Deterministic generation of three-dimensional entanglement for two atoms separately trapped in two optical cavities

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A scheme is proposed for the deterministic generation of three-dimensional entanglement of two distant atoms separately trapped in two optical cavities connected by an optical fiber. Employing adiabatic passage along dark states, the atoms are always in ground states, in particular, the fiber mode remains in the vacuum state due to the quantum destructive interference, and the population of the cavities being excited can be negligible under certain conditions. In this sense, our scheme constructs an effective way to avoid the atomic spontaneous emission and the decays of fiber and cavities.

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Entanglement, one of the most interesting features in quantum mechanics, plays a significant role in quantum mechanics, as it not only holds the power for demonstration of the quantum nonlocality against local hidden variable theory [1], but also provides promising and wide applications in quantum information processing, such as quantum cryptography [2], teleportation [3], and dense coding [4]. Thus intense theoretical efforts have been devoted to the generation of entangled states, and remarkable improvements have been made in experiments in two-state systems, for example, such entanglements have been demonstrated in cavity QED [5] and ion traps [6].

Recently, high-dimensional entangled states have received a broad interest, since it has been demonstrated that the violations of local realism and the security of quantum cryptography can be enhanced by high-dimensional entanglement [7]. For photons, such entangled states have been experimentally demonstrated [8]. For atoms, schemes have been proposed in the context of cavity QED. For instance, Zou *et al.* [9] have proposed a scheme for the entanglement generation of two three-level atoms via cavity-assisted collisions, which is insensitive to the cavity decay, but requires individual addressing of the two identical atoms in a cavity and thus is experimentally problematic. Zheng [10] also has proposed a scheme for generating entangled many multilevel atoms in a thermal cavity, which does not require individual addressing of atoms in a cavity and even is insensitive to both cavity decay and thermal field. However, the scheme requires the atoms simultaneously interacting with a cavity mode and a strong classical field, which has not been experimentally demonstrated yet. Recently, Zheng [11] has proposed a scheme for entanglement generation of multiple three-level atoms by sending atoms one by one through a two-mode cavity with the resonant interaction with two field modes sequentially. In the above-mentioned schemes, the atoms all have to interact with the same cavity.

Since entanglement between two separate subsystems is very useful for quantum communication, in particular, Zheng [12] first proposed a scheme for the generation of two three-dimensional entangled atoms trapped in two spatially separated cavities. However, it is a probabilistic scheme as it depends on the detection of the photons decaying from two leaking cavities and thus high efficient photon detectors are required. The ideal success probability is only $1/3$. Inspired by Ref. [13], which employs an optical fiber to deterministically realize quantum gates between two separated atoms, in this Brief Report we present a scheme for deterministically producing three-dimensional entanglement for two atoms separately trapped in two optical cavities. Based on adiabatic passage along dark states, our scheme has the following advances: (i) the field mode remains in vacuum state, and in a certain range the coupling strengths between the fiber and cavities only have slight effects on the fidelity; (ii) all the atoms are always in ground states; (iii) the cavities' modes being excited can be negligible under the condition that all the laser Rabi frequencies are much smaller than the cavity Rabi frequency; and (iv) it goes beyond the Lamb-Dicke limit, and is robust against the small fluctuations of experimental parameters.

First, we introduce the atom-cavity-fiber system. As shown in Fig. 1(a), two identical atoms A and B are separately trapped in two optical cavities 1 and 2 coupled by an optical fiber. Each atom has three ground states $|e\rangle$, $|f\rangle$, and $|g\rangle$, which are, respectively, coupled to one excited state $|r\rangle$ by pulsed laser fields with coupling strengths $\Omega_e^{(k)}(t)$ and $\Omega_f^{(k)}(t)$, and a cavity mode with coupling strength $g^{(k)}$ ($k=1,2$), as shown in Fig. 1(b). An additional ground state $|a\rangle$ will be used. The interaction Hamiltonian for the optical fiber coupled to the modes of the two cavities can be modeled as [14]

$$H_{IF}=\sum_{j=1}^{\infty}\nu_j\{b_j[a_1^\dagger+(-1)^j e^{i\varphi}a_2^\dagger]+\text{H.c.}\},\qquad(1)$$

where $a_1^\dagger$ and $a_2^\dagger$ are the creation operators for the cavities' modes, b_j is the annihilation operator for the mode j of the fiber, ν_j is the coupling strength between the modes of cavities and the fiber mode j, and the phase φ is caused by the propagation of the field through the fiber of length l: $\varphi=2\pi wl/c$ [15], where w is the frequency of the cavities.

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Let $\bar{\nu}$ be the decay rate of the cavities' fields into a continuum of fiber modes. In the short fiber limit $(l\bar{\nu})/(2\pi c) \leqslant 1$, only one resonant mode of the fiber will interact with the cavity modes [13]. This reduces the Hamiltonian H_{IF} to [16]

$$H_{IF} = \nu[b(a_1^\dagger + a_2^\dagger) + \text{H.c.}], \tag{2}$$

where b is the annihilation operator of resonant mode of the fiber. We use laser fields with Rabi frequency $\Omega_l^{(1)}(t)$ and $\Omega_m^{(2)}(t)$ (l, $m=e$ or f) in cavities 1 and 2, respectively. In the interaction picture, the Hamiltonian for the total atom-cavity-fiber system under consideration is

$$H_{IN} = [\Omega_l^{(1)}(t)|r\rangle_A\langle l|_A + g^{(1)}a_1|r\rangle_A\langle g|_A + \Omega_m^{(2)}(t)|r\rangle_B\langle m|_B + g^{(2)}a_2|r\rangle_B\langle g|_B + \text{H.c.}] + H_{IF}. \tag{3}$$

For simplicity, we assume $g^{(1)}=g^{(2)}=g$ in the following. For an initial state $|l\rangle_A|g\rangle_B|000\rangle_f$, the state evolution remains in the subspace spanned by the basis state vectors $\{|g\rangle_A|m\rangle_B|000\rangle_f, |g\rangle_A|r\rangle_B|000\rangle_f, |g\rangle_A|g\rangle_B|010\rangle_f, |g\rangle_A|g\rangle_B|001\rangle_f, |g\rangle_A|g\rangle_B|100\rangle_f, |r\rangle_A|g\rangle_B|000\rangle_f, |l\rangle_A|g\rangle_B|000\rangle_f\}$. Here, the numbers n_1, n_2, and n_f in the state $|n_1 n_2 n_f\rangle_f$ denote the photon numbers in the modes of cavity 1, cavity 2, and fiber, respectively. There is an interesting dark state with null eigenvalue

$$|\phi\rangle_{dark} \propto g\Omega_m^{(2)}(t)|l\rangle_A|g\rangle_B|000\rangle_f - \Omega_l^{(1)}(t)\Omega_m^{(2)}(t)|g\rangle_A|g\rangle_B \times(|100\rangle_f - |010\rangle_f) - g\Omega_l^{(1)}(t)|g\rangle_A|m\rangle_B|000\rangle_f. \tag{4}$$

We note that if the system remains in the dark state, the two atoms are always in ground states. Most notably, the fiber links two spatially separate QED subsystems but the fiber mode is in vacuum state. Indeed, such phenomenon seems incredible. However, essentially, it can be interpreted as the result of quantum destructive interference. Here, the two states $|g\rangle_A|g\rangle_B|100\rangle_f$ and $|g\rangle_A|g\rangle_B|010\rangle_f$ are mediated by the intermediate state $|g\rangle_A|g\rangle_B|001\rangle_f$. Since the two transition paths $|g\rangle_A|g\rangle_B|100\rangle_f \to |g\rangle_A|g\rangle_B|001\rangle_f$ and $|g\rangle_A|g\rangle_B|010\rangle_f \to |g\rangle_A|g\rangle_B|001\rangle_f$ interfere destructively, the fiber mode remains in vacuum state. In other words, the fiber plays an important role as a mediate to connect two separated subsystems. How the coupling strength ν between the fiber and cavity influences the fidelity of our scheme will be discussed later in this Brief Report. Moreover, to make the population of the cavity modes in excited states negligible, we can assume that the condition $g \gg \Omega_l^{(k)}(t)$, $\Omega_m^{(k)}(t)$ is always satisfied during the whole procedure in our scheme.

Now we show how to generate three-dimensional entanglement of the atoms A and B. First, atom A is prepared in the state $|e\rangle_A$, atom B in the superposition state $(\sqrt{2}|g\rangle_B + |a\rangle_B)/\sqrt{3}$, and all the field modes in vacuum state. Then, the state $|e\rangle_A|g\rangle_B|000\rangle_f$ is adiabatically transferred into $-|g\rangle_A|f\rangle_B|000\rangle_f$ along dark states by using the lasers $\Omega_e^{(1)}(t)$ and $\Omega_f^{(2)}(t)$, while the state $|e\rangle_A|a\rangle_B|000\rangle_f$ remains unchanged. Here, we analyze the adiabatic evolution in detail. For the initial state $|e\rangle_A|g\rangle_B|000\rangle_f$, according to Eq. (4), the dark state is

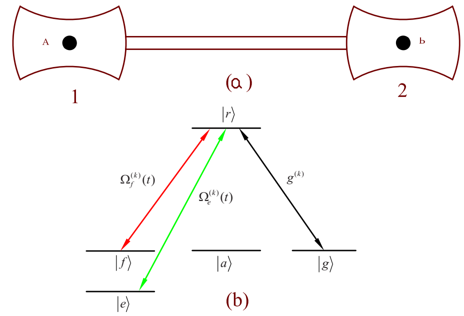

FIG. 1. (Color online) (a) The experimental setup. (b) The level configuration for each atom. The transitions $|f\rangle \to |r\rangle$ and $|e\rangle \to |r\rangle$ are, respectively, driven by two pulsed laser fields with coupling strength $\Omega_e^{(k)}(t)$ and $\Omega_f^{(k)}(t)$, while the transition $|g\rangle \to |r\rangle$ is coupled to a cavity field with the constant strength $g^{(k)}$.

$$|\Phi\rangle_{dark}^{(1)} \propto g\Omega_f^{(2)}(t)|e\rangle_A|g\rangle_B|000\rangle_f - \Omega_e^{(1)}(t)\Omega_f^{(2)}(t)|g\rangle_A|g\rangle_B \times(|100\rangle_f - |010\rangle_f) - g\Omega_e^{(1)}(t)|g\rangle_A|f\rangle_B|000\rangle_f. \tag{5}$$

Initially, we assume that $\Omega_e^{(1)}(0) \ll \Omega_f^{(2)}(0)$, and $\Omega_e^{(1)}(0)$, $\Omega_f^{(2)}(0) \ll g$. In this case, the dark state is approximately $|e\rangle_A|g\rangle_B|000\rangle_f$. To adiabatically transfer the state $|e\rangle_A|g\rangle_B|000\rangle_f$ into $-|g\rangle_A|f\rangle_B|000\rangle_f$, we slowly increase $\Omega_e^{(1)}(t)$ and decrease $\Omega_f^{(2)}(t)$ so that $\Omega_e^{(1)}/\Omega_f^{(2)} \gg 1$ at the time t_1 [17]. Meanwhile, for the initial state $|e\rangle_A|a\rangle_B|000\rangle_f$, the state evolution remains in the subspace spanned by the basis state vectors $\{|e\rangle_A|a\rangle_B|000\rangle_f, |r\rangle_A|a\rangle_B|000\rangle_f, |g\rangle_A|a\rangle_B|100\rangle_f, |g\rangle_A|a\rangle_B|001\rangle_f, |g\rangle_A|a\rangle_B|010\rangle_f\}$. The corresponding dark state is

$$|\Phi\rangle_{dark}^{(2)} \propto g|e\rangle_A|a\rangle_B|000\rangle_f - \Omega_e^{(1)}(t)|g\rangle_A|a\rangle_B(|100\rangle_f - |010\rangle_f). \tag{6}$$

Similar to the previous dark state $|\phi\rangle_{dark}$, here for the dark state $|\Phi\rangle_{dark}^{(2)}$ the fiber mode is in vacuum state as well due to the quantum destructive interference. Remember that the condition $g \gg \Omega_l^{(k)}(t)$, $\Omega_m^{(k)}(t)$ $(k=1,2)$ is always fulfilled in our scheme, and thus the state $|e\rangle_A|a\rangle_B|000\rangle_f$ remains unchanged under the adiabatic condition. Therefore the atomic state adiabatically evolves into $|\Psi(t_1)\rangle_{A+B} = (-\sqrt{2}|g\rangle_A|f\rangle_B + |e\rangle_A|a\rangle_B)/\sqrt{3}$, leaving all the field modes in vacuum state under the condition $g \gg \Omega_e^{(1)}(t)$, $\Omega_f^{(2)}(t)$.

Next we change the state $|\Psi(t_1)\rangle_{A+B}$ to $(|g\rangle_A|f\rangle_B - |g\rangle_A|e\rangle_B + |e\rangle_A|a\rangle_B)/\sqrt{3}$ by performing the transformation $|f\rangle_B \to (-|f\rangle_B + |e\rangle_B)/\sqrt{2}$ on atom B in cavity 2. Then, similar to the previous step, we apply pulses $\Omega_f^{(1)}(t)$ and $\Omega_e^{(2)}(t)$ to make the transformation $|g\rangle_A|e\rangle_B|000\rangle_f \to -|f\rangle_A|g\rangle_B|000\rangle_f$ adiabatically. This can be achieved by applying the pulse $\Omega_f^{(1)}(t)$ on atom 1 preceding the pulse $\Omega_e^{(2)}(t)$ on atom 2, leading from $\Omega_f^{(1)}/\Omega_e^{(2)} \gg 1$ to $\Omega_f^{(1)}/\Omega_e^{(2)} \ll 1$ at the time t_2. At the same time, the states $|g\rangle_A|f\rangle_B|000\rangle_f$ and $|e\rangle_A|a\rangle_B|000\rangle_f$ undergo no evolution and are unchanged during the stage. Therefore after the procedure atoms A and B evolve to the state

$$|\Psi\rangle_{A+B}=(|g\rangle_A|f\rangle_B+|f\rangle_A|g\rangle_B+|e\rangle_A|a\rangle_B)/\sqrt{3}, \tag{7}$$

leaving all the field modes in vacuum state. We note that during the whole procedure, the condition $g \gg \Omega_f^{(k)}(t)$, $\Omega_e^{(k)}(t)$ should always be satisfied to make the population of the cavity modes in excited states negligible. Performing the following transformations, $|f\rangle_B \to |g\rangle_B$, $|g\rangle_B \to |f\rangle_B$, and $|a\rangle_B \to |e\rangle_B$, through external classical fields, we obtain the three-dimensional maximally entangled state of normal form

$$|\Psi''\rangle_{A+B}=(|g\rangle_A|g\rangle_B+|f\rangle_A|f\rangle_B+|e\rangle_A|e\rangle_B)/\sqrt{3}, \tag{8}$$

where the quantum information is encoded in the three levels $|g\rangle$, $|f\rangle$, and $|e\rangle$ for both atoms A and B. We note that, from now on, the dynamics of the two atoms is restricted to such levels: no other levels will be coupled to $|g\rangle$, $|f\rangle$, and $|e\rangle$, neither by coherent couplings nor through incoherent decays [18]. Therefore the two atoms can rightly be considered as effective three-dimensional systems, and the state $|\Psi''\rangle_{A+B}$ is maximally entangled.

Finally, we discuss the experimental feasibility of the present scheme. For the atomic level structure, we can take Cs as our choice. The states $|f\rangle$, $|a\rangle$, $|g\rangle$, and $|e\rangle$ correspond to the hyperfine levels $|F=4, m=-1\rangle$, $|F=4, m=0\rangle$, $|F=4, m=1\rangle$, and $|F=3, m=-1\rangle$ of $6S_{1/2}$, respectively, while the state $|r\rangle$ corresponds to $|F=4, m=0\rangle$ of $6P_{3/2}$. The transitions $|f\rangle \to |r\rangle$ and $|e\rangle \to |r\rangle$ are driven by right-circular polarized laser pulses, while the transition $|g\rangle \to |r\rangle$ is coupled to a left-circular polarized cavity mode. All the transitions are resonant. In the above analyses, we have assumed that the atoms are trapped well in specific locations in cavities, but in real experiments, it is very challenging to control atoms precisely to meet the Lamb-Dicke condition. Therefore to make our scheme more robust against the randomness of the atoms' positions, we here employ the idea of Ref. [19]: All the applied laser pulses are incident from one mirror of their corresponding cavities and collinear with their respective cavities' axes, and thus share the same spatial mode structure with their respective cavities. As the adiabatic evolution in our scheme only depends on the ratios $\Omega_e^{(k)}(\vec{r},t)/g(\vec{r})$, $\Omega_f^{(k)}(\vec{r},t)/g(\vec{r})$, and $\Omega_l^{(k)}(\vec{r},t)/\Omega_m^{(k')}(\vec{r},t)$ ($k\neq k'$; k, $k'=1,2$; l, $m=e$, f), which are all independent on the random atom position $\vec{r}$, our scheme is not restricted to the Lamb-Dicke condition.

In the above discussion, the cavity modes are never excited as we have assumed that $g \gg \Omega_f^{(k)}(t)$, $\Omega_e^{(k)}(t)$ are always fulfilled throughout the whole procedure. However, if the condition $g \gg \Omega_f^{(k)}(t)$, $\Omega_e^{(k)}(t)$ is not ideally fulfilled, there is a probability of the cavity modes being excited. Here we set $\Omega_{f\,\max}^{(k)}=\Omega_{e\,\max}^{(k)}=g/4$ (in experiment, g more than four times larger than $\Omega_{e,f\,\max}^{(k)}$ has been achieved [20]) to estimate such probability. In this case, during the adiabatically transferring procedure $|e\rangle_A|g\rangle_B|000\rangle_f \to -|g\rangle_A|f\rangle_B|000\rangle_f$, according to the Eqs. (5) and (6), there is a probability of the states $|g\rangle_A|g\rangle_B(|100\rangle_f-|010\rangle_f)/\sqrt{2}$ and $|g\rangle_A|a\rangle_B(|100\rangle_f-|010\rangle_f)/\sqrt{2}$ being populated. The average probability for the state $|g\rangle_A|g\rangle_B(|100\rangle_f-|010\rangle_f)/\sqrt{2}$ being populated during the transferring procedure can be estimated as

$$P_1=\frac{2}{\theta_{\max}}\int_0^{\theta_{\max}}\sin^2\theta d\theta, \tag{9}$$

where $\sin\theta=\Omega_e^{(1)}\Omega_f^{(2)}/\sqrt{2(\Omega_e^{(1)}\Omega_f^{(2)})^2+g^2[(\Omega_e^{(1)})^2+(\Omega_f^{(2)})^2]}$. When $\Omega_e^{(1)}=\Omega_f^{(2)}=g/8$, we have $\theta=\theta_{\max}=\arcsin(1/\sqrt{130})$ and thus $P_1\simeq 0.005$. The average probability for the state $|g\rangle_A|a\rangle_B(|100\rangle_f-|010\rangle_f)/\sqrt{2}$ being populated during the transferring procedure can be estimated as

$$P_2=\frac{2}{\alpha_{\max}}\int_0^{\alpha_{\max}}\sin^2\alpha d\alpha, \tag{10}$$

where $\sin\alpha=\Omega_e^{(1)}/\sqrt{2(\Omega_e^{(1)})^2+g^2}$. When $\Omega_e^{(1)}=g/4$, we have $\alpha=\alpha_{\max}=\arcsin(1/\sqrt{18})$ and thus $P_2\simeq 0.037$. [At the end of the procedure (at the time t_1) the state $|e\rangle_A|a\rangle_B|000\rangle_f$ evolves to the superposition state $[4|e\rangle_A|a\rangle_B|000\rangle_f-|g\rangle_A|a\rangle_B(|100\rangle_f-|010\rangle_f)]/\sqrt{18}$ instead of remaining in the state $|e\rangle_A|a\rangle_B|000\rangle_f$.] Thus due to the cavity decay, the states $|g\rangle_A|g\rangle_B(|100\rangle_f-|010\rangle_f)/\sqrt{2}$ and $|g\rangle_A|a\rangle_B(|100\rangle_f-|010\rangle_f)/\sqrt{2}$ may evolve into the states $|g\rangle_A|g\rangle_B|000\rangle_f$ and $|g\rangle_A|a\rangle_B|000\rangle_f$, respectively, which will reduce the fidelity of our scheme. To see to what extent of the cavity decay effects the fidelity, we first estimate the time for the adiabatic transfer $|e\rangle_A|g\rangle_B|000\rangle_f \to -|g\rangle_A|f\rangle_B|000\rangle_f$. Here, the energy gap between the dark state $|\Phi\rangle_{dark}^{(1)}$ and its most closest bright state is denoted by ΔE_1, and that between the dark state $|\Phi\rangle_{dark}^{(2)}$ and its most closest bright state is denoted by ΔE_2. It is interesting that as ν goes from $10g$ to $100g$, ΔE_1 rises slightly from $0.249g$ to $0.250g$ and similarly ΔE_2 rises slightly from $0.749g$ to $0.750g$. Since to fulfill the adiabatic condition the transferring time should be larger than the inverse of the minimum gap [21], such phenomenon implies that the variety in the coupling between the fiber and the cavity (for ν ranging from $10g$ to $100g$) has slight effects on the required operating time and thus only influences the fidelity of our scheme slightly. Set $g=1$ GHz [22] and $\nu=10g$ [23], the adiabatic transferring time is $t_1\simeq 10/0.2g=50$ ns, which means that the total time for the adiabatic generation of a three-dimensional entangled state is about $2t_1=100$ ns. Set $\kappa=0.01g$ [24], where κ is the cavity decay rate. Due to the cavity decay, the probability for the state $|e\rangle_A|g\rangle_B|000\rangle_f$ evolving to $|g\rangle_A|g\rangle_B|000\rangle_f$ through the intermediate state $|g\rangle_A|g\rangle_B(|100\rangle_f-|010\rangle_f)/\sqrt{2}$ is approximately $\varepsilon_1 \simeq \kappa t_1 P_1=0.0025$, and that for the state $|e\rangle_A|a\rangle_B|000\rangle_f$ evolving to $|g\rangle_A|a\rangle_B|000\rangle_f$ through the intermediate state $|g\rangle_A|a\rangle_B(|100\rangle_f-|010\rangle_f)/\sqrt{2}$ is approximately $\varepsilon_2\simeq\kappa t_1 P_2=0.0185$. Similarly, during the transferring procedure $|g\rangle_A|e\rangle_B|000\rangle_f \to -|f\rangle_A|g\rangle_B|000\rangle_f$, due to the cavity decay the probability for the state $|g\rangle_A|e\rangle_B|000\rangle_f$ evolving to $|g\rangle_A|g\rangle_B|000\rangle_f$ through the intermediate state $|g\rangle_A|g\rangle_B(|100\rangle_f-|010\rangle_f)/\sqrt{2}$ is approximately ε_1 as well. Thus the final state can be approximately given by [25]

$$\rho=|\psi\rangle\langle\psi|+\frac{\varepsilon_1(3-\varepsilon_1)}{3}|g\rangle_A|f\rangle_B\langle g|_A\langle f|_B+\frac{8\varepsilon_2+1}{27}|g\rangle_A|e\rangle_B\langle g|_A\langle e|_B, \tag{11}$$

where

$$|\psi\rangle = \frac{1}{\sqrt{3}}\left(\sqrt{1-\varepsilon_1}|g\rangle_A|g\rangle_B + (1-\varepsilon_1)|f\rangle_A|f\rangle_B + \frac{4\sqrt{1-\varepsilon_2}}{\sqrt{18}}|e\rangle_A|e\rangle_B \right). \quad (12)$$

Thus under the condition $\Omega_{f\,\max}^{(k)} = \Omega_{e\,\max}^{(k)} = g/4$, the fidelity is

$$F = {}_{A+B}\langle''\Psi|\rho|\Psi''\rangle_{A+B} = \frac{1}{9}\left[\sqrt{1-\varepsilon_1} + (1-\varepsilon_1) + \frac{4\sqrt{1-\varepsilon_2}}{\sqrt{18}} \right]^2 \simeq 0.95. \quad (13)$$

In conclusion, a scheme is presented for generating a three-dimensional entangled state for two atoms separately trapped in two optical cavities connected by an optical fiber. It is based on the adiabatic passages by using a sequence of pulsed laser fields, where the atoms are all in ground states and the fiber mode remains in vacuum state and thus the atomic spontaneous emission and the decay of optical fiber can be greatly avoided. Under the condition that all the laser Rabi frequencies are much smaller than the cavity Rabi frequency, the scheme is insensitive to the decay of the optical cavities. Furthermore, taking advantage of adiabatic passage, it is insensitive to small fluctuations of experimental parameters [26,27]. We also show that it is not restricted to the Lamb-Dicke limit and make an estimation on the fidelity under the condition that the cavity Rabi frequency is only four times the maximal laser Rabi frequencies.

[1] A. Einstein, B. Podolsky, and N. Rosen, Phys. Rev. **47**, 777 (1935); J. S. Bell, Physics (Long Island City, N.Y.) **1**, 195 (1965); D. M. Greenberger, M. Horne, A. Shimony, and A. Zeilinger, Am. J. Phys. **58**, 1131 (1990).

[2] A. K. Ekert, Phys. Rev. Lett. **67**, 661 (1991).

[3] C. H. Bennett, G. Brassard, C. Crepéau, R. Jozsa, A. Peres, and W. K. Wootters, Phys. Rev. Lett. **70**, 1895 (1993).

[4] C. H. Bennett and S. J. Wiesner, Phys. Rev. Lett. **69**, 2881 (1992).

[5] S. Osnaghi, P. Bertet, A. Auffeves, P. Maioli, M. Brune, J. M. Raimond, and S. Haroche, Phys. Rev. Lett. **87**, 037902 (2001); A. Rauschenbeutel, G. Nogues, S. Osnaghi, P. Bertet, M. Brune, J. M. Raimond, and S. Haroche, Science **288**, 2024 (2000).

[6] Q. A. Turchette, C. S. Wood, B. E. King, C. J. Myatt, D. Leibfried, W. M. Itano, C. Monroe, and D. J. Wineland, Phys. Rev. Lett. **81**, 3631 (1998); C. A. Sackett, D. Kielpinski, B. E. King, G. Langer, V. Meyer, C. J. Myatt, M. Rowe, Q. A. Turchette, W. M. Itano, D. J. Wineland, and C. Monroe, Nature (London) **404**, 256 (2000).

[7] D. Kaszlikowski, P. Gnacinski, M. Zukowski, W. Miklaszewski, and A. Zeilinger, Phys. Rev. Lett. **85**, 4418 (2000); M. Bourennane, A. Karlsson, and G. Björk, Phys. Rev. A **64**, 012306 (2001); D. Bruss and C. Macchiavello, Phys. Rev. Lett. **88**, 127901 (2002); N. J. Cerf, M. Bourennane, A. Karlsson, and N. Gisin, *ibid.* **88**, 127902 (2002).

[8] A. Mair, A. Vaziri, G. Weihs, and A. Zeilinger, Nature (London) **412**, 313 (2001); A. Lamas-Linares, J. C. Howell, and D. Bouwmeester, *ibid.* **412**, 887 (2001); H. de Riedmatten, I. Marcikic, H. Zbinden, and N. Gisin, Quantum Inf. Comput. **2**, 425 (2002); J. C. Howell, A. Lamas-Linares, and D. Bouwmeester, Phys. Rev. Lett. **88**, 030401 (2002).

[9] X. B. Zou, K. Pahlke, and W. Mathis, Phys. Rev. A **67**, 044301 (2003).

[10] S. B. Zheng, Phys. Rev. A **68**, 035801 (2003).

[11] S. B. Zheng, Commun. Theor. Phys. **45**, 539 (2006).

[12] S. B. Zheng, Chin. Phys. Lett. **22**, 3064 (2005).

[13] A. Serafini, S. Mancini, and S. Bose, Phys. Rev. Lett. **96**, 010503 (2006).

[14] T. Pellizzari, Phys. Rev. Lett. **79**, 5242 (1997).

[15] S. J. van Enk, H. J. Kimble, J. I. Cirac, and P. Zoller, Phys. Rev. A **59**, 2659 (1999).

[16] Z. Q. Yin and F. L. Li, Phys. Rev. A **75**, 012324 (2007).

[17] A. S. Parkins, P. Marte, P. Zoller, and H. J. Kimble, Phys. Rev. Lett. **71**, 3095 (1993); T. Pellizzari, S. A. Gardiner, J. I. Cirac, and P. Zoller, *ibid.* **75**, 3788 (1995).

[18] Remember that the levels $|g\rangle$, $|f\rangle$, $|e\rangle$, and $|a\rangle$ used here are all ground states. Thus without classical or cavity fields, no coherence coupling will happen, and decoherence (spontaneous emission) also will not induce the coupling of the levels $|g\rangle$, $|f\rangle$, $|e\rangle$ to $|a\rangle$.

[19] L. M. Duan, A. Kuzmich, and H. J. Kimble, Phys. Rev. A **67**, 032305 (2003); B. Yu, Z. W. Zhou, Y. Zhang, G. Y. Xiang, and G. C. Guo, *ibid.* **70**, 014302 (2004).

[20] N. V. Vitanov, M. Fleischhauer, B. W. Shore, and K. Bergmann, Adv. At., Mol., Opt. Phys. **46**, 55 (2001); J. McKeever, A. Boca, A. D. Boozer, J. R. Buck, and H. J. Kimble, Nature (London) **425**, 268 (2003); R. Miller, T. E. Northup, K. M. Birnbaum, A. Boca, A. D. Boozer, and H. J. Kimble, J. Phys. B **38**, S551 (2005).

[21] L.-M Duan, J. I. Cirac, and P. Zoller, Science **292**, 1695 (2001). Here, the specific condition is $t_1 \gg 1/\min\{\Delta E_1, \Delta E_2\}$.

[22] M. Trupke, E. A. Hinds, S. Eriksson, E. A. Curtis, Z. Moktadir, E. Kukharenka, and M. Kraft, Appl. Phys. Lett. **87**, 211106 (2005).

[23] The coupling strength ν can be increased by decreasing the reflectivity of the cavity mirror connected to the fiber. See Ref. [13].

[24] We note that the coupling strength g could be significantly larger than the cavity decay rate κ by smoothing the substrate surface. See also Ref. [22].

[25] It is noted that here the transformations $|f\rangle_B \to |g\rangle_B$, $|g\rangle_B \to |f\rangle_B$, and $|a\rangle_B \to |e\rangle_B$ have been performed on atom B to obtain Eq. (8).

[26] N. V. Vitanov, T. Halfmann, B. W. Shore, and K. Bergmann, Annu. Rev. Phys. Chem. **52**, 763 (2001).

[27] S. B. Zheng, Phys. Rev. Lett. **95**, 080502 (2005).