# Effective tunneling coefficient of a coupled double-well system modulated by anharmonic periodic potentials

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We numerically study coherent tunneling oscillations of the particles between two levels in a double-well potential in the presence of anharmonic periodic potentials. Extremely short driving pulses modify the tunneling coefficient to $\kappa_{eff}=\kappa\cos A$, where $\kappa$ is the bare tunneling coefficient without the driving field and $A$ is the pulse area of the driving wave form. The modulation amplitude of the $\kappa_{eff}$ gradually decreases as the driving wave form becomes broad and is given by $\kappa_{eff}=\kappa J_0(A)$ for the sinusoidal modulation, where $J_0(x)$ denotes the ordinary Bessel function of order zero. Theoretical derivation of the effective tunneling coefficient $\kappa_{eff}=\kappa\cos A$ is also shown for a periodic $\delta$ kick with alternating sign by means of the transfer matrix formula.

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## I. INTRODUCTION

The problem of coherent tunneling through the barrier of a double well or a superlattice in the presence of an external oscillating (ac) field has received a considerable amount of attention for more than a decade [1–8]. Dynamical localization of a moving particle in an infinite chain of lattice sites under the action of a time-periodic field has reported earlier by Dunlap and Kenkre [1]. Their key finding was the localization of the particle that occurs when the ratio of the strength of the imposed time-periodic electric field to its frequency becomes equal to certain discrete values—i.e., zeros of the ordinary Bessel function of order zero when the driving field is sinusoidal. Grossmann *et al.* [2] reported a similar effect on an electron in a double well. If the electron is initially localized in one of the two wells and if the laser power and frequency are chosen appropriately, the radiation field can prevent the coherent tunneling of the electron. According to this phenomenon electrons in a symmetric double-well potential can be localized in one of the wells if the following condition is satisfied: $J_0(\varepsilon/\omega)=0$, where $J_0(x)$ denotes an ordinary Bessel function of order zero, and $\varepsilon$ and $\omega$ are the amplitude and the angular frequency of the driving field, respectively. They called this phenomenon coherent destruction of tunneling (CDT). This phenomenon has been studied in detail in many papers [2–8]. Furthermore, the case of a combined application of ac and dc fields in the system has also been studied [9–11]. In this case, the localization condition becomes $\Omega_0=n\omega$ and $J_n(\varepsilon/\omega)=0$, where $\Omega_0$ is the detuning of the concerning levels induced by the dc field, $n$ is an integer, and $J_n(x)$ is the $n$th-order Bessel function. It has been revealed through these investigations that the dynamical localization in the infinite chain of lattice sites predicted by Dunlap and Kenkre [1], miniband collapse of the superlattices reported by Holthause *et al.* [3,8], and the coherent destruction of tunneling by Grossmann are physically an identical concept. Bavli and Metiu [12] have reported on a more complex situation. They showed by utilizing the concept of CDT that a semi-infinite driving field acts on a ground-state (delocalzed) electron, localizes it in one of the wells, and then confines it there (see [11]). Recently, the CDT in Bose-Einstein condensates (BECs) in an optical lattice has attracted much attention and has been investigated in the context of the phase transition between the superfluid states and the Mott insulator [13–15]. In addition, it is interesting to note that the $J_0$-type rescaling also underlies the renormalization of atomic $g$ factor in oscillating magnetic fields [16–18] and the modulation of the evanescent coupling constants of the optical directional coupler by long-period gratings [19].

This study addresses the theoretical description of the coherent tunneling process for electrons in a double-well quantum structure which is subjected to various driving fields with different wave forms and gives the effective tunneling coefficient as a function of the pulse area of the driving field.

## II. DOUBLE-WELL SYSTEM AND DRIVING FIELD

Let us start with particles (atoms or electrons) which are placed into a double-well structure and exposed to an external ac field with a nonsinusoidal wave form. The time evolution of the wave function $\phi(x,t)$ is managed by the time-dependent Schrödinger equation

$$i\hbar\frac{\partial\phi(x,t)}{\partial t}=\left[-\frac{\hbar^2}{2m}\nabla^2+V_c(x)+xE(t)\right]\phi(x,t), \qquad (1)$$

where $V_c(x)$ is the confining potential, $E(t)$ is an external periodic field of zero mean, $x$ being one of the spatial coordinates of the two wells, and $m$ the atomic mass. We assume that the energy separation between the ground state and the first excited energy is much larger than the maximum modulation frequency which is contained in the driving field. In the two-mode approximation in which only state $|R\rangle$ in the right well and state $|L\rangle$ in the left well are considered, we write $\phi_N(x,t)=c_R(t)u_R(x)+c_L(t)u_L(x)$, satisfying the simple normalization $|c_R(t)|^2+|c_L(t)|^2=1$, where $u_{R,L}(x)$ are the local-mode solutions of the individual wells. Substituting this into Eq. (1), we can obtain coupled-mode equations as follows with $\hbar=1$:

$$i\frac{dc_R}{dt}=[\omega_R^0+x_RE(t)]c_R+\kappa c_L, \qquad (2a)$$

 

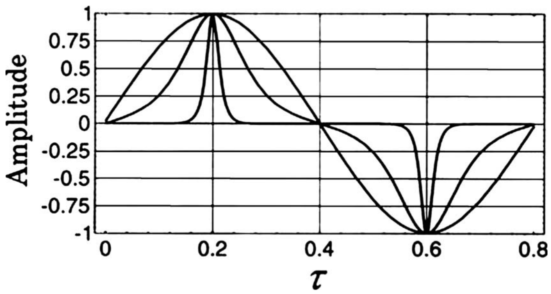


FIG. 1. The three driving wave forms derived from Eq. (3) for $\tau_p$=0.01, 0.05, and 0.2 with a constant pulse interval $\tau_{in}$=0.4. The driving wave form for $\tau_p$=0.01 is hardly different from the hyperbolic secant pulse. Those for $\tau_p$=0.05 and 0.2 show a trianglelike wave form and a sinusoidal wave form, respectively.

$$i\frac{dc_L}{dt} = [\omega_L^0 + x_L E(t)]c_L + \kappa c_R, \quad (2b)$$

where $\omega_{R,L}^0$ ($x_{R,L}$) are the zero-point energies (spatial coordinate) and of the right and left wells, respectively, and $E(t)$ is the periodic driving field of zero mean. $\kappa$ is the tunneling coefficient between the two wells given by $\kappa = \int[(1/2m)(\nabla u_R \nabla u_L) + u_R V_c(x) u_L]dx$.

Hereafter we will restrict ourselves to a symmetric double well, where the zero-point energies in the two wells are equal; i.e., $\omega_R^0 = \omega_L^0 = \omega^0$. The particle number in the right (left) well is given by $N_R = |c_R|^2 N$ ($N_L = |c_L|^2 N$), and the total number of particles in the system $N$ is maintained; i.e., $N_R + N_L = N$. If all particles are initially localized in one of the wells, they will undergo tunneling oscillations having a period $T_{tun} = \pi/\kappa$.

We introduce a driving field which is created by a hyperbolic secant pulse train ($n$ pulses) as follows:

$$E(t) = Ef(t) = Eg(t_{in}, t_p)\sum_n (-1)^n \operatorname{sech}\{(t - nt_{in})/t_p\}, \quad (3)$$

where $E$ is the amplitude of the driving field and $g(t_{in}, t_p)$ is a supplementary coefficient which is introduced to maintain the driving field amplitude to be a constant value $E$ regardless of the different driving wave forms, which are obtained for different values of $t_{in}$ and/or $t_p$, because the summation of the hyperbolic-secant functions in Eq. (3) decreases the modulation amplitude as the pulse interval $t_{in}$ is decreased and/or the pulse width $t_p$ is increased. The alternating driving potential induced between the two wells is thus given by $|x_R - x_L|Ef(t) = Vf(t)$, where $V = |x_R - x_L|E$.

## III. MODULATION OF TUNNELING BY DRIVING FIELDS WITH DIFFERENT WAVE FORMS

The coupled equation (1) can be solved numerically by use of the Runge-Kutta algorithm. We introduce a dimensionless time $\tau \equiv \kappa t$, $\tau_{in} \equiv \kappa t_{in}$, and $\tau_p \equiv \kappa t_p$ and hence the dimensionless tunneling time is given by $\tau_{tun} \equiv \kappa T_{tun} = \pi$. Direct numerical integration of Eq. (1) is performed with an initial condition $c_R(0) = 1$ and $c_L(0) = 0$.

In Fig. 1, we show three wave forms of the driving field obtained for $\tau_p$=0.01, 0.05, and 0.2 with a constant pulse interval $\tau_{in}$=0.4. The driving wave form is nearly identical to the hyperbolic-secant wave for $\tau_p$=0.01, trianglelike wave for $\tau_p$=0.05, and the sinusoidal wave for $\tau_p$=0.2. The driving wave form for $\tau_{in}$=0.2 nearly coincides with the sinusoidal one. Their discrepancy is less than 2% for all points of one period. The supplementary coefficient $g(\tau_{in}, \tau_p)$ is 1.000, 1.001, and 1.863 for $\tau_p$=0.01, 0.05, and 0.2, respectively.

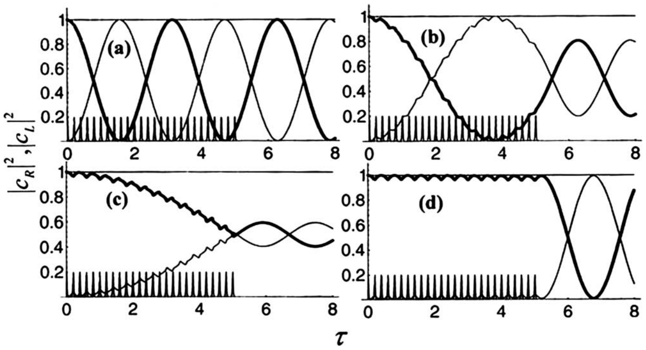


FIG. 2. Typical numerical results that show the modification of the tunneling oscillations by the driving field derived from Eq. (3) for $\tau_{in}$=0.2 and $\tau_p$=0.01. The pulse interval $\tau_{in}$ is very small compared with the tunneling time $\tau_{tun}$; i.e., $\tau_{tun}/\tau_{in} = \pi/0.2 = 15.7$, which satisfies the high-frequency condition.

In Fig. 2, we show typical numerical results which show the modification of the tunneling oscillations by the driving field which is given by Eq. (3) for $\tau_{in}$=0.2 and $\tau_p$=0.01, which give $g(\tau_{in}, \tau_p) = 1.000$. The pulse interval $\tau_{in}$ is very small compared with the tunneling time $\tau_{tun}$; i.e., $\tau_{tun}/\tau_{in} = 15.7$, which satisfies the high-frequency condition required to realize the CDT [1–11]. Here we define pulse area as used in the research field of quantum optics to study coherent dynamic phenomena [20]:

$$A = \int_0^{\tau_{in}} |Vf(t)| dt. \quad (4)$$

The coherent tunneling oscillations in the absence of the driving field are shown in Fig. 2(a), in which the sequence of 26 pulses (13 pairs of plus and minus pulses) is shown on the abscissa as positive pulses. Thick and thin curves show electron populations in the right well ($N_R$) and left well ($N_L$), respectively. We can see that the period of the coherent oscillations increases as the pulse area $A$ (or the pulse height $V$) is increased as shown in Figs. 2(b) ($A = 0.6\pi$) and (c) ($A = \pi$), and the CDT is realized at $A = 1.1\pi$ [see Fig. 2(d)]. The change of the oscillation period means that the tunneling coefficient is effectively modulated by the driving field. Here, we introduce the effective tunneling coefficient (ETC) which is defined by the ratio of the oscillation frequencies of the coherent tunneling oscillations with and without the driving field.

In Fig. 3, we show the ETC as a function of the pulse area of the driving fields for three different driving wave forms shown in Fig. 1. It is obvious that the modulation of the tunneling coefficient significantly depends on the wave form of the driving field. The maximum modulation amplitude of the ETC is obtained for a pulselike driving field with $\tau_p$

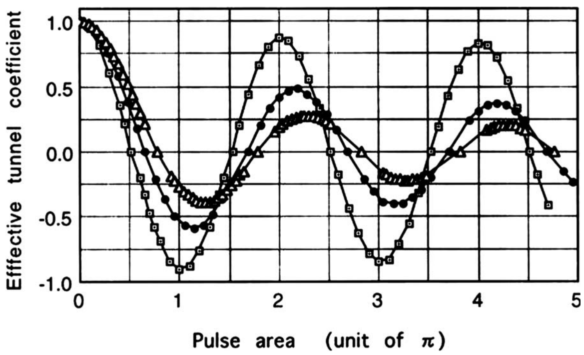


FIG. 3. The effective tunneling coefficient (ETC) $\kappa_{eff}/\kappa$, which is derived from the numerical period of the coherent tunneling oscillations, is shown as a function of the pulse area for the three wave forms of the driving fields shown in Fig. 1: $\tau_p=0.01$ (open squares), 0.05 (solid circles), and 0.2 (open triangles) with $\tau_{in}=0.4$. The solid lines are for guide to eyes.

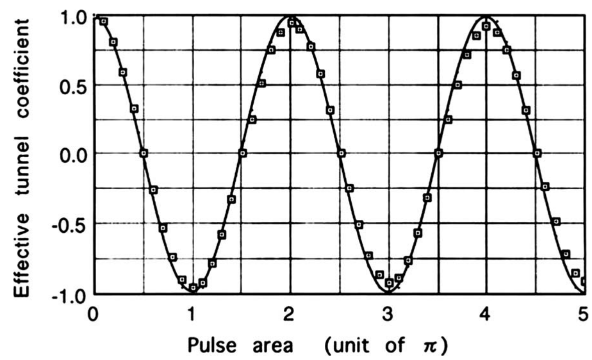


FIG. 4. The ETC as a function of the pulse area of the extremely narrow driving field which is given by Eq. (3) for $\tau_p=0.005$ and $\tau_{in}=0.4$. The squares indicate the numerical results, and the solid curve shows a function $\kappa_{eff}/\kappa=\cos A$, where $A$ is the pulse area.

$=0.01$. The sinusoidal modulation ($\tau_p=0.2$), on the other hand, leads to the minimum modulation of the tunneling coefficient. As we will discuss below, this modulation is given by the Bessel function $J_0(x)$. The ETC driven by the triangle-like wave ($\tau_p=0.05$) shows the middle characteristics between them. We can see that the value of the pulse area which gives the zeros of the ETC gradually increases as the driving wave form becomes broad. Here, we stress that the modulation of the tunneling coefficient is realized not only by the sinusoidal driving field, but also by alternating driving fields which have wave forms other than the sinusoidal wave, and large modulation of the ETC is obtained for narrow driving fields.

Here we discuss the first zeros of the ETC from a viewpoint of the pulse area for the driving fields of both the hyperbolic secant pulse and the sinusoidal wave. The pulse area of the hyperbolic secant pulse satisfying $\tau_p \ll \tau_{in}$ is given by $A_p=V\tau_p\pi$, and that of the sinusoidal wave is given by $A_s=Vg(\tau_{in},\tau_p)\tau_{in}/\pi$. We can see that first zero of the ETC is obtained at $A_p \approx \pi/2$ for the pulselike driving field ($\tau_p=0.01$ and $\tau_{in}=0.4$). On the other hand, we can see that the numerical results for the sinusoidal modulation ($\tau_p=0.2$ and $\tau_{in}=0.4$) give the first zero of the ETC at $A\approx 2.4$. It is well known that for the sinusoidal driving field, if $\varepsilon/\omega=x_i^0$, $i=1,2,\ldots$, with $x_i^0$ the $i$th root of the ordinary Bessel function of order zero, $J_0(\varepsilon/\omega)$, the ETC effectively becomes zero and therefore the CDT can be realized. The first root is given at $\varepsilon/\omega=2.4$ which should be related to the pulse area $A$. In fact, we note that the argument $\varepsilon/\omega$ corresponds to the pulse area of the sinusoidal wave $A_s$, which is derived from $\varepsilon/\omega=\varepsilon/2\pi f=\varepsilon T/2\pi=Vg(\tau_{in},\tau_p)\tau_{in}/\pi=A_s$, where $f$ and $T$ ($=2\tau_{in}$) are the frequency and the period of the sinusoidal wave, respectively. The coincidence of the first zeros between the numerical results and $J_0(\varepsilon/\omega)$ is quite good; i.e., $\varepsilon/\omega=A_s\approx 2.4$.

In order to understand the basic mechanism of the ETC and the CDT, we study the tunneling dynamics driven by extremely narrow pulse trains. In Fig. 4, we show the ETC as a function of the pulse area for the driving field with $\tau_p=0.005$ and $\tau_{in}=0.4$. The squares indicate the numerical results, and the solid curve represents $\cos A$. Although the modulation amplitude of the numerical result is slightly smaller than $\kappa_{eff}=\kappa\cos A$, we see that the coincidence between them is quite good. Comparing this result with those obtained for $\tau_p=0.01$ shown in Fig. 3, we can anticipate that ultimate narrow driving pulses must bring the result which completely coincides with $\kappa_{eff}=\kappa\cos A$. The ETC becomes zero at the pulse areas of $(m+1/2)\pi$, where $m=0,1,2\ldots$, and shows maximal modulation peaks at the pulse areas of $m\pi$.

The zeros of the ETC give rise to the CDT. We qualitatively explain the CDT phenomena shown in Fig. 2(d) which is the CDT obtained for the driving field parameters $\tau_{in}=0.2$ and $\tau_p=0.01$ and the pulse area $A=0.55\pi$, which is slightly larger than $\pi/2$. It is easy to explain the CDT at $A=(m+1/2)\pi$ by visualizing the Bloch vector which is defined by $u=c_R^*c_L+c_Rc_L^*$ ($x$ component), $v=-i(c_R^*c_L-c_Rc_L^*)$ ($y$ component), and $w=c_Rc_R^*-c_Lc_L^*$ ($z$ component) on the unit sphere (Bloch sphere). The Bloch vector initially directs to the north pole ($z$ axis), rotates around the tunneling vector ($x$ axis) by making a trace of the free tunneling oscillation, which is shown in Fig. 2(a), on $y$-$z$ plane in the interval of the first and second pulses, and the Bloch vector directs to the point (0, $v_1$, $w_1$), then turns around the $z$ axis by an amount of $\pi$ (a driving pulse causes $\pi/2$ pulse area for the level in the right well and $-\pi/2$ pulse area for the level in the left well, totally $\pi$) by the driving pulse, resulting in the Bloch vector being in the opposite point (0, $-v_1$, $w_1$), and then rotates again around $x$ axis, resulting in recovering the initial state. In other words, the $\pi/2$ pulse ($\pi$-phase shift) acts as a time-reversal operator. The third pulse does not effect on the oscillation any more because the initial state is not affected by the phase shift, and then the oscillation dynamics repeats again, resulting in the population dynamics which shows rectified sinusoidal-like waves with small amplitude as seen in Fig. 2(d).

To obtain the effective CDT, therefore, we have to use short repetition time $\tau_{in}$. This is the reason why the high frequencies compared to the free tunneling frequency—i.e., the condition $\omega \gg \kappa/2\pi$—are required for the effective modulation of the tunneling coefficient or the CDT. This requirement is well known for the sinusoidal modulations

[1–11]. The suppression of the coherent tunneling in a three-coupled system by a sequence of laser pulses, which has an intimate connection with the CDT discussed here, has been reported [21].

In Fig. 5, we compare the numerical result of the ETC, which is obtained for the sinusoidal-like driving field, with the ordinary Bessel function of order zero $J_0(\varepsilon/\omega)$ as a function of $\varepsilon/\omega$. The triangles indicate the numerical results, and the solid curve shows $J_0(\varepsilon/\omega)$. The coincidence between them is quite good for $\varepsilon/\omega \leqslant 13$, but some difference is seen for $\varepsilon/\omega \geqslant 13$. This is due to the discrepancy between the real sinusoidal wave and the driving field which is created by the superposition of hyperbolic secant pulses given by Eq. (3). It is interesting to note that tiny difference of the wave forms gives rise to large discrepancy of the ETC for large values of $\varepsilon/\omega$ (or pulse area).

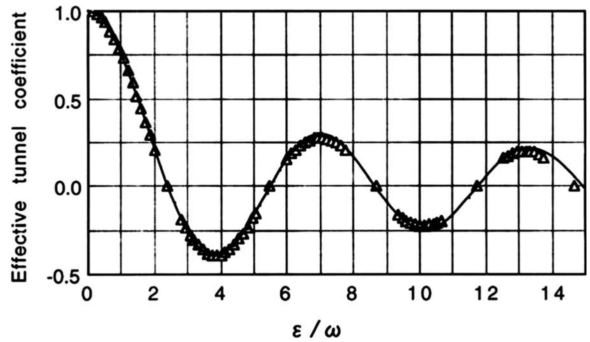


FIG. 5. The ETC which is obtained for the sinusoidal-like driving field and the ordinary Bessel function of order zero, $J_0(\varepsilon/\omega)$, are shown as a function of $\varepsilon/\omega$, where $\varepsilon$ and $\omega$ are amplitude and angular frequency of the sinusoidal driving field, respectively. The triangles indicate the numerical results obtained for the driving field for $\tau_{in}$=0.4 and $\tau_p$=0.2, which is already shown in Fig. 3, and the solid line shows $J_0(\varepsilon/\omega)$.

## IV. THEORETICAL CONSIDERATIONS

In this section, we theoretically study the coherent population dynamics by use of the transfer matrix formula [19,21]. During the intervals $\tau_{in}$ between the pulse excitations, the system undergoes free tunneling oscillations. For arbitrary initial amplitudes $c_R(0)$ and $c_L(0)$, the solution of the equation of motion can be written in the matrix form

$$\begin{pmatrix} c_R(\tau) \\ c_L(\tau) \end{pmatrix} = \begin{pmatrix} \cos\tau_{in} & i\sin\tau_{in} \\ i\sin\tau_{in} & \cos\tau_{in} \end{pmatrix} \begin{pmatrix} c_R(0) \\ c_L(0) \end{pmatrix} = M_I \begin{pmatrix} c_R(0) \\ c_L(0) \end{pmatrix}, \tag{5}$$

which expresses free tunneling (Bloch) oscillations between two wells, while the modulation field which has a very narrow pulselike shape ($\delta$ kick) induces the phase shifts between the two states $|R\rangle$ and $|L\rangle$, and the transfer matrix is given by

$$M_P^{\pm} = \begin{pmatrix} e^{\pm iA} & 0 \\ 0 & e^{\mp iA} \end{pmatrix}, \tag{6}$$

where upper and lower signs correspond to the periodic kicks with alternating sign. The one cycle of the modulation sequence is thus given by

$$M_u = M_P^- M_I M_P^+ M_I = \begin{pmatrix} \cos^2\tau_{in} - e^{-i2A}\sin^2\tau_{in} & i\sin\tau_{in}\cos\tau_{in}(1+e^{-i2A}) \\ i\sin\tau_{in}\cos\tau_{in}(1+e^{i2A}) & \cos^2\tau_{in} - e^{i2A}\sin^2\tau_{in} \end{pmatrix}. \tag{7}$$

For $A=(m+1/2)\pi(m=0,1,2,\ldots)$, $M_u$ becomes a unit matrix. Therefore, after one cycle of the modulation sequence the initial state completely recovers. This corresponds to the CDT phenomenon which is obtained at $A=(m+1/2)\pi$ shown in Fig. 3. The tunneling coefficient effectively becomes zero at those points. On the other hand, for $A=m\pi$, $M_P^{\pm}$ becomes a unit matrix and $M_u=M_I M_I$ which expresses the free tunneling (Bloch) oscillations and thus the ETC becomes definitely identical to the bare tunneling coefficient $\kappa$.

Finally, we consider a more general case for an arbitrary value of the pulse area $A$ with the initial condition used in the numerical calculations; i.e., $c_R(0)=1$ and $c_L(0)=0$. We can obtain the probability amplitudes of the states $|R\rangle$ and $|L\rangle$ after one cycle of the modulation sequence as follows;

$$M_u \begin{pmatrix} c_R(0) \\ c_L(0) \end{pmatrix} = \begin{pmatrix} c_R(2\tau_{in}) \\ c_L(2\tau_{in}) \end{pmatrix} = \begin{pmatrix} \cos^2\tau_{in} - e^{-i2A}\sin^2\tau_{in} \\ i\sin\tau_{in}\cos\tau_{in}(1+e^{i2A}) \end{pmatrix}. \tag{8}$$

The population transfer rate from state $|R\rangle$ to state $|L\rangle$ during the one cycle ($2\tau_{in}$) is thus given by $|c_L(2\tau_{in})|^2 = 2\sin^2\tau_{in}\cos^2\tau_{in}(1+\cos 2A)$. Comparing this with the population transfer rate for free tunneling, $|c_L(2\tau_{in})|^2_{free} = 4\sin^2\tau_{in}\cos^2\tau_{in}$ which is obtained by setting $A=0$, we can derive the population transfer rate which corresponds to the ETC as follows:

$$\frac{|c_L(2\tau_{in})|}{|c_L(2\tau_{in})|_{free}} = \frac{\kappa_{eff}}{\kappa} = |\cos A|. \tag{9}$$

Finally we obtain $\kappa_{eff}=\kappa|\cos A|$. The simple theoretical considerations shown here give a complete understanding of the ETC which is given by the numerical calculations in Sec. III. Unfortunately, the sign of the ETC is not determined by this treatment. To do so, we have to study on trajectories of the Bloch vector on the Bloch sphere and have confirmed that the ETC is really given by $\kappa_{eff}=\kappa\cos A$.

## V. CONCLUSION

We have studied the coherent tunneling oscillations between two levels in a double-well system in the presence of anharmonic periodic potentials. We found that the behavior of the modification of the tunneling oscillations is strongly depended on the wave form of the driving field. Extremely short driving pulses with alternating sign modify the ETC to be given by $\kappa_{eff}=\kappa\cos A$. The CDT can, therefore, be realized for the condition $\cos A=0$. Theoretical derivation of the effective tunneling coefficient is also shown for a periodic delta kick with alternating sign by means of the transfer matrix formula.

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