Compensation of noise in optical lattices via feedback: Low-temperature limit

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We consider the problem of suppression of noise acting on atomic ensembles trapped in optical lattices in the low-energy limit. Noise affecting external degrees of freedom of each atom independently and noise influencing only the center-of-mass (c.m.) mode of the ensemble are addressed. Taking into account the quantum character of the atomic motion, we show that negative feedback loop acting on the c.m. coordinate of the atomic ensemble is able to partially compensate both noise sources mentioned above.

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To date several schemes to realize quantum computation have been proposed [1–3]. Some of them use internal degrees of freedom of neutral atoms trapped in a far-off-resonance optical lattice [4–6]. For successful computation one should ensure that the decoherence time of the internal degrees of freedom of the atoms is much longer than the time necessary to produce the required number of operations. Also, the trap should be stable enough so that atoms do not escape it during the computation.

One of the loss channels arises from elastic collisions of atoms in the optical lattice with the background gas. The effect of this noise can be significantly diminished by reducing the background pressure. However, it can hardly be completely eliminated. The other channel is due to experimental imperfections resulting in space and amplitude fluctuations of the lattice field [7,8]. The amplitude fluctuations can be suppressed in principle, providing for the perfect stabilization of laser sources; whereas the elimination of the space fluctuations appearing mainly due to mechanical instabilities in different parts of the experimental setup is a serious technological problem.

In this paper, we show that the storage time of atoms in an optical lattice can be increased to some extent by stabilizing their motion via an appropriate feedback loop. It is worth noting that this method does not introduce additional sources of decoherence, since the feedback acts on external degrees of freedom not affecting internal ones.

Below we address two types of noise source. The first one acts on each atom independently and can be thought of as being the result of background collisions. The second one acts on all atoms simultaneously and can be referred to fluctuations of the positions of the lattice sites. Later on we will call the first type of noise a short-wavelength noise and the second a long-wavelength one.

First, we address the short-wavelength noise. Let us consider N identical atoms trapped in a one-dimensional optical lattice so that each lattice site contains only one atom. Distances between neighboring atoms in the optical lattice are about one-half of the wavelength of the lasers forming the lattice, which is equal to several hundreds of nanometers. The scattering length, which determines collision cross section, varies in the limits of 1–10 nm for alkali-metal atoms. Therefore, one can assume that in each act of scattering an atom of the background gas interacts only with one atom in the lattice. Thus, it can be suggested that each atom in the lattice interacts with its own reservoir, and the reservoirs of different atoms are independent. In this case, atoms in the lattice can be considered as being independent Brownian particles [9].

The derivation of the master equation for the reduced density operator $\hat{\rho}_s$ of atoms interacting with independent reservoirs is in direct analogy with that for a single quantum Brownian particle. It can be found, for example, in Ref. [10]. Therefore, here we omit the derivation and write down the final equation[1]

$$\frac{d\hat{\rho}_s}{dt} = -i[\hat{H}_s,\hat{\rho}_s] - i\gamma\sum_{\mu=1}^{N}[\hat{x}_\mu,\{\hat{p}_\mu,\hat{\rho}_s\}] - 2m\gamma k_B T_b\sum_{\mu=1}^{N}[\hat{x}_\mu,[\hat{x}_\mu,\hat{\rho}_s]] - \frac{\gamma}{8mk_B T_b}\sum_{\mu=1}^{N}[\hat{p}_\mu,[\hat{p}_\mu,\hat{\rho}_s]]. \quad (1)$$

Here $\hat{H}_s$ is the Hamiltonian of the atoms, and $\hat{x}_\mu$ and $\hat{p}_\mu$ are the coordinate and momentum of the μth atom, respectively. The parameter γ describes the coupling of the atoms to the reservoir; it determines the trapping time of atoms in the optical lattice, which can be estimated as $\tau_{\mathrm{trap}}=1/(2\gamma)$. The coupling constant is chosen to be the same for all atoms. Each reservoir is assumed to be in thermal equilibrium with temperature T_b, the mass of each atom is denoted by m, and k_B is the Boltzmann constant. Note that the last term in the right side of Eq. (1) does not appear in the standard derivation of the master equation for Brownian motion. This term is added to cast the master equation into the Lindblad form.

The evolution of atoms subject to noise has been studied in terms of second moments: $\mathcal{X}_{\alpha\beta}=\langle\hat{x}_\alpha\hat{x}_\beta\rangle$, $\mathcal{P}_{\alpha\beta}=\langle\hat{p}_\alpha\hat{p}_\beta\rangle$, $\Upsilon_{\alpha\beta}=\frac{1}{2}\langle\{\hat{x}_\alpha,\hat{p}_\beta\}\rangle$, and $\mathcal{Y}_{\alpha\beta}=\Upsilon_{\alpha\beta}+\Upsilon_{\beta\alpha}$, where α and β label atoms in the optical lattice. These quantities form $N\times N$ matrices.

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[1]Hereinafter we use $\hbar=1$.

Using master equation (1) the closed system of equations for matrices $\mathcal{X}$, $\mathcal{P}$, and $\mathcal{Y}$ can be straightforwardly found and reads

$$\dot{\mathcal{X}} = \mathcal{Y} + (\gamma/\xi_b)\mathcal{I},$$

$$\dot{\mathcal{P}} = -4\gamma\mathcal{P} - \mathcal{Y} + 4\gamma\xi_b\mathcal{I},$$

$$\dot{\mathcal{Y}} = -2\mathcal{X} + 2\mathcal{P} - 2\gamma\mathcal{Y}. \tag{2}$$

Here, $\mathcal{I}$ is the unit matrix. For the sake of convenience the time in Eq. (2) is written in units of $\tau/(2\pi)$, where τ is the oscillation period of the atom in the lattice potential. The decay constant γ is measured in units of angular frequency ω_0 of these oscillations. We also assume the coordinates and momenta of the atoms to be normalized by the corresponding ground-state uncertainties $\Delta x_0 = 1/\sqrt{2m\omega_0}$ and $\Delta p_0 = \sqrt{m\omega_0/2}$. $\xi_b = k_B T_b / E_0$ denotes the average energy of the reservoir in units of the ground-state energy $E_0 = \omega_0/2$. The system (2) of ordinary differential equations with time-independent coefficients can be solved analytically, revealing the time evolution of the average energy of atoms $[E = \Sigma_{\alpha=1}^{N}(\mathcal{X}_{\alpha\alpha} + \mathcal{P}_{\alpha\alpha})/2]$ and two-atom correlations.

Let us now consider the long-wavelength noise. This noise is caused by random vibrations of different elements of the experimental setup. Assuming that all the atoms experience the same vibrations, the master equation can be derived as

$$\frac{d\hat{\rho}_s}{dt} = -i[\hat{H}_s, \hat{\rho}_s] - i\gamma \sum_{\mu,\nu=1}^{N} [\hat{x}_\mu, \{\hat{p}_\nu, \hat{\rho}_s\}] - 2m\gamma k_B T_b \sum_{\mu,\nu=1}^{N} [\hat{x}_\mu, [\hat{x}_\nu, \hat{\rho}_s]] - \frac{\gamma}{8mk_B T_b} \sum_{\mu,\nu=1}^{N} [\hat{p}_\mu, [\hat{p}_\nu, \hat{\rho}_s]]. \tag{3}$$

Using this master equation, one obtains the following equations of motion for the matrices of second moments $\mathcal{X}$, $\mathcal{P}$, and $\mathcal{Y}$:

$$\dot{\mathcal{X}} = \mathcal{Y} + (\gamma/\xi_b)\mathcal{E},$$

$$\dot{\mathcal{P}} = -2\gamma(\mathcal{P}\mathcal{E} + \mathcal{E}\mathcal{P}) - \mathcal{Y} + 4\gamma\xi_b\mathcal{E},$$

$$\dot{\mathcal{Y}} = -2\mathcal{X} + 2\mathcal{P} - 2\gamma(\Upsilon\mathcal{E} + \mathcal{E}\Upsilon^{\dagger}). \tag{4}$$

Here, $\mathcal{E}$ is an $N \times N$ matrix with all unit elements. In addition to the correlations we are interested in, these equations contain terms proportional to $\mathcal{P}\mathcal{E} + \mathcal{E}\mathcal{P} = \Sigma_{\mu=1}^{N}(\mathcal{P}_{\alpha\mu} + \mathcal{P}_{\beta\mu})$ and $\Upsilon\mathcal{E} + \mathcal{E}\Upsilon^{\dagger} = \Sigma_{\mu=1}^{N}(\Upsilon_{\alpha\mu} + \Upsilon_{\beta\mu})$, which have the meaning of correlations between single atoms and the center of mass of all of them.

To solve Eq. (4) we apply the following procedure. We introduce new variable matrices $\mathcal{Q} = \mathcal{X}\mathcal{E} + \mathcal{E}\mathcal{X}$, $\mathcal{M} = \mathcal{P}\mathcal{E} + \mathcal{E}\mathcal{P}$, $\mathcal{G} = \Upsilon\mathcal{E} + \mathcal{E}\Upsilon^{\dagger}$, and $\mathcal{Z} = \mathcal{Y}\mathcal{E} + \mathcal{E}\mathcal{Y}$. The evolution equations for these matrices can easily be found using Eq. (4):

$$\dot{\mathcal{Q}} = \mathcal{Z} + (2\gamma N/\xi_b)\mathcal{E},$$

$$\dot{\mathcal{M}} = -2\gamma N\mathcal{M} - 4\gamma\mathcal{E}\mathcal{P}\mathcal{E} - \mathcal{Z} + 8\gamma\xi_b N\mathcal{E},$$

$$\dot{\mathcal{G}} = -\mathcal{Q} + \mathcal{M} - 2\gamma N\mathcal{G},$$

$$\dot{\mathcal{Z}} = -2\mathcal{Q} + 2\mathcal{M} - 2\gamma N\mathcal{G} - 4\gamma\mathcal{E}\mathcal{Y}\mathcal{E}. \tag{5}$$

This system of equations is also not closed since it contains degenerate matrices like $\mathcal{E}\mathcal{P}\mathcal{E}$ and $\mathcal{E}\mathcal{Y}\mathcal{E}$, whose elements are $\Sigma_{\mu,\nu=1}^{N}\mathcal{P}_{\mu\nu}$ and $\Sigma_{\mu,\nu=1}^{N}\mathcal{Y}_{\mu\nu}$, respectively. However, for these second moments of the center-of-mass variables, which we denote by $\mathcal{W} = \mathcal{E}\mathcal{X}\mathcal{E}$, $\mathcal{L} = \mathcal{E}\mathcal{P}\mathcal{E}$, and $\mathcal{D} = \mathcal{E}\Upsilon\mathcal{E}$, one finally obtains the following closed system:

$$\dot{\mathcal{W}} = 2\mathcal{D} + (\gamma N^2/\xi_b)\mathcal{E},$$

$$\dot{\mathcal{L}} = -4\gamma N\mathcal{L} - 2\mathcal{D} + 4\gamma\xi_b N^2\mathcal{E},$$

$$\dot{\mathcal{D}} = -\mathcal{W} + \mathcal{L} - 2\gamma N\mathcal{D}. \tag{6}$$

The solution to the system (6) can be found in a standard way. Then this solution is inserted in (5), which now contains only the correlations between a single atom and the center of mass (c.m.) and can be easily solved. Finally, the sought two-atom correlations are found from Eq. (4) using the solution of Eq. (5).

In both considered cases the coupling to the reservoirs leads to gradual heating of atoms until an equilibrium is reached. In the case of independent reservoirs (short-wavelength noise) the ensemble of atoms approaches the thermal equilibrium state with zero atom-atom correlations. The equilibrium energy of the atoms corresponds to the temperature of the reservoirs.

The situation is different for long-wavelength noise, where the correlations between atoms are generated in the course of interaction and the stationary state of the ensemble is far from the thermal equilibrium. Since only the c.m. of the ensemble is well coupled to the reservoir, the relative motion of the atoms remains unaffected. Thus the energy the ensemble can receive from the reservoir is much smaller than in the case of short-wavelength noise.

To reduce the heating and avoid the loss of atoms from an optical lattice without affecting their internal degrees of freedom one can use the method of feedback cooling. This approach has been proposed to cool trapped single atoms [11,12] and atomic ensembles [13–16].

To study the action of feedback cooling against the noise sources described above, we assume the following model of the feedback loop. The observable that is measured and affected is the collective coordinate $\hat{q} = (1/N)\Sigma_{\alpha=1}^{N}\hat{x}_\alpha$. Here, $\hat{x}_\alpha$ is the deviation of the atom from the equilibrium position in the αth lattice site. The collective coordinate can be measured by detecting the imbalance in intensities of the laser beams forming the lattice due to the interaction with atoms [15,17]. The result of the measurement of $\hat{q}$ is then sent to a feedback circuit, which produces the spatial translation of the

optical lattice potential to compensate for the measured coordinate. The compensation of this coordinate leads to a decrease in the total potential energy of atoms and, hence, protects the system against noise-induced heating.

Here we consider a series of instantaneous feedback actions consisting of the coordinate measurement and its shift via the lattice translation. Another approach where the system is continuously monitored and controlled is possible [18,19]. This situation can be regarded as a limiting case of a discrete series of feedbacks with vanishing time intervals between them. This suggests that certain features of continuous feedback can be revealed within the discrete model used by taking sufficiently small time intervals.

A detailed quantum-mechanical analysis of the feedback action on atoms in an optical lattice including effects due to the measurement-induced noise and the feedback-induced atom-atom correlations can be found in [20,21].

In terms of the matrices of the second moments introduced above, the action of feedback is given by the following transformations [20,21]:

$$\mathcal{X}' = \sigma^2\mathcal{E} + \mathcal{X} - \frac{1}{N}(\mathcal{X}\mathcal{E} + \mathcal{E}\mathcal{X}) + \frac{1}{N^2}\mathcal{E}\mathcal{X}\mathcal{E},$$

$$\mathcal{P}' = \frac{1}{N^2\sigma^2}\mathcal{E} + \mathcal{P},$$

$$\mathcal{G}' = \mathcal{G} - \frac{1}{N}\mathcal{E}\mathcal{Y}\mathcal{E},$$

$$\mathcal{Y}' = \mathcal{Y} - \frac{1}{N}\tilde{\mathcal{G}}, \quad \tilde{\mathcal{G}}' = 0. \tag{7}$$

Here, symbols with and without prime denote the correlation matrices after and before the feedback, respectively. The new matrix $\tilde{\mathcal{G}}=\mathcal{E}\Upsilon+\Upsilon^{\dagger}\mathcal{E}$ is required to obtain a closed set of equations. The parameter σ denotes the resolution of the measurement. The limit $\sigma\to 0$ corresponds to an ideal measurement with infinite accuracy. The measurement of the collective coordinate with the finite accuracy σ results in the measurement-induced noise, which reveals itself as a term proportional to σ in the equation for the coordinate correlation matrix $\mathcal{X}$. The term inversely proportional to σ in the equation for the momentum correlation matrix $\mathcal{P}$ describes the measurement back action. It is seen that due to the back action the feedback will inevitably generate correlations between momenta of different atoms.

Using Eq. (7) and taking into account that the total average energy of atoms before the feedback equals $E=\Sigma_{\alpha=1}^{N}(\mathcal{X}_{\alpha\alpha}+\mathcal{P}_{\alpha\alpha})/2$, one can easily obtain the energy change due to the feedback action,

$$\Delta E = \frac{1}{2}\left(N\sigma^2 + \frac{1}{N\sigma^2} - \frac{1}{N}\sum_{\alpha=1}^{N}\mathcal{X}_{\alpha\alpha} - \frac{2}{N}\sum_{\mu<\nu}\mathcal{X}_{\mu\nu}\right). \tag{8}$$

Here, the energy is measured in units of E_0. The positive terms in Eq. (8) arise due to the measurement-induced noise. This noise cannot be completely eliminated; however, it can be reduced to the ground-state energy. Choosing the optimal measurement resolution is equivalent to $\sigma_{\mathrm{opt}}=1/\sqrt{N}$. The third term in Eq. (8) is the potential energy of a single atom before the feedback. Being always positive, this term provides the sought effect of the noise suppression. The last term in Eq. (8) is determined by the two-atom coordinate correlations. Depending on the state of the atoms, this term can be positive or negative, increasing or decreasing the efficiency of the noise suppression, respectively.

To have the energy of the atoms below a certain level, the time interval between feedbacks should not be very large. In closed systems the maximum energy is subtracted by two feedbacks separated by $\Delta t=0.25\tau$. The first feedback compensates for the coordinate and the second compensates for the initial momentum that is transformed into the coordinate due to free rotation in a harmonic potential. Taking this into account, we choose this time interval to study the simultaneous action of the feedback and the noise sources.

To demonstrate noise suppression (stabilization of the atoms) by means of the feedback procedure described above, the evolution of the system has been modeled according to the following scheme. First, we choose the initial state of atoms and calculate the evolution of the two-atom correla-

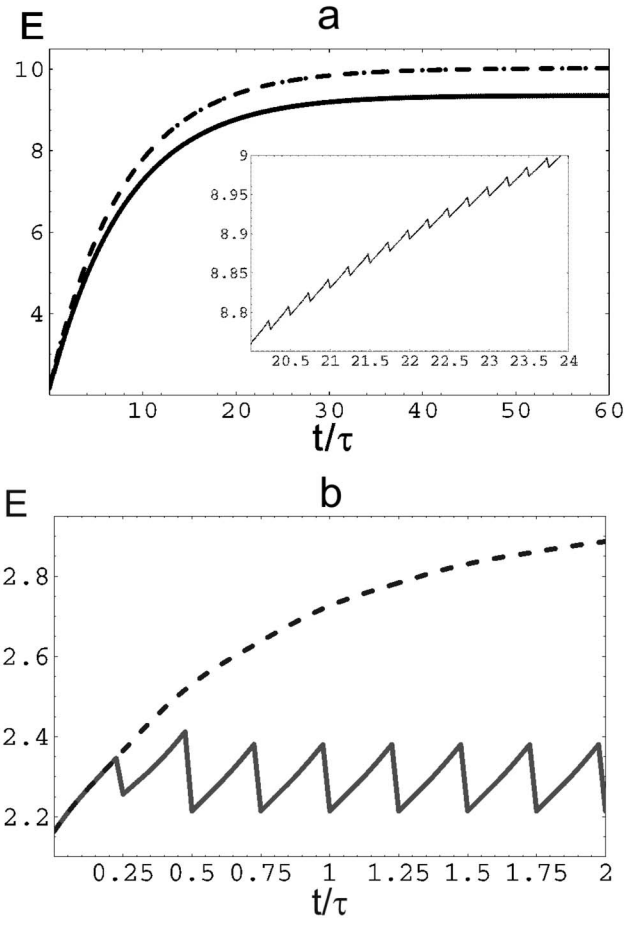

FIG. 1. Dependence of the average energy of a single atom on time for an atomic ensemble subject simultaneously to feedback and short-wavelength (a) or long-wavelength (b) noise. This dependence is shown with the solid curve. The dashed curves show the dynamics of the atoms without feedback.

tion matrices without feedback using Eqs. (2) for the short-wavelength noise and Eqs. (4)–(6) for the long-wavelength one. In addition we find the time dependence of $\tilde{\mathcal{G}}$, which is given by $\tilde{\mathcal{G}}(t)=\tilde{\mathcal{G}}(0)+\int_0^t(-4\gamma\mathcal{D}+\mathcal{M}-\mathcal{Q})d\tau$. Then, at some instant of time the feedback is applied, resulting in the instantaneous change of the correlations according to Eq. (7). The obtained correlations are then used as initial values to calculate the further evolution of atoms under the action of the noise. This procedure is repeated as many times as necessary, revealing the time dependence of the two-atom correlations and the atomic energy.

Figure 1 shows the time dependence of the energy of a single atom for the ensemble of $N=10$ atoms initially in a thermal equilibrium state with the temperature $\xi_a=2$. The upper and lower parts of the figure correspond to the short- and long-wavelength noise, respectively. In both cases the temperature of the reservoir is chosen to be $\xi_b=10$, and $\gamma=0.01$. The measurement resolution is equal to the optimal value. The solid curves show the energy if the feedback is switched on, while the dashed curves correspond to the situation without feedback.

As can be seen from Fig. 1 the application of the feedback allows one to reduce the effect of both types of noise. However, in the case of the long-wavelength noise, the average energy of atoms can be kept at a level well below that for the short-wavelength one. This can be explained by the fact that the former affects only collective observables of the atomic ensemble, and exactly these observables are subject to the feedback.

Furthermore, there is a qualitative difference in the dynamics of atoms influenced by short- and long-wavelength noise. Namely, the energy of the atoms subject to long-wavelength noise and feedback oscillates in time, decreasing almost to the initial value when the feedback is applied and increasing due to the coupling with the reservoir between the feedbacks. In contrast, for the short-wavelength noise similar dynamics is observed with simultaneous gradual increase of the energy [see the inset in Fig. 1(a)]. The difference in the dynamics of the atoms can be understood by analyzing the behavior of the two-atom correlations. In the case of long-wavelength noise the coupling with the common reservoir generates positive two-atom correlations. These correlations indicate the presence of the collective motion of atoms, which is effectively compensated by the feedback. Indeed, the last term in Eq. (8) is large and positive, which results in essential energy subtraction.

In the case of independent reservoirs, the destruction of the negative correlations generated by the feedback is less effective. Therefore, most of the energy is concentrated in the modes of the relative motion and cannot be effectively subtracted. Nevertheless, the stationary energy of the atoms in this case also is clearly below the equilibrium value.

In conclusion, we have demonstrated the possibility of reducing loss of atoms from a far-off-resonance optical lattice caused by external noise sources. It can be achieved by applying feedback compensating for the collective coordinate of the atoms. Since the feedback acts on the external degrees of freedom, this method might find applications for quantum computation or quantum memory. Moreover, for the same reason the method can be applied to molecules. Short- and long-wavelength noise has been addressed. It has been found that feedback can protect the atoms against both of these types of noise. In the case of the long-wavelength noise the stabilization of atoms is, however, more effective.

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