibly small in current experiments [26,27]. Mode mismatch effects may be minimized by proper spectral and spatial filtering as discussed in Ref. [28].

V. EXTENDING POLARIZATION ENTANGLED GHZ STATES

In our proposed gate, a W state is essentially produced before the PS in Fig. 2, which merely applies a local unitary operation. We then notice that the essential passive components (the two BSs) are polarization independent, and the polarization dependence of the gate stems solely from the polarization of the ancilla photons, i.e., the expansion of a W state is achieved even if we rotate the polarization of the photons in input modes 1 and 2 by the same angle. Interestingly, this indicates the possibility of expanding states other than W states by changing the polarization of ancillas. Indeed, the same set of the two BSs can also be used for the extension of the GHZ states, just by replacing the ancillary state $|2_H\rangle_2$ with $|1_H 1_V\rangle_2$. If the photon in mode 1 is V polarized, destructive two-photon interference kills the events with only one V-polarized photon in mode 4. Hence, under the condition that each of the output modes 4, 5, and 6 has a photon, the transformation is given by $|1_V\rangle_1 \rightarrow 2^{-3/2}|1_V\rangle_4|1_V\rangle_5|1_H\rangle_6$. Similarly, an H-polarized input will be transformed as $|1_H\rangle_1 \rightarrow 2^{-3/2}|1_H\rangle_4|1_H\rangle_5|1_V\rangle_6$. It is then obvious that this gate achieves $|\mathrm{GHZ}_N\rangle \rightarrow |\mathrm{GHZ}_{N+2}\rangle$ up to a local unitary, with a success probability of 1/8.

VI. CONCLUSION

In summary, we have proposed a simple elementary optical gate based on postselection for expanding the symmetrically shared entanglement in polarization entangled *W* states. With a proper seeding, the gate can also be used for preparation of *W* states, and it has a larger success probability than other preparation methods. We believe that the proposed gate is easy to implement and feasible with the current experimental technologies. In our gate, polarization-dependent components play no essential role, and the desired transformation is achieved by multiphoton interference between the input photon and the ancilla photons. Note that this does not require subwavelength adjustments. We have also shown that just by changing the state of the ancilla photons, we obtain a gate for extending GHZ states.

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