Complete tunneling of light through impedance-mismatched barrier layers

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We investigate the conditions for and properties of the complete tunneling of light through a composite barrier made of impedance-mismatched metamaterial layers. It is shown that two kinds of complete tunneling phenomena exist: phase-unmodulated and phase-modulated complete tunneling. The local surface modes formed near the interfaces between the metamaterial barrier layers play key roles in complete tunneling. Using the terminology of coupled-mode theory, phase-unmodulated complete tunneling occurs through successive mode couplings: from the incident light to the local surface mode and then to the other local surface mode and finally to the light mode in the transmission layer. Phase-modulated complete tunneling results from the complete transfer of the incident optical power to the transmission layer through the direct mediation of the symmetrically and antisymmetrically coupled supermodes of the local surface modes.

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I. INTRODUCTION

Artificially structured metamaterials have opened a new era in electromagnetism, and their unprecedented design flexibility has enabled us to achieve new electromagnetic devices unimaginable in conventional positive index media, such as superlenses [1], phase-compensated microcavities [2], photonic-band-gap structures having an average index of zero [3], and the cloak of invisibility [4–6], to name a few. Tunneling and related characteristics of evanescent waves in such metamaterials have attracted much attention over the last decade [1,7–15] and have shown many interesting and sometimes quite surprising features, one of which is so-called complete tunneling or transparent propagation of light [7,9,13–15]. We say complete tunneling occurs when the transmission coefficient of light incident upon a tunneling barrier becomes exactly 1, in which case the incident light can tunnel through a very long distance without any phase delay or loss of power (if the barrier layers are lossless). Complete tunneling could not be implemented with only positive index media and thus provides a good example of how previously unimaginable optical phenomena or functions can be realized with the advent of metamaterials.

II. THEORETICAL FRAMEWORK

Let us consider the frustrated total internal reflection (FTIR) structure shown in Fig. 1. The composite tunneling barrier is assumed to be composed of multiple mutually impedance-mismatched layers, with the nth layer having relative permittivity ε_n, relative permeability μ_n, and length d_n. The left (incident) and right (transmission) high-index layers surrounding the barrier are assumed to have ε_l, μ_l and ε_r, μ_r, respectively. Here we will consider the case when the number of barrier layers is 3.

The incident light [either the TE-mode $\mathbf{E}$ field (E_y) or the TM-mode $\mathbf{H}$ field (H_y)] can be written as $\psi_{\text{inc}}=\exp(ik_l x)+r\exp(-ik_l x)$, where $k_l^2=k_0^2\varepsilon_l\mu_l\cos^2\theta_l$ and $k_0=2\pi/\lambda$ (λ being the wavelength of light in vacuum). r denotes the reflection amplitude. In the transmission layer, we can write the transmitted light as $\psi_{\text{trans}}=t\exp[ik_r(x-\Sigma_{k=1}^{3}d_k)]$ where t is the transmission amplitude and $k_r^2=k_0^2(\varepsilon_r\mu_r-\varepsilon_l\mu_l\sin^2\theta_l)=k_0^2\varepsilon_r\mu_r\cos^2\theta_r$. k_l and k_r are assumed to have positive and negative values if the corresponding layers have positive and negative refractive indices, respectively. The different signs of k_l and k_r are due to the fact that the direction of the wave vector ($\mathbf{k}$) and that of the Poynting vector ($\mathbf{S}$) are antiparallel in negative index materials [13]. In the barrier, the assumed solution is $\psi_n=A_n\exp[-\kappa_n(x-\Sigma_{k=1}^{n-1}d_k)]+B_n\exp[\kappa_n(x-\Sigma_{k=1}^{n-1}d_k)]$, where $\kappa_n^2=k_0^2(\varepsilon_l\mu_l\sin^2\theta_l-\varepsilon_n\mu_n)$. We assume all κ_j's have positive values.

The solutions in the respective layers must meet appropriate boundary conditions at every interface between layers. The required conditions are the continuity of ψ and that of $(1/\sigma)(\partial\psi/\partial x)$, where σ denotes the relative permeability μ and the relative permittivity ε when the incident light is TE and TM polarized, respectively [13,14]. Using the transfer matrix formulation, we can arrange these continuity conditions into the following compact form:

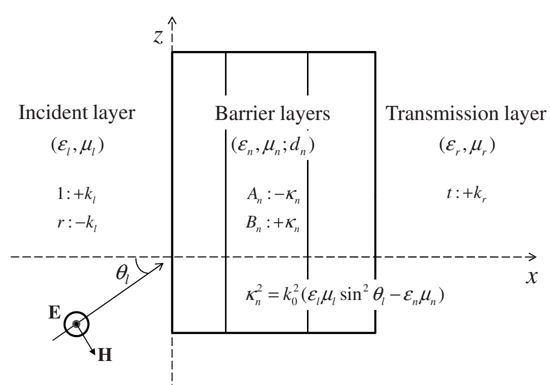

FIG. 1. Tunneling of incident light through a composite barrier made of impedance-mismatched layers.

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$$\begin{pmatrix} t \\ 0 \end{pmatrix} = \mathbf{P}\begin{pmatrix} 1 \\ r \end{pmatrix} = \mathbf{R}\cdot\mathbf{M}\cdot\mathbf{L}\begin{pmatrix} 1 \\ r \end{pmatrix}. \tag{1}$$

where $\mathbf{L}$ and $\mathbf{R}$ link the incident light $(1,r)$ to $\psi_1(A_1,B_1)$ and $\psi_3(A_3,B_3)$ to the transmitted light (t), respectively. $\mathbf{M}$ can be written as

$$\begin{pmatrix} A_3 \\ B_3 \end{pmatrix} = \mathbf{M}\begin{pmatrix} A_1 \\ B_1 \end{pmatrix} = \prod_{n=1}^{2} \mathbf{M}_n^{n+1}\begin{pmatrix} A_1 \\ B_1 \end{pmatrix}, \tag{2}$$

where $\mathbf{M}_n^{n+1}$ connects $\psi_n(A_n,B_n)$ to $\psi_{n+1}(A_{n+1},B_{n+1})$. Using Eq. (1), we can obtain the transmission and reflection amplitudes as $t=\det(\mathbf{P})/P_{22}$ and $r=-P_{21}/P_{22}$. The reflection and transmission coefficients are given by $R=|r|^2$ and $T=1-R$.

III. CONDITIONS FOR COMPLETE TUNNELING

A. Derivation

If we set $\kappa_1 d_1=\kappa_3 d_3=\Xi$ (see Sec. IV for the physical meaning of this condition), the actual transmission and reflection amplitudes can be written as [16]

$$t = \frac{8i\varsigma_{r3}k_l\kappa_1\kappa_2\kappa_3}{\Gamma + i\Omega}, \tag{3}$$

$$r = \frac{\Lambda + i\Sigma}{\Gamma + i\Omega}, \tag{4}$$

where

$$\begin{aligned}\Gamma = {} & (\kappa_1+\varsigma_{12}\kappa_2)(\kappa_2+\varsigma_{23}\kappa_3)(k_lk_r-\varsigma_{l1}\varsigma_{r3}\kappa_1\kappa_3) \\ & \times\sinh(2\Xi+\kappa_2d_2)+2(\varsigma_{23}\kappa_1\kappa_3-\varsigma_{12}\kappa_2^2)(k_lk_r+\varsigma_{l1}\varsigma_{r3}\kappa_1\kappa_3) \\ & \times\sinh(\kappa_2d_2)+(\kappa_1-\varsigma_{12}\kappa_2)(\kappa_2-\varsigma_{23}\kappa_3)(k_lk_r-\varsigma_{l1}\varsigma_{r3}\kappa_1\kappa_3) \\ & \times\sinh(2\Xi-\kappa_2d_2),\end{aligned} \tag{5}$$

$$\begin{aligned}\Omega = {} & (\kappa_1+\varsigma_{12}\kappa_2)(\kappa_2+\varsigma_{23}\kappa_3)(\varsigma_{r3}k_l\kappa_3+\varsigma_{l1}k_r\kappa_1) \\ & \times\cosh(2\Xi+\kappa_2d_2)+2\kappa_2(\kappa_1-\varsigma_{13}\kappa_3)(\varsigma_{r3}k_l\kappa_3-\varsigma_{l1}k_r\kappa_1) \\ & \times\cosh(\kappa_2d_2)+(\kappa_1-\varsigma_{12}\kappa_2)(\kappa_2-\varsigma_{23}\kappa_3)(\varsigma_{r3}k_l\kappa_3+\varsigma_{l1}k_r\kappa_1) \\ & \times\cosh(2\Xi-\kappa_2d_2),\end{aligned} \tag{6}$$

$$\begin{aligned}\Lambda = {} & (\kappa_1+\varsigma_{12}\kappa_2)(\kappa_2+\varsigma_{23}\kappa_3)(k_lk_r+\varsigma_{l1}\varsigma_{r3}\kappa_1\kappa_3) \\ & \times\sinh(2\Xi+\kappa_2d_2)+2(\varsigma_{23}\kappa_1\kappa_3-\varsigma_{12}\kappa_2^2)(k_lk_r-\varsigma_{l1}\varsigma_{r3}\kappa_1\kappa_3) \\ & \times\sinh(\kappa_2d_2)+(\kappa_1-\varsigma_{12}\kappa_2)(\kappa_2-\varsigma_{23}\kappa_3)(k_lk_r+\varsigma_{l1}\varsigma_{r3}\kappa_1\kappa_3) \\ & \times\sinh(2\Xi-\kappa_2d_2),\end{aligned} \tag{7}$$

$$\begin{aligned}\Sigma = {} & (\kappa_1+\varsigma_{12}\kappa_2)(\kappa_2+\varsigma_{23}\kappa_3)(\varsigma_{r3}k_l\kappa_3-\varsigma_{l1}k_r\kappa_1) \\ & \times\cosh(2\Xi+\kappa_2d_2)+2\kappa_2(\kappa_1-\varsigma_{13}\kappa_3)(\varsigma_{r3}k_l\kappa_3+\varsigma_{l1}k_r\kappa_1) \\ & \times\cosh(\kappa_2d_2)+(\kappa_1-\varsigma_{12}\kappa_2)(\kappa_2-\varsigma_{23}\kappa_3)(\varsigma_{r3}k_l\kappa_3-\varsigma_{l1}k_r\kappa_1) \\ & \times\cosh(2\Xi-\kappa_2d_2).\end{aligned} \tag{8}$$

ς_{lm} is given by $\varsigma_{lm}=\mu_l/\mu_m$ for TE-mode incident light and $\varsigma_{lm}=\varepsilon_l/\varepsilon_m$ for TM light. From Eqs. (7) and (8), we can see that if we can configure the FTIR structure so that $\kappa_1=\varsigma_{13}\kappa_3$ and $k_l=\varsigma_{lr}k_r$, Σ vanishes and Λ becomes

$$\Lambda = (2/\varsigma_{lr}\varsigma_{12})[\alpha\cosh(\kappa_2d_2)+\beta\sinh(\kappa_2d_2)], \tag{9}$$

where

$$\alpha = 2\varsigma_{12}\kappa_1\kappa_2(k_l^2+\varsigma_{l1}^2\kappa_1^2)\sinh(2\Xi), \tag{10}$$

$$\begin{aligned}\beta = {} & (\kappa_1^2+\varsigma_{12}^2\kappa_2^2)(k_l^2+\varsigma_{l1}^2\kappa_1^2)\cosh(2\Xi) \\ & +(\kappa_1^2-\varsigma_{12}^2\kappa_2^2)(k_l^2-\varsigma_{l1}^2\kappa_1^2).\end{aligned} \tag{11}$$

If we can further meet the condition $\tanh(\kappa_2d_2)=-\alpha/\beta$ or $\exp(2\kappa_2d_2)=(\beta-\alpha)/(\beta+\alpha)$, Λ becomes zero as well, and finally complete tunneling can occur. Therefore, we can conclude that complete tunneling occurs when the following conditions are satisfied:

$$\kappa_1d_1=\kappa_3d_3=\Xi, \tag{12}$$

$$k_l=\varsigma_{lr}k_r, \tag{13}$$

$$\kappa_1=\varsigma_{13}\kappa_3, \tag{14}$$

$$\tanh(\kappa_2d_2) = -\frac{\alpha}{\beta} \quad \text{or} \quad \exp(2\kappa_2d_2)=(\beta-\alpha)/(\beta+\alpha). \tag{15}$$

B. Some comments

We want to make a few remarks about the complete tunneling under the conditions of Eqs. (12)–(15).

(1) For the realization of complete tunneling, ς_{12} must be negative. Positive ς_{12} or positive α results in $\beta-\alpha<\beta+\alpha$, which requires $\exp(2\kappa_2d_2)<1$ or $\kappa_2d_2<0$ because $\beta>0$ [17]. Since it is impossible to meet this requirement, we must make $\varsigma_{12}<0$, i.e., we need at least one metamaterial layer for complete tunneling. $\varsigma_{12}<0$ compels ς_{23} to be negative also due to the condition $\varsigma_{13}>0$ [from Eq. (14)].

(2) The most distinct characteristic of complete tunneling through impedance-mismatched layers compared to that through impedance-matched layers is that it can occur only at specific incident angles (θ_{CT}) which can simultaneously satisfy Eqs. (12)–(15). When the barrier is symmetric, Eqs. (12) and (14)— are satisfied automatically (because $\varsigma_{13}=1$ and $d_1=d_3$) and do not impose any restrictions on the incident angles. However, if $\varsigma_{13}\neq 1$, for example, there can be at most one θ_{CT}, which is given from Eq. (14) as $\sin^2\theta_{CT}=(\varepsilon_1\mu_1-\varsigma_{13}^2\varepsilon_3\mu_3)/[(1-\varsigma_{13}^2)\varepsilon_l\mu_l]$. If this value meets other conditions [Eqs. (12), (13), and (15)] as well, complete tunneling indeed occurs. However, if it does not, complete tunneling never occurs. Similarly, when the surrounding layers are mutually impedance matched (which produces $\varsigma_{lr}=\pm 1$), Eq. (13) does not confine the incidence angles. However, if $\varsigma_{lr}\neq 1$, complete tunneling can occur only at one θ_{CT} which satisfies $\sin^2\theta_{CT}=(\varepsilon_l\mu_l-\varsigma_{lr}^2\varepsilon_r\mu_r)/[(1-\varsigma_{lr}^2)\varepsilon_l\mu_l]$ [18]. If this value does not satisfy any one of Eqs. (12), (14), and (15), complete tunneling cannot take place.

(3) Equation (13) is an extended version of the Brewster angle condition. That is, if we neglect the tunneling barrier,

we have just one interface between the incidence and transmission layers. In this situation, total transmission (zero reflection or complete power transfer) occurs only when Eq. (13) is satisfied. Therefore, we can view Eq. (13) as an *effective* impedance-matching condition between the incidence and transmission layers.

C. Phase-modulated and phase-unmodulated complete tunneling

A specific solution of Eq. (14) is given as $\kappa_1+\varsigma_{12}\kappa_2=\kappa_2+\varsigma_{23}\kappa_3=0$. In this case, Γ becomes zero in addition to Σ and Λ while Ω does not (see Appendix A for proof). This makes t in Eq. (3) become exactly 1, and the tunneling does not generate a phase delay. We will refer to this kind of complete tunneling as phase-unmodulated complete tunneling. However, other solutions of Eq. (14) cause a nonzero phase delay of $\tau_\phi=-[\pi/2-\tan^{-1}(\Omega/\Gamma)]/k_l$. We will refer to this mode as phase-modulated complete tunneling. Therefore, in addition to the effective impedance-matching condition between the surrounding layers, we need the following conditions:

(1) for phase-modulated complete tunneling,

$$\kappa_1=\varsigma_{13}\kappa_3\neq-\varsigma_{12}\kappa_2, \tag{16}$$

$$\tanh(\kappa_2 d_2)=-\frac{\alpha}{\beta}, \tag{17}$$

$$\kappa_1 d_1=\kappa_3 d_3=\Xi; \tag{18}$$

(2) for phase-unmodulated complete tunneling [19],

$$\kappa_1+\varsigma_{12}\kappa_2=\kappa_2+\varsigma_{23}\kappa_3=0, \tag{19}$$

$$\kappa_2 d_2=2\kappa_1 d_1=2\kappa_3 d_3. \tag{20}$$

It is notable that what matters in the phase-unmodulated complete tunneling are not the absolute lengths of barrier layers but the ratios between them. If we scale all the lengths of the barrier layers by a common factor, the complete tunneling characteristics will not change.

IV. CONSIDERATION OF THE CONDITIONS FOR COMPLETE TUNNELING

A. Local surface modes in barrier layers and phase-modulated complete tunneling

Let us set the lengths of the first and third barrier layers to be sufficiently long (see Fig. 2). If we consider the two remaining interfaces separately, we can see that local surface modes φ_{12} and φ_{23} can be formed near the interfaces between the barrier layers 1 and 2, and between the barrier layers 2 and 3, respectively [see Fig. 3(a)]. They denote the guided surface modes along the z axis with propagation constants of $\beta_{12}^{(0)}=k_0[(\varepsilon_1\mu_1-\varsigma_{12}^2\varepsilon_2\mu_2)/(1-\varsigma_{12}^2)]^{1/2}$ and $\beta_{23}^{(0)}=k_0[(\varepsilon_2\mu_2-\varsigma_{23}^2\varepsilon_3\mu_3)/(1-\varsigma_{23}^2)]^{1/2}$. It is notable that these local surface modes are the same kind of modes as surface plasmon polariton (SPP) waves [20].

If we consider both interfaces or include the coupling effects between them [see Fig. 3(b)], we can show that the

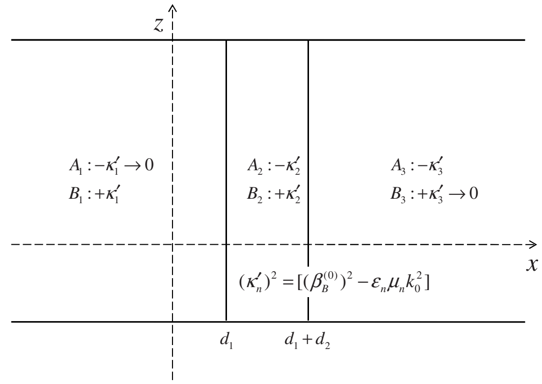

FIG. 2. Barrier layers after setting the lengths of the first and third barrier layers to be sufficiently long.

guided mode $\psi_B^{(0)}$ having the propagation constant of $\beta_B^{(0)}$ along the z axis becomes identical to the symmetrically and antisymmetrically coupled supermodes of φ_{12} and φ_{23}, i.e., $\psi_B^{(0)}=\psi_{s(a)}^{(0)}=(\varphi_{12}\pm\varphi_{23})/\sqrt{2}$ (see Appendix B for details) when $\kappa_1'=\varsigma_{13}\kappa_3'\neq-\varsigma_{12}\kappa_2'$, where $\kappa_n'=[(\beta_B^{(0)})^2-\varepsilon_n\mu_n k_0^2]^{1/2}$. Their propagation constants will be denoted as $\beta_s^{(0)}$ and $\beta_a^{(0)}$, respectively, and their dispersion relation is given by $\tanh(\kappa_2' d_2)=-2\varsigma_{12}\kappa_1'\kappa_2'/[(\kappa_1')^2+\varsigma_{12}^2(\kappa_2')^2]$. This is just the condition given by Eq. (17) with $\Xi\to\infty$ and $\kappa_n'=\kappa_n$ or $\beta_B^{(0)}=\beta_{s(a)}^{(0)}=k_0\sqrt{\varepsilon_l\mu_l}\sin\theta_l$, which makes $\kappa_1'=\varsigma_{13}\kappa_3'\neq-\varsigma_{12}\kappa_2'$ equal to Eq. (16).

Therefore, what Eqs. (16) and (17) demand for phase-modulated complete tunneling is that there must exist symmetrically or antisymmetrically coupled supermodes ($\psi_{s(a)}$) and that the z directional component of the wave vector of the incident light must be equal to their propagation constants, i.e., $\beta_{s(a)}=k_0\sqrt{\varepsilon_l\mu_l}\sin\theta_l$. The dispersion relation of $\psi_{s(a)}$, i.e., Eq. (17), is different from that of $\psi_{s(a)}^{(0)}$ mentioned above, because of the influence of the finite lengths of the first and third barrier layers, and the presence of the incidence and transmission layers. The propagation constant $\beta_{s(a)}$ also becomes different from $\beta_{s(a)}^{(0)}$ for the same reason (we will try to verify this argument numerically using perturbation theory in Sec. V) [21].

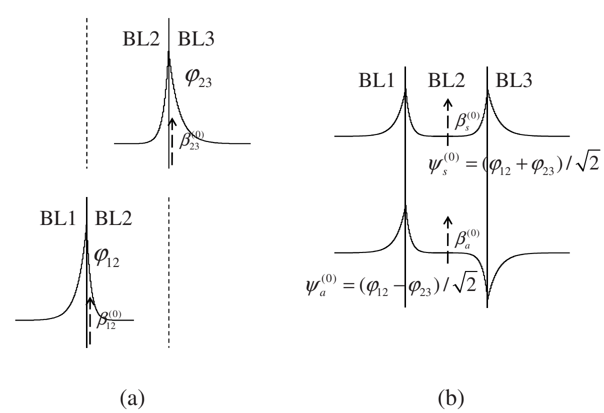

FIG. 3. (a) Local surface modes φ_{12} and φ_{23}, and (b) their symmetrically and antisymmetrically coupled supermodes $\psi_{s(a)}^{(0)}=(\varphi_{12}\pm\varphi_{23})/\sqrt{2}$.

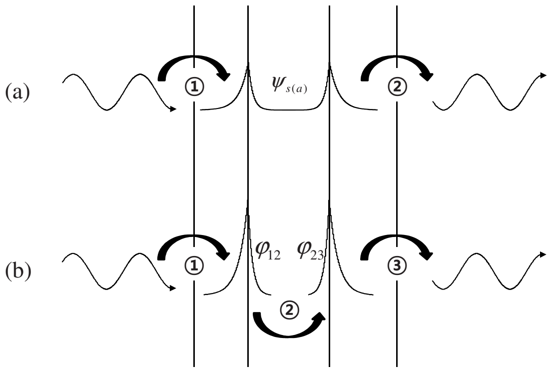

FIG. 4. Successive mode couplings in (a) the phase-modulated and (b) the phase-unmodulated complete tunneling phenomena.

The condition $\beta_{s(a)}=k_0\sqrt{\varepsilon_l\mu_l}\sin\theta_l$ is a kind of phase-matching condition between the incident plane wave and the supermodes in the *finite* barrier ($\psi_{s(a)}$) so that the optical power of the incident light can be completely transferred to the supermodes. This power can be completely transferred to the light mode in the transmission layer if the surrounding layers are effectively impedance matched.

Therefore, we can conclude that phase-modulated complete tunneling is a kind of successive mode-coupling phenomenon: from the incident plane wave to the guided supermodes in the barrier and then to the plane wave in the transmission layer [see Fig. 4(a)]. However, if the transfer rates of optical power through these two mode couplings are different, the incident optical power cannot be transferred completely to the transmission layer. Therefore, we need Eq. (18) for complete tunneling, which makes the two transfer rates identical, so that the optical power transfers smoothly and completely from the incident light to the light mode in the transmission layer.

B. Phase-unmodulated complete tunneling

The dispersion relations of the local surface modes φ_{12} and φ_{23} can be written as $\kappa_1'+\varsigma_{12}\kappa_2'=0$ and $\kappa_2''+\varsigma_{23}\kappa_3''=0$, where $\kappa_{1(2)}'=[(\beta_{12}^{(0)})^2-\varepsilon_{1(2)}\mu_{1(2)}k_0^2]^{1/2}$ and $\kappa_{2(3)}''=[(\beta_{23}^{(0)})^2-\varepsilon_{2(3)}\mu_{2(3)}k_0^2]^{1/2}$. They become identical to Eq. (19) when $\kappa_1=\kappa_1'$, $\kappa_3=\kappa_3''$, and $\kappa_2=\kappa_2'=\kappa_2''$, which can be rewritten as the following simple equation: $\beta_{12}^{(0)}=\beta_{23}^{(0)}=k_0\sqrt{\varepsilon_l\mu_l}\sin\theta_l$. Therefore, what Eq. (19) demands for phase-unmodulated complete tunneling is that (1) the propagation constants of φ_{12} and φ_{23} are the same and (2) the z directional component of the wave vector of the incident light must be equal to this propagation constant [22]. This is also a kind of phase-matching condition between the incident plane wave and the local surface modes (φ_{12} and φ_{23}). Since $\beta_{12}^{(0)}=\beta_{23}^{(0)}$, the power guided along the z axis in the form of φ_{12} can be completely transferred to φ_{23}, and vice versa. That is, phase-unmodulated complete tunneling occurs through successive mode couplings: from the incident plane wave to the local surface mode (φ_{12}) and then to the other local surface mode (φ_{23}) and finally, if the surrounding layers are effectively impedance matched, to the plane wave in the transmission layer [see Fig. 4(b)].

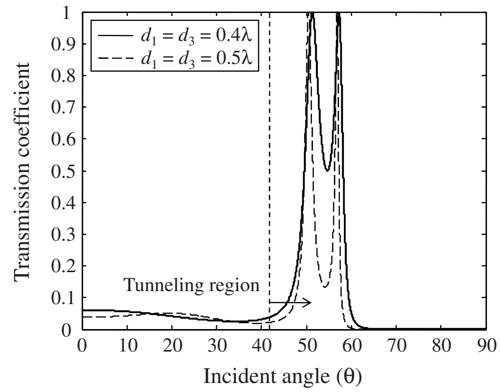

FIG. 5. Transmission coefficients of the TM incident light through a FTIR structure whose incident and transmission layers are made of glass and air acts as the first layer of the composite tunneling barrier, $\varepsilon_2=-3.33$, $\mu_2=1$, and $\varepsilon_3=1$, $\mu_3=1$, and $d_2=0.2\lambda$.

As is the case in phase-modulated complete tunneling, if the transfer rates of optical power through these three mode couplings are different, the incident optical power cannot be transferred to the transmission layer completely. Equation (20) makes these transfer rates the same so that the optical power transfers smoothly and completely from the incident light to the light mode in the transmission layer.

V. NUMERICAL CALCULATION RESULTS

For a proof-of-principle example, we calculated the transmission coefficients of incident light ($\lambda=1550$ nm) through a FTIR structure whose incidence and transmission layers are composed of glass ($\varepsilon_l=\varepsilon_r=2.25$, $\mu_l=\mu_r=1$) and where air ($\varepsilon_1=1$, $\mu_1=1$) acts as the first layer of the composite tunneling barrier. If we choose the material parameters as $\varepsilon_2=-3.33$, $\mu_2=1$ and $\varepsilon_3=1$, $\mu_3=1$, and set $d_2=0.2\lambda$ and $d_1=d_3=0.4\lambda$, phase-unmodulated complete tunneling cannot take place because we cannot satisfy Eqs. (19) and (20): $d_2/d_1=2\kappa_1/\kappa_2=-2\varsigma_{12}$. We show the calculated transmission coefficients (for TM-mode incident light) in Fig. 5; from them we obtained two incident angles at which complete tunneling occurs: $\theta_{\mathrm{CT},1}=51.35°$ and $\theta_{\mathrm{CT},2}=57.30°$. Please note that more than one θ_{CT} is available since the barrier was assumed to be symmetric and the surrounding layers were impedance matched. Both $\theta_{\mathrm{CT},1}$ and $\theta_{\mathrm{CT},2}$ correspond to phase-modulated complete tunneling, i.e., $\theta_{\mathrm{CT},1}$ and $\theta_{\mathrm{CT},2}$ are related to the symmetrically and antisymmetrically coupled supermodes ψ_s and ψ_a, respectively [see Fig. 6, where we show the **H**-field (H_y) intensity distributions when complete tunneling occurs]. The phase delays were -0.125 and 0.610 μm, respectively. We note that the tunneling characteristics are highly sensitive to the incident direction of the light, a feature that can be applied to incident-direction-selective transmission filters.

To see in more detail whether $\theta_{\mathrm{CT},1}$ and $\theta_{\mathrm{CT},2}$ have their origin in the symmetrically and antisymmetrically coupled supermodes ($\psi_{s(a)}$), we calculated the transmission coefficients when only the thickness of the first and third barrier

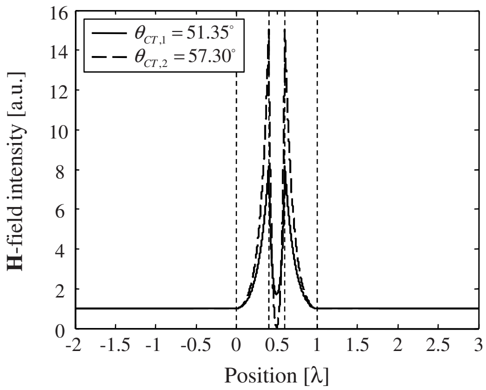

FIG. 6. **H**-field intensity distributions when complete tunneling occurs.

layers was varied and plotted the derived values of θ_{CT} in Fig. 7 as triangles and squares (the transmission coefficients when $d_1=d_3=0.5\lambda$ are plotted in Fig. 5). In addition, we also show the incident angles at which the z directional wave vector of the incident light is equal to $\beta_{s(a)}$, calculated using perturbation theory (see solid lines) [20]. They are very similar, and nearly identical when the first and third barrier layers are relatively thick, which demonstrates clearly that the phase-modulated complete tunneling has its origin in $\psi_{s(a)}$.

From Fig. 7, we can see that, when $d_2=\lambda/3$, phase-unmodulated complete tunneling occurs, since we can meet the condition $d_2/d_1=-2\varsigma_{12}=0.6$. It must be mentioned that there is another way to interpret phase-unmodulated complete tunneling. It can be regarded as a special case of phase-modulated complete tunneling. It occurs through two sequential mode couplings: from the incident plane wave to the supermodes in the finite barrier ($\psi_{s(a)}$, not $\psi_{s(a)}^{(0)}$) and then to the light mode in the transmission layer, when the propagation constants $\beta_{s(a)}$ of such supermodes (including the effects of the finite lengths of the first and third barrier layers) become equal to $\beta_{12}^{(0)}$ and $\beta_{23}^{(0)}$, which are the propagation constants of the local surface modes φ_{12} and φ_{23} determined by

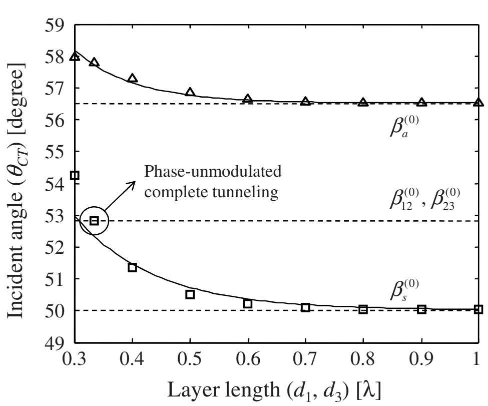

FIG. 7. Values of the incident angle (θ_{CT}) at which complete tunneling occurs (triangles and squares). Dotted lines show the incident angles at which the z directional wave vector of the incident light is equal to $\beta_{12(23)}^{(0)}$ and $\beta_{s(a)}^{(0)}$. Solid lines show the incident angles at which the z directional wave vector of the incident light is equal to $\beta_{s(a)}$ calculated using perturbation theory, Eq. (B14).

neglecting multi-interface coupling effects as well as the influence of the finite lengths of the first and third barrier layers.

VI. SUMMARY

In this paper, the conditions for and properties of the complete tunneling of light through impedance-mismatched barrier layers were investigated. We showed that there are two kinds of complete tunneling, phase-unmodulated and phase-modulated complete tunneling, whose origins are related to the local surface modes formed near the interfaces between metamaterial barrier layers and their coupled supermodes, respectively. While phase-unmodulated complete tunneling occurs through successive mode couplings, from the incident light to the local surface mode and then to the other local surface mode and finally to the light mode in the transmission layer, phase-modulated complete tunneling involves two mode couplings, from the incident plane wave to the plane wave in the transmission layer through the direct mediation of the supermodes of the local surface modes.

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APPENDIX A

Under the conditions of Eqs. (12)–(15), Γ and Ω reduce to

$$\Gamma = (2/\varsigma_{lr}\varsigma_{12})\cosh(\kappa_2 d_2)[\alpha_\Gamma - (\alpha/\beta)\beta_\Gamma], \tag{A1}$$

$$\Omega = (4/\varsigma_{lr}\varsigma_{12})\varsigma_{l1}k_l\kappa_1 \cosh(\kappa_2 d_2)[\alpha_\Omega - (\alpha/\beta)\beta_\Omega], \tag{A2}$$

where

$$\alpha_\Gamma = 2\varsigma_{12}\kappa_1\kappa_2(k_l^2 - \varsigma_{l1}^2\kappa_1^2)\sinh(2\Xi), \tag{A3}$$

$$\beta_\Gamma = (\kappa_1^2 + \varsigma_{12}^2\kappa_2^2)(k_l^2 - \varsigma_{l1}^2\kappa_1^2)\cosh(2\Xi) + (\kappa_1^2 - \varsigma_{12}^2\kappa_2^2)(k_l^2 + \varsigma_{l1}^2\kappa_1^2), \tag{A4}$$

$$\alpha_\Omega = 2\varsigma_{12}\kappa_1\kappa_2 \cosh(2\Xi), \tag{A5}$$

$$\beta_\Omega = (\kappa_1^2 + \varsigma_{12}^2\kappa_2^2)\sinh(2\Xi). \tag{A6}$$

If we look for the condition for $\Gamma=0$, i.e., $\alpha_\Gamma\beta=\alpha\beta_\Gamma$, we obtain $\kappa_1^2=\varsigma_{12}^2\kappa_2^2$. Since ς_{12} must be negative for complete tunneling as was mentioned in the main text, we finally have $\kappa_1+\varsigma_{12}\kappa_2=0$. That is, if $\kappa_1+\varsigma_{12}\kappa_2=0$, Γ becomes zero. Then what about Ω? If we calculate $\alpha_\Omega\beta-\alpha\beta_\Omega$, it becomes

$$\alpha_\Omega\beta - \alpha\beta_\Omega = 2\varsigma_{12}\kappa_1\kappa_2[(\kappa_1^2 + \varsigma_{12}^2\kappa_2^2)(k_l^2 + \varsigma_{l1}^2\kappa_1^2) + (\kappa_1^2 - \varsigma_{12}^2\kappa_2^2) \times (k_l^2 - \varsigma_{l1}^2\kappa_1^2)\cosh(2\Xi)]. \tag{A7}$$

Therefore, we can be sure that Ω does not vanish when $\kappa_1+\varsigma_{12}\kappa_2=0$.

APPENDIX B

If we consider the layers shown in Fig. 2, we can obtain the dispersion relation of the guided mode $\psi_B^{(0)}$ in the barrier having the propagation constant $\beta_B^{(0)}$ as

$$-\frac{\kappa_2' - \varsigma_{23}\kappa_3'}{\kappa_1' + \varsigma_{12}\kappa_2'}\exp(-\kappa_2' d_2) = \frac{\kappa_2' + \varsigma_{23}\kappa_3'}{\kappa_1' - \varsigma_{12}\kappa_2'}\exp(\kappa_2' d_2), \tag{B1}$$

when $\kappa_1' \pm \varsigma_{12}\kappa_2' \neq 0$ and $\kappa_2' \pm \varsigma_{23}\kappa_3' \neq 0$. We can assume $\kappa_1' - \varsigma_{12}\kappa_2' \neq 0$ and $\kappa_2' - \varsigma_{23}\kappa_3' \neq 0$ since we are interested only in the case when $\varsigma_{12} < 0$. If $\kappa_1' = \varsigma_{13}\kappa_3' \neq -\varsigma_{12}\kappa_2'$, Eq. (B1) becomes

$$\exp(\kappa_2' d_2) = \pm \frac{\kappa_1' - \varsigma_{12}\kappa_2'}{\kappa_1' + \varsigma_{12}\kappa_2'}, \tag{B2}$$

and reduces further to $\coth(\kappa_2' d_2/2) = -\kappa_1'/\varsigma_{12}\kappa_2'$ and $\tanh(\kappa_2' d_2/2) = -\kappa_1'/\varsigma_{12}\kappa_2'$.

The mode field in the second barrier layer can be written as

$$\psi_2 = A_2 \exp[-\kappa_2'(x - d_1)] + B_2' \exp[\kappa_2'(x - d_1 - d_2)], \tag{B3}$$

where $B_2' = B_2 \exp(\kappa_2' d_2)$. From the continuity conditions at $x = d_1$, we have

$$B_1 \exp(\kappa_1' d_1) = A_2 + B_2' \exp(-\kappa_2' d_2), \tag{B4}$$

$$\kappa_1' B_1 \exp(\kappa_1' d_1) = \varsigma_{12}\kappa_2'[-A_2 + B_2' \exp(-\kappa_2' d_2)], \tag{B5}$$

and from these we can obtain

$$B_2'/A_2 = -\frac{\kappa_1' + \varsigma_{12}\kappa_2'}{\kappa_1' - \varsigma_{12}\kappa_2'}\exp(\kappa_2' d_2). \tag{B6}$$

We can also get Eq. (B6) from the continuity conditions at $x = d_1 + d_2$. If $\kappa_1' = \varsigma_{13}\kappa_3' \neq -\varsigma_{12}\kappa_2'$ (which results in $\kappa_2' + \varsigma_{23}\kappa_3' \neq 0$), we have $B_2'/A_2 = \pm 1$. Since A_2 and B_2' express the contributions of local surface modes φ_{12} and φ_{23} to the mode field distribution in layer 2, respectively, we can see that the contributions of φ_{12} and φ_{23} are either in phase or out of phase, while their magnitudes are always the same in the barrier layer 2. Therefore, we can rewrite Eq. (B3) as

$$\psi_2 = A_2\{\exp[-\kappa_2'(x - d_1)] \pm \exp[\kappa_2'(x - d_1 - d_2)]\}$$
$$= \frac{1}{\sqrt{2}}(\varphi_{12} \pm \varphi_{23}), \tag{B7}$$

which clearly demonstrates that symmetric and antisymmetric supermodes of φ_{12} and φ_{23} are formed when $\kappa_1' = \varsigma_{13}\kappa_3'$ and $\kappa_1' + \varsigma_{12}\kappa_2' \neq 0$ (or $\kappa_2' + \varsigma_{23}\kappa_3' \neq 0$).

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[16] If we assume $\kappa_1 d_1 = \kappa_2 d_2 = \Xi$ instead, we have instead of Eqs. (5)–(8)

$$\Gamma = (\kappa_1 + \varsigma_{12}\kappa_2)(\kappa_2 + \varsigma_{23}\kappa_3)(k_l k_r - \varsigma_{l1}\varsigma_{r3}\kappa_1\kappa_3)\sinh(2\Xi + \kappa_3 d_3)$$
$$+ 2(\kappa_1 - \varsigma_{12}\kappa_2)(k_l k_r \kappa_2 + \varsigma_{l1}\varsigma_{23}\varsigma_{r3}\kappa_1\kappa_3^2)\sinh(\kappa_3 d_3)$$
$$- (\kappa_1 + \varsigma_{12}\kappa_2)(\kappa_2 - \varsigma_{23}\kappa_3)(k_l k_r + \varsigma_{l1}\varsigma_{r3}\kappa_1\kappa_3)$$
$$\times \sinh(2\Xi - \kappa_3 d_3), \tag{B8}$$

$$\Omega = (\kappa_1 + \varsigma_{12}\kappa_2)(\kappa_2 + \varsigma_{23}\kappa_3)(\varsigma_{r3}k_l\kappa_3 + \varsigma_{l1}k_r\kappa_1)\cosh(2\Xi + \kappa_3 d_3)$$
$$+ 2\kappa_3(\kappa_1 - \varsigma_{12}\kappa_2)(\varsigma_{r3}k_l\kappa_2 - \varsigma_{l1}\varsigma_{23}k_r\kappa_1)\cosh(\kappa_3 d_3)$$
$$+ (\kappa_1 + \varsigma_{12}\kappa_2)(\kappa_2 - \varsigma_{23}\kappa_3)(\varsigma_{r3}k_l\kappa_3 - \varsigma_{l1}k_r\kappa_1)$$
$$\times \cosh(2\Xi - \kappa_3 d_3), \tag{B9}$$

$$\Lambda = (\kappa_1 + \varsigma_{12}\kappa_2)(\kappa_2 + \varsigma_{23}\kappa_3)(k_l k_r + \varsigma_{l1}\varsigma_{r3}\kappa_1\kappa_3)\sinh(2\Xi + \kappa_3 d_3)$$
$$+ 2(\kappa_1 - \varsigma_{12}\kappa_2)(k_l k_r \kappa_2 - \varsigma_{l1}\varsigma_{23}\varsigma_{r3}\kappa_1\kappa_3^2)\sinh(\kappa_3 d_3)$$
$$- (\kappa_1 + \varsigma_{12}\kappa_2)(\kappa_2 - \varsigma_{23}\kappa_3)(k_l k_r - \varsigma_{l1}\varsigma_{r3}\kappa_1\kappa_3)$$
$$\times \sinh(2\Xi - \kappa_3 d_3), \tag{B10}$$

$$\Sigma = (\kappa_1 + \varsigma_{12}\kappa_2)(\kappa_2 + \varsigma_{23}\kappa_3)(\varsigma_{r3}k_l\kappa_3 - \varsigma_{l1}k_r\kappa_1)\cosh(2\Xi + \kappa_3 d_3)$$
$$+ 2\kappa_3(\kappa_1 - \varsigma_{12}\kappa_2)(\varsigma_{r3}k_l\kappa_2 + \varsigma_{l1}\varsigma_{23}k_r\kappa_1)\cosh(\kappa_3 d_3)$$
$$+ (\kappa_1 + \varsigma_{12}\kappa_2)(\kappa_2 - \varsigma_{23}\kappa_3)(\varsigma_{r3}k_l\kappa_3 + \varsigma_{l1}k_r\kappa_1)$$
$$\times \cosh(2\Xi - \kappa_3 d_3), \tag{B11}$$

and we cannot find any conditions for complete tunneling. A similar conclusion to this can be drawn when $\kappa_2 d_2 = \kappa_3 d_3 = \Xi$ is assumed.
[17] Since $\cosh(2\Xi) > 1$, we have $\beta > (\kappa_1^2 + \varsigma_{12}^2\kappa_2^2)(k_l^2 + \varsigma_{l1}^2\kappa_1^2) + (\kappa_1^2 - \varsigma_{12}^2\kappa_2^2)(k_l^2 - \varsigma_{l1}^2\kappa_1^2) > 0$.

[18] For θ_{CT} to exist, the relative permittivity and relative permeability values of the surrounding layers must satisfy the following inequalities:

$$\mu_l^2 \leqslant (\varepsilon_r/\varepsilon_l)\mu_r\mu_l \leqslant \mu_r^2 \quad \text{or} \quad \mu_r^2 \leqslant (\varepsilon_r/\varepsilon_l)\mu_r\mu_l \leqslant \mu_l^2 \quad \text{(TE mode)}, \tag{B12}$$

$$\varepsilon_l^2 \leqslant (\mu_r/\mu_l)\varepsilon_r\varepsilon_l \leqslant \varepsilon_r^2 \quad \text{or} \quad \varepsilon_r^2 \leqslant (\mu_r/\mu_l)\varepsilon_r\varepsilon_l \leqslant \varepsilon_l^2 \quad \text{(TM mode)}. \tag{B13}$$

[19] If $\kappa_1+\varsigma_{12}\kappa_2=\kappa_2+\varsigma_{23}\kappa_3=0$, Eqs. (10) and (11) become $\alpha=-2\kappa_1^2(k_l^2+\varsigma_{l1}^2\kappa_1^2)\sinh(2\Xi)$ and $\beta=2\kappa_1^2(k_l^2+\varsigma_{l1}^2\kappa_1^2)\cosh(2\Xi)$, respectively, and Eq. (15) reduces to $\kappa_2 d_2=2\Xi$.

[20] If we set $\mu=1$ in all layers, the propagation constants of the TM local surface modes reduce to $\beta_{12}^{(0)}=k_0[\varepsilon_1\varepsilon_2/(\varepsilon_1+\varepsilon_2)]^{1/2}$ and $\beta_{23}^{(0)}=k_0[\varepsilon_2\varepsilon_3/(\varepsilon_2+\varepsilon_3)]^{1/2}$, which are identical to the propagation constants of the SPP waves, neglecting the loss in the metals.

[21] If we use perturbation theory, $\beta_{s(a)}$ can be calculated from

$$(\beta_{s(a)})^2=(\beta_{s(a)}^{(0)})^2+k_0^2\left\{\int_{-\infty}^{0}[\varepsilon_l\mu_l-\varepsilon_1\mu_1]|\psi_{s(a)}^{(0)}|^2dx + \int_{d_1+d_2+d_3}^{\infty}[\varepsilon_r\mu_r-\varepsilon_3\mu_3]|\psi_{s(a)}^{(0)}|^2dx\right\}. \tag{B14}$$

Therefore, we can see that, if $\varepsilon_l\mu_l>\varepsilon_1\mu_1$ and $\varepsilon_r\mu_r>\varepsilon_3\mu_3$, we have $\beta_{s(a)}>\beta_{s(a)}^{(0)}$.

[22] We must note that, if the barrier is symmetric, i.e., $\varsigma_{13}=1$, $\kappa_1=\kappa_3$, and $d_1=d_3$, Eq. (19) can be easily satisfied. However, this does not mean that Eq. (19) cannot be satisfied if the barrier is not symmetric. The barrier must be symmetric when either the permeability or the permittivity has a symmetric distribution through the barrier layers (including a constant one).