

Atomic entanglement versus visibility of photon interference for quantum criticality of a hybrid system

M. X. Huo,¹ Ying Li,¹ Z. Song,^{1,*} and C. P. Sun^{2,†}

¹*Department of Physics, Nankai University, and the Key Laboratory of Weak Light Nonlinear Photonics, Ministry of Education, Tianjin 300071, China*

²*Institute of Theoretical Physics, Chinese Academy of Sciences, Beijing, 100080, China*

(Received 15 February 2007; revised manuscript received 21 November 2007; published 6 February 2008)

We use the two-atom concurrence and the visibility of interference fringes of photons to characterize a quantum phase transition for a hybrid system consisting of an array of coupled cavities and each cavity doped with a two-level atom. We compare them with the total excitation number fluctuation. Analytical and numerical simulation results show quantum critical phenomena similar to the Mott insulator to superfluid transition. Here, the contour lines of the atomic concurrence, the visibility of interference fringes of photons, and the excitation number fluctuation in the phase diagram are consistent in the vicinity of the nonanalytic locus of the atomic concurrence.

DOI: [10.1103/PhysRevA.77.022103](https://doi.org/10.1103/PhysRevA.77.022103)

PACS number(s): 03.65.Ud, 42.50.Dv, 73.43.Nq, 03.67.-a

I. INTRODUCTION

The concept of the local order parameter, which characterizes different phases with symmetry breaking, is very crucial to the modern theory of the second order phase transitions. For the quantum phase transition [1], however, the conventional Ginzburg-Landau-Wilson paradigm does not work well in some cases where there is no appropriate local order parameter to correctly characterize the emergent phenomena due to quantum criticality [2].

In this paper we study the quantum phase transition for a hybrid system consisting of a coupled waveguide resonator array (CWRA) doped with atoms in each cavity, which has been studied intensively [3–11]. We show that, though we cannot make sure what is the appropriate order parameter for the hybrid system, some physical observable quantities can be used to witness its quantum critical phenomena. We notice that this hybrid architecture has been suggested as a quantum coherent device to transfer and store quantum information as well as to create the laserlike output [3,12,13] and can be implemented with the defect array in the photonic crystal by doping artificial atoms [14] or the Josephson junction array in the cavity [3].

It was shown that such a doped CWRA can simulate the Mott-like transition of light from “Mott insulator (MI) to superfluid (SF)” [4] since a doped atom can induce an effective photon-photon interaction in each cavity. Together with the intercavity hopping of localized photons, this nonlinear photon-photon coupling can result in the Bose-Hubbard [15] model for the Mott phase transition [16]. Here, the average of the annihilation operator is employed as the order parameter while the number fluctuation is also used to probe the quantum phase transition [17–19]. However, the study of the quantum phase transition of light [4] in such a hybrid system still assumes the same order parameter for photons, while a more strict approach based on order parameter [5,6] was implicitly made with respect to polaritons, the mixtures of photons and atoms.

The Bose-Hubbard model is the minimal model of strongly interacting bosons on a lattice, and is believed to be simulated by the cold-atom optical lattice system. However, the doped CWRA as a hybrid system consists of two kinds of excitations: Internal atomic excited states and photons. Quantum phase transition from Mott insulator to superfluid is contained within the solutions to both models of the Bose-Hubbard and the hybrid system. The coupling between the photons and the atom in a cavity can induce the nonlinearity of photons. People try to seek the equivalence between such two models. There are two mechanisms to accomplish this task. The first one [4–7] considers a cavity array with each one containing a single atom. In this scheme, the polariton is treated as a boson. However, the effective repulsive strength between polaritons is excitation number dependent. Then, in this sense, the equivalence between two systems is qualitative. The second one [8] is based on a cavity array with each one containing an ensemble of four-level atoms. The photon nonlinearities give rise to very pure photon-photon interactions rather than polariton-polariton interactions in the first scheme.

We focus on the observables as signatures of the quantum phase transition (QPT) for the first scheme. The observables, the local number fluctuation, and the visibility of interference fringes, proposed for the QPT in the Bose-Hubbard model, are borrowed to investigate such a hybrid system. In the following section we shall introduce the concepts of the atomic entanglement and the visibility of photon interference. In Sec. III we present the model setup and the concept of the quasiexcitation fluctuation. In Sec. IV we investigate observable characterizations for the hybrid system. In Sec. V the observables are computed analytically and numerically to characterize the quantum criticality. Finally, a summary and discussion are given in Sec. VI.

II. ATOMIC ENTANGLEMENT AND VISIBILITY OF PHOTON INTERFERENCE

Now, we adopt some observable quantities to characterize the critical phenomena of a hybrid system. Comparing with the Bose-Hubbard model, the mechanism of the QPT in a

*songtc@nankai.edu.cn

†suncp@itp.ac.cn; URL: <http://www.itp.ac.cn/~suncp>

hybrid system is a little different. In the Bose-Hubbard model, the competition between the on-site repulsion and the hopping integral induces the QPT. For a hybrid system, the fundamental excitation is the polariton which is a mixed excitation of the cavity QED and the atom. The statistics of the polariton depends on the coupling strength and the detuning between the cavity and the atom, respectively. It is natural to employ the properties of atomic and photonic observables to investigate the critical behavior. In fact, in two limits of larger detuning, the polariton reduces to a pure photon (boson) and an atomic excited state (hardcore boson). This feature is the root of the QPT. Obviously, the visibility of photon interference fringes can be employed to witness the occurrence of the superfluid. But the photon number fluctuation should be replaced by the excitation number fluctuation. On the other hand, around the critical point, the tunneling of photons between adjacent cavities can induce an entanglement of atoms when the kinetic energy of photons starts to overcome the photon blockade. Then the entanglement of atoms should also show a critical behavior.

To this end, we make use of two-atom entanglement [20] from the viewpoint of quantum information, as well as the visibility of interference fringes of photons (VIFP) [21]. As a measure of entanglement of two two-level systems i and j , the two-atom concurrence (TAC) is defined as

$$C_{ij} = \max\{0, \lambda_1 - \lambda_2 - \lambda_3 - \lambda_4\} \quad (1)$$

in terms of the square roots $\{\lambda_i\}$ ($\lambda_1 = \max\{\lambda_i\}$) of eigenvalues of $\rho^{(ij)}(\sigma_y \otimes \sigma_y) \rho^{(ij)*}(\sigma_y \otimes \sigma_y)$ [22]. Here, $\rho^{(ij)}$ is the reduced density matrix defined for two local atoms i and j ; σ_y is the Pauli matrix with respect to the ground and excited states $|g\rangle_i$ and $|e\rangle_i$ of the local atom. The VIFP is defined as

$$\mathcal{V} = \frac{V_{\max} - V_{\min}}{V_{\max} + V_{\min}}, \quad (2)$$

for characterizing the coherence of photons, where V_{\max} and V_{\min} are the maximum and minimum of the photon number distribution of the ground state in k space. Both C_{ij} and \mathcal{V} are observables and so can act as possible candidates to determine the critical point.

In the following, we examine the excitation number fluctuation (ENF) in comparison with the above-mentioned two observable quantities, TAC and VIFP, for lattice atom-photon hybrid systems of small size by analytical and numerical methods, respectively. Our results reveal nontrivial connections among the three quantities in such an intriguing way: Contour lines of three quantities in the phase diagram are approximately consistent with each other when the nonanalyticity of TAC occurs. It firmly shows that such three quantities are signatures for the MI to SF transition in the present hybrid system.

III. MODEL SETUP AND THE QUASIEXCITATION FLUCTUATION

We consider an array of N coupled cavities with each one containing a single two-level atom [3,4,6,9,10,12,13], which is illustrated in Fig. 1. The model Hamiltonian

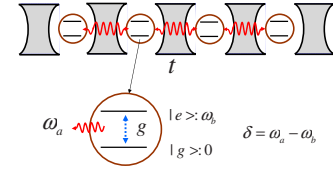


FIG. 1. (Color online) Schematic setup of a cavity array with each one containing a two-level atom. Photons of mode ω_a can tunnel between adjacent cavities with the hopping integral t and couple to the atoms with the strength g . This atom-photon lattice is expected to simulate the Mott insulator to superfluid transition.

$$H = H_{\text{free}} + H_{\text{int}} + H_{\text{hop}} \quad (3)$$

is decomposed as three parts: Free Hamiltonians of light and atom,

$$H_{\text{free}} = \omega_a \sum_{i=1}^N a_i^\dagger a_i + \omega_b \sum_{i=1}^N |e\rangle_i \langle e|, \quad (4)$$

the cavity-mode-atom interaction,

$$H_{\text{int}} = g \sum_{i=1}^N (a_i^\dagger |g\rangle_i \langle e| + \text{H.c.}), \quad (5)$$

with strength g , and the photon hopping between nearest-neighbor (NN) defects,

$$H_{\text{hop}} = -t \sum_{i=1}^{N-1} (a_i^\dagger a_{i+1} + \text{H.c.}), \quad (6)$$

with hopping integral constant t for the tunneling between adjacent cavities. Here, a_i^\dagger and a_i are the creation and annihilation operators of the photon at defect i . Obviously the total excitation number

$$\hat{\mathcal{N}} = \sum_{i=1}^N \hat{\mathcal{N}}_i = \sum_{i=1}^N (a_i^\dagger a_i + S_i^z + 1/2) \quad (7)$$

is a conserved quantity for the Hamiltonian H , i.e.,

$$[H, \hat{\mathcal{N}}] = 0, \quad (8)$$

where $2S_i^z |e\rangle_i = |e\rangle_i$ and $2S_i^z |g\rangle_i = -|g\rangle_i$.

It can be seen that $\hat{\mathcal{N}}$ is just the excitation number of polaritons. It is well known that the conventional MI to SF phase transition occurs in a Bose-Hubbard model. Here, when the on-site repulsive interaction between bosons is large enough in the Mott phase, the number fluctuation would become energetically unfavorable, forcing the system into a number state and exhibiting a vanishing particle number fluctuation. In the SF regime, bosons are delocalized with the nonvanishing particle number fluctuation. As for the present hybrid system, the fundamental excitations are polaritons [6] and the mechanism of the Mott transition is due to the effect of photon blockade. Since the photon number is not conserved in such a system, the photon number fluctuation $\Delta n_i = \Delta(a_i^\dagger a_i)$ is not appropriate to characterize the SF phase as that for a pure Bose-Hubbard model. This is because Δn_i does not vanish even in the MI regime due to the

coupling between photons and atoms. Hereafter, we define the variance ΔA by

$$(\Delta A)^2 = \langle (A)^2 \rangle - \langle A \rangle^2. \quad (9)$$

It is noticed that one can take the ENF per site $\Delta \mathcal{N}_i$ as an order parameter to characterize the Mott transition [5]. In the large detuning limit $\delta = \omega_a - \omega_b \gg 0$, all atoms are in excited states, which are perfectly number squeezed states, i.e., $\Delta \mathcal{N}_i = 0$ for all sites. In the other limit $\delta \ll 0$, all atoms are in ground states. Obviously the TAC vanishes and the ENF per site becomes

$$\Delta \mathcal{N}_i = \sqrt{\langle a_i^\dagger a_i^\dagger a_i a_i \rangle} = \sqrt{(N-1)/N} \approx 1 \quad (10)$$

due to $\mathcal{N} = N$ in this case. On the other hand, the ENF in one individual cavity can be measured via resonance fluorescence [5] This experimental scheme can be applied to the current system to distinguish between the Mott insulating and superfluid phases. In the next step, we will investigate the ENF in comparison with other two observable quantities: TAC and VIFP.

IV. OBSERVABLE CHARACTERIZATIONS

Intuitively, two atoms in two adjacent cavities should entangle with each other due to the delocalization of photons, which also enhances to coherence of photons. Then the observable quantities TAC and VIFP should be sensitive to the critical point. The TAC can be expressed in terms of observable quantities such as correlation functions of two atoms. The creation and measure of an entanglement for two atoms in separated cavities have been widely studied and regarded as a resource for quantum-information processing [23].

The complete basis vectors of the total system are denoted by

$$| \{n_j, s_j\} \rangle = |n_1, \dots, n_N; s_1, \dots, s_N\rangle = \prod_{j=1}^N |n_j\rangle \otimes |s_j\rangle, \quad (11)$$

where $|n_j\rangle$ is the Fock state of photons and $|s_j\rangle = |g\rangle_j, |e\rangle_j$ for $s_j = 0, 1$, respectively. The fact that $\hat{\mathcal{N}}$ is conserved can be reflected by the vanishing matrix element of the density operator $\rho = \rho(H)$ on the above basis for any state (pure or thermal equilibrium state) of the hybrid system; that is,

$$\rho_{n'_j, s'_j; n_j, s_j} \propto \delta \left[\sum (n_j + s_j - n'_j - s'_j) \right]. \quad (12)$$

The reduced density matrix

$$\rho^{(12)} = \text{Tr}_p \text{Tr}_{3\dots N} [\rho(H)] \quad (13)$$

for two atomic quasispins 1 and 2, e.g., s_1 and s_2 , are obtained as

$$\begin{aligned} [\rho^{(12)}]_{s'_1 s'_2; s_1 s_2} &= \sum_{[n_j; s_3 \dots s_N]} \rho_{n_j, s_1 s_2 s_3 \dots s_N}^{n'_j, s'_1 s'_2 s_3 \dots s_N} \delta \left[\sum (s_j - s'_j) \right] \\ &= \delta(s_1 + s_2 - s'_1 - s'_2) \sum_{[n_j; s_3 \dots s_N]} \rho_{n_j, s_1 s_2 s_3 \dots s_N}^{n'_j, s'_1 s'_2 s_3 \dots s_N} \end{aligned} \quad (14)$$

by tracing over all photon variables (with Tr_p) and atomic variables except for s_1 and s_2 .

The corresponding reduced density matrix for two atoms i and j is of the form

$$\rho^{(ij)} = \begin{pmatrix} u_{ij}^+ & 0 & 0 & 0 \\ 0 & w_{ij}^1 & z_{ij}^* & 0 \\ 0 & z_{ij} & w_{ij}^2 & 0 \\ 0 & 0 & 0 & u_{ij}^- \end{pmatrix}, \quad (15)$$

which is the same as that for a pure spin-1/2 system [24] in the absence of photons. Using the observable quantities, the quantum correlation

$$z_{ij} = \langle \psi | S_i^+ S_j^- | \psi \rangle,$$

$$u_{ij}^\pm = \langle \psi | (1/2 \pm S_i^z)(1/2 \pm S_j^z) | \psi \rangle, \quad (16)$$

the TAC is rewritten as a computable form

$$C_{ij} = 2 \max(0, |z_{ij}| - \sqrt{u_{ij}^+ u_{ij}^-}). \quad (17)$$

The nonanalyticity of TAC arises from the abrupt switch of the sign of quantity $|z_{ij}| - \sqrt{u_{ij}^+ u_{ij}^-}$ and can be used to determine the quantum phase transition.

Similar to the transition of MI to SF in the Bose-Hubbard model [21], two phases of the atom-photon hybrid system can also be delimited through the quantum coherence of the ground state. In the MI phase, the quantum coherence of photons is completely destroyed due to the photon blockade. In the SF phase limit, the quantum coherence of photons gets its maximum. Therefore, the quantum coherence of photons can be employed to indicate phases, which is characterized by an observable quantity VIFP (2). In a lattice model, the photon number distribution in k space is

$$V(k) = \frac{1}{N} \sum_{j,l} e^{ik(j-l)} \langle a_j^\dagger a_l \rangle. \quad (18)$$

In the strong photon blockade limit, $\mathcal{V} = 0$, while in the SF limit, $\mathcal{V} = 1$. Comparing with the local parameter $\psi = \langle a_i^\dagger \rangle = \langle a_i \rangle$ induced by the mean-field approach [4], the VIFP is more appropriate to discriminate two phases experimentally.

V. CHARACTERIZING QUANTUM CRITICALITY

For a lattice atom-photon system, we now consider connections among three quantities $\Delta \mathcal{N}_i$, C_{ij} , and \mathcal{V} around the critical point. It is hard to get the solution of the Hamiltonian (3) for large N . We will investigate a minimal model which contains the key physics of the lattice atom-photon system and allows us to understand the character of the quantum phase transition. The simplified Hamiltonian reads

$$\mathcal{H} = \mathcal{H}_a + \mathcal{H}_b + \mathcal{H}', \quad (19)$$

where

$$\mathcal{H}_a = \omega_a \sum_{i=1}^N a_i^\dagger a_i - t \sum_{i=1}^{N-1} (a_i^\dagger a_{i+1} + \text{H.c.}),$$

$$\mathcal{H}_b = \omega_b \sum_{i=1}^N |e\rangle_i \langle e|,$$

$$\mathcal{H}' = g(a_A^\dagger|g\rangle_A\langle e| + a_B^\dagger|g\rangle_B\langle e| + \text{H.c.}). \quad (20)$$

Here, \mathcal{H}_a and \mathcal{H}_b describe two kinds of excitations, photons and excited atoms, while \mathcal{H}' represents the photon-atom coupling *only* in two cavities A and B . Here the positions of A and B are chosen arbitrarily. The purpose of this simplification is to investigate the mechanism of creating entanglement between two atoms in cavities A and B . In the following, we will consider the case with $\mathcal{N}=N$ and small g , so that \mathcal{H}' can be treated as a perturbation term.

We start with the case of $g=0$, i.e., switching off \mathcal{H}' first. Since there is no coupling between photons and atoms, the ground state of $\mathcal{H}_a+\mathcal{H}_b$ is determined by the competition of the ground state energies of \mathcal{H}_a and \mathcal{H}_b . In the region $\delta > 2t$, the ground state is $\Pi_l|e\rangle_l$, where l represents each cavity l . Such a state is an insulating state with $\Delta\mathcal{N}_i=0$, $\mathcal{V}=0$, and $C_{ij}=0$. On the other hand, in the region $\delta < 2t$, the ground state is $|N\rangle_{k=0}\Pi_l|g\rangle_l$, where $|n\rangle_k$ denotes the photon Fock state in k space. It is a superfluid state with $\Delta\mathcal{N}_i=\sqrt{(N-1)/N}$, $\mathcal{V}=1$, and $C_{ij}=0$. At line $\delta=2t$, the model admits multifold degenerate ground states with energy $\varepsilon^{(0)}=N\omega_b$, which can be expressed as

$$|\Phi\rangle = |n\rangle_{k=0}\prod_{l=1}^N |s_l\rangle, \quad (21)$$

where $n+\sum_{l=1}^N s_l^+ + N/2 = N$ to ensure the conservation of the excitation number and $|s_l\rangle = |g\rangle_l$, $|e\rangle_l$ for $s_l=0, 1$, respectively. Then the ENF and VIFP experience a big jump, while the TAC is ‘‘uncertain’’ due to the energy-level crossing.

Next, we will consider the entanglement between two atoms in cavities A and B when \mathcal{H}' is taken into account as a perturbation term with a small $g \ll 4\pi^2 t/N^2$ [25]. In the region $|\delta-2t| \gg g$, \mathcal{H}' should not affect the ground states significantly. Thus the TAC remains vanishing. However, when the interaction \mathcal{H}' is switched on at $\delta=2t$, it leads to an avoided energy-level crossing. This fact results in a quantum fluctuation which can drive the transition from the MI to SF phase. This transition also corresponds to the nonvanishing TAC. In fact, for a small $g \ll 4\pi^2 t/N^2$, all the unperturbed ground states with excitation number $\mathcal{N}=N$ are Eq. (21). Among them, states

$$\begin{aligned} |\phi_1\rangle &= |N\rangle_{k=0}|g\rangle_A|g\rangle_B|G_{A,B}\rangle, \\ |\phi_2\rangle &= |N-1\rangle_{k=0}|e\rangle_A|g\rangle_B|G_{A,B}\rangle, \\ |\phi_3\rangle &= |N-1\rangle_{k=0}|g\rangle_A|e\rangle_B|G_{A,B}\rangle, \\ |\phi_4\rangle &= |N-2\rangle_{k=0}|e\rangle_A|e\rangle_B|G_{A,B}\rangle \end{aligned} \quad (22)$$

span an invariant subspace, in which the first-order perturbation ground state can be obtained. Here state

$$|G_{A,B}\rangle = \prod_{l \neq A,B} |g\rangle_l \quad (23)$$

denotes the product of the ground states of all atoms except the two in cavities A and B .

Up to the first-order perturbation we have the ground state energy

$$\varepsilon = \varepsilon^{(0)} + \varepsilon^{(1)},$$

$$\varepsilon^{(0)} = -2t = -\delta,$$

$$\varepsilon^{(1)} = -g\sqrt{2(1+\beta^2)} = -\frac{g}{\eta}, \quad (24)$$

where

$$\eta = \frac{1}{\sqrt{2(1+\beta^2)}},$$

$$\beta = \sqrt{(N-1)/N}. \quad (25)$$

The perturbed ground state is

$$|\psi_g\rangle = \eta(|\phi_1\rangle + \beta|\phi_4\rangle) - \frac{1}{2}(|\phi_2\rangle + |\phi_3\rangle). \quad (26)$$

Then the corresponding TAC between two atoms in two cavities A and B can be calculated as

$$C_{AB} = \frac{(1-\beta)^2}{2(1+\beta^2)}. \quad (27)$$

Note that $|\psi_g\rangle$ and C_{AB} only depend on the size of the system N . This is because they are just the first-order perturbation results for the simplified Hamiltonian (19) at $\delta=2t$. Obviously, it is not true for finite g and the full Hamiltonian (3). This can be seen from the following numerical results for small size systems. Nevertheless, it qualitatively indicates that the photon-atom coupling can induce entanglement between atoms. As δ is apart from the degenerate point, C_{AB} decreases due to the energy competition of two ground states of \mathcal{H}_a and \mathcal{H}_b . Therefore, this heuristic analysis has shown a simple relation among TAC, VIFP, and ENF around the quantum phase transition critical point: The ENF and the VIFP both exhibit an abrupt jump while the TAC has a sharp maximum. It can be predicted that, as g increases, changes of the three quantities will be slow due to the strong coupling between atoms and photons, i.e., the abrupt jump of the ENF and the VIFP; the pointlike shape of the TAC should be smooth for a finite system. In the following, these will be investigated for small systems in a wide range of parameters by numerical simulations.

Next we investigate three quantities in small size systems by the exact diagonalization method. For open chain cavity array systems, the VIFP \mathcal{V} can be calculated by

$$V(k) = \frac{2}{N+1} \sum_{i,j} \sin(ki)\sin(kj)\langle a_i^\dagger a_j \rangle, \quad (28)$$

where $k=n\pi/(N+1)$, $n \in [1, N]$, while the TAC and the ENF can be characterized as average TAC

$$\bar{C} = \frac{1}{N} \sum_{i < j} C_{ij} \quad (29)$$

and average ENF

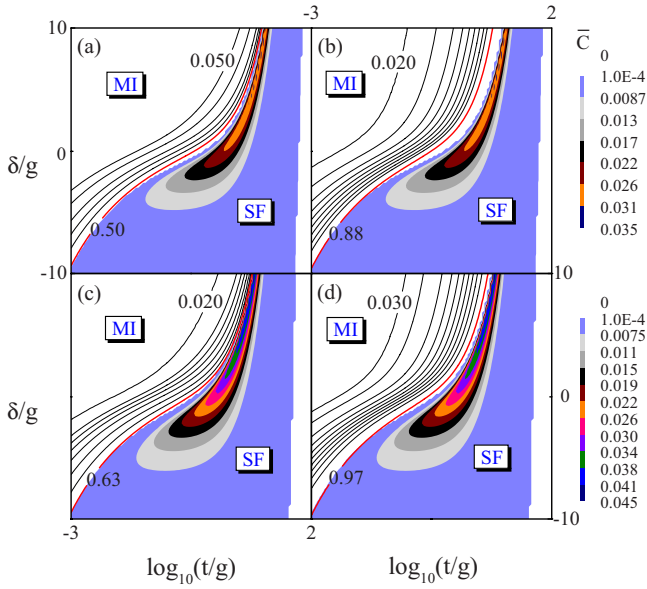


FIG. 2. (Color online) Contours of three quantities, $\overline{\Delta\mathcal{N}}$ [dark lines in (a),(c)], \mathcal{V} [dark lines in (b),(d)], and \overline{C} [color maps in (a)–(d)], obtained by the exact diagonalization for (a),(b) two- and (c),(d) four-cavity systems. Red lines in (a)–(d) denote closer contour lines of $\overline{\Delta\mathcal{N}}$ and \mathcal{V} to the nonanalytical curve of \overline{C} . It shows that contour lines of three quantities are consistent in the vicinity of the nonanalytic locus at which the nonanalyticity of \overline{C} occurs.

$$\overline{\Delta\mathcal{N}} = \frac{1}{N} \sum_i \Delta\mathcal{N}_i. \quad (30)$$

For given parameters δ , t , and g , the ground state wave function of the Hamiltonian for $N=2,4$ can be computed by the exact diagonalization. Then the corresponding quantities \mathcal{V} , \overline{C} , and $\overline{\Delta\mathcal{N}}$ can be obtained. In Fig. 2, contours of three quantities obtained by the exact diagonalization are plotted in the δ/g - t/g plane for two- [Figs. 2(a) and 2(b)] and four- [Figs. 2(c) and 2(d)] cavity systems. Contours of the average ENF $\overline{\Delta\mathcal{N}}$ [dark lines in Figs. 2(a) and 2(c)] and the VIFP \mathcal{V} [dark lines in Figs. 2(b) and 2(d)] are compared with the average TAC \overline{C} [color maps in Figs. 2(a)–2(d)] as functions of the scaled detuning δ/g and the photon hopping integral t/g . We see that contour lines of three quantities are consistent in the vicinity of the locus at which the nonanalyticity of \overline{C} occurs. The nonanalytic locus of \overline{C} in the δ/g - t/g plane is defined by the equation $|z_{ij}| - \sqrt{u_{ij}^+ u_{ij}^-} = 0$. Red lines in Figs. 2(a)–2(d) denote closer contour lines of $\overline{\Delta\mathcal{N}}$ and \mathcal{V} to the nonanalytical curve of \overline{C} . It also shows that $\overline{\Delta\mathcal{N}}$ and \mathcal{V} start to jump at the nonanalytic locus of \overline{C} . There is a slight difference between profiles of two- and four-cavity systems. The red contour line of \mathcal{V} in the four-cavity system is closer to the nonanalytic locus of \overline{C} than that in the two-cavity system. It indicates that contour lines of three quantities will cover in the vicinity of the nonanalytic locus of \overline{C} in the

thermodynamics limit. Although quantum phase transitions occur in infinite lattice systems, our results indicate that the critical signatures of such a transition are robust and evident even in small systems, which may create the possibility of simulating the transition via a nanoquantum device.

The numerical results of \overline{C} for small systems confirm the sketch description of the behavior of the TAC by the perturbation process. In fact, in the strong photon blockade regime, the whole ground state is a simple direct product of the polariton state of each cavity. So the TAC should be zero due to the confinement of photons in each cavity. As the kinetic energy of photons increases, the tunneling of photons leads to a nonzero TAC. In the deep superfluid region, the polaritons reduce to photons and leave all the atoms in their ground states. Although the whole ground state is entangled, the TAC is vanishing. Then the TAC experiences a maximum around the critical point. In addition, results in Fig. 2 show that the relationship among the signatures does not seem sensitive to the size of the system.

VI. SUMMARY AND DISCUSSION

In summary, we have investigated the MI to SF transition in a hybrid system consisting of an array of coupled cavities doped with two-level atoms. We investigate two nonlocal observable quantities, TAC and VIFP, in comparison with a local order parameter, ENF, for the MI to SF transition. Numerical results obtained the complete information about the relationship among the signatures for small size systems. Although these results cannot be extrapolated, they predict a possibility of larger systems. It can be observed from analytical and numerical simulation results that the TAC and the VIFP in the phase diagram indeed can reflect the quantum critical phenomena signatored by the total ENF. In principle, such nonlocal observable quantities can be used to detect the critical point in experiments for all atom-photon systems with strong couplings. The higher quality factor Q of the cavity and the long coherent time of the qubit are necessary. Many systems such as the photonic crystal [14], the superconducting circuit QED [26], etc. can be potential candidates to demonstrate the above observations. Among them, as pointed out by Ref. [4], a diamond based on cavity array with individual negatively charged nitrogen-vacancy (NV) centers placed in each cavity is a promising hybrid system for detecting the criticality via the observable quantities due to its applicable nature (for a review, see [27] and references therein). On the other hand, signatures simulated for a small finite number cavity array in this paper could also be exploited for quantum coherent control of on-chip devices in quantum-information processing.

ACKNOWLEDGMENTS

This work is supported by the NSFC with Grant Nos. 90203018, 10474104, and 60433050, and NFRPC with Nos. 2006CB921206 and 2005CB724508.

- [1] S. Sachdev, *Quantum Phase Transitions* (Cambridge University Press, Cambridge, England, 1999).
- [2] T. Senthil *et al.*, *Science* **303**, 1490 (2004).
- [3] L. Zhou *et al.*, e-print arXiv:cond-mat/0608577.
- [4] A. D. Greentree *et al.*, *Nat. Phys.* **2**, 856 (2006).
- [5] M. J. Hartmann *et al.*, *Nat. Phys.* **2**, 849 (2006).
- [6] D. G. Angelakis, M. F. Santos, and S. Bose, *Phys. Rev. A* **76**, 031805(R) (2007).
- [7] M. J. Hartmann *et al.*, e-print arXiv:0706.2251.
- [8] M. J. Hartmann and M. B. Plenio, *Phys. Rev. Lett.* **99**, 103601 (2007).
- [9] D. Rossini and R. Fazio, *Phys. Rev. Lett.* **99**, 186401 (2007).
- [10] D. G. Angelakis *et al.*, e-print arXiv:quant-ph/0702133.
- [11] M. J. Hartmann, F. G. S. L. Brandao, and M. B. Plenio, *Phys. Rev. Lett.* **99**, 160501 (2007).
- [12] F. M. Hu *et al.*, e-print arXiv:quant-ph/0610250.
- [13] L. Zhou, J. Lu, and C. P. Sun, *Phys. Rev. A* **76**, 012313 (2007).
- [14] M. F. Yanik and S. Fan, *Phys. Rev. Lett.* **92**, 083901 (2004); Q. Xu, S. Sandhu, M. L. Povinelli, J. Shakya, S. Fan, and M. Lipson, *ibid.* **96**, 123901 (2006).
- [15] D. Jaksch, C. Bruder, J. I. Cirac, C. W. Gardiner, and P. Zoller, *Phys. Rev. Lett.* **81**, 3108 (1998).
- [16] M. P. A. Fisher, P. B. Weichman, G. Grinstein, and D. S. Fisher, *Phys. Rev. B* **40**, 546 (1989).
- [17] M. Greiner *et al.*, *Nature (London)* **415**, 39 (2002).
- [18] C. Orzel *et al.*, *Science* **291**, 2386 (2001).
- [19] F. Gerbier, S. Folling, A. Widera, O. Mandel, and I. Bloch, *Phys. Rev. Lett.* **96**, 090401 (2006).
- [20] A. Osterloh *et al.*, *Nature (London)* **416**, 608 (2002).
- [21] F. Gerbier, A. Widera, S. Folling, O. Mandel, T. Gericke, and I. Bloch, *Phys. Rev. Lett.* **95**, 050404 (2005); *Phys. Rev. A* **72**, 053606 (2005).
- [22] W. K. Wootters, *Phys. Rev. Lett.* **80**, 2245 (1998).
- [23] L. M. Duan *et al.*, *Nature (London)* **414**, 413 (2001); C. Cabillo, J. I. Cirac, P. Garcia-Fernandez, and P. Zoller, *Phys. Rev. A* **59**, 1025 (1999); S. Bose, P. L. Knight, M. B. Plenio, and V. Vedral, *Phys. Rev. Lett.* **83**, 5158 (1999); D. E. Browne, M. B. Plenio, and S. F. Huelga, *ibid.* **91**, 067901 (2003); X. L. Feng, Z. M. Zhang, X. D. Li, S. Q. Gong, and Z. Z. Xu, *ibid.* **90**, 217902 (2003); L. M. Duan and H. J. Kimble, *ibid.* **90**, 253601 (2003); C. Simon and W. T. M. Irvine, *ibid.* **91**, 110405 (2003).
- [24] X. Wang and P. Zanardi, *Phys. Lett. A* **301**, 1 (2002); X. Wang, *Phys. Rev. A* **66**, 034302 (2002).
- [25] The degenerate perturbation is applied under the condition $g \ll \Delta E$, where ΔE is the energy gap between the first excited and ground states. From the tight-binding spectrum of photon, $E = -2t \cos 2\pi m/N$ ($m=0, 1, \dots, N-1$), we have $\Delta E = -2t \cos 2\pi/N + 2t \approx 4\pi^2 t/N^2$.
- [26] A. Wallraff *et al.*, *Nature (London)* **431**, 162 (2004); A. Blais, R. S. Huang, A. Wallraff, S. M. Girvin, and R. J. Schoelkopf, *Phys. Rev. A* **69**, 062320 (2004).
- [27] A. D. Greentree *et al.*, *J. Phys.: Condens. Matter* **18**, S825 (2006).