Coherent output of photons from coupled superconducting transmission line resonators controlled by charge qubits

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We study the coherent control of microwave photons propagating in a superconducting waveguide consisting of coupled transmission line resonators, each of which is connected to a tunable charge qubit. While these coupled line resonators form an artificial photonic crystal with an engineered photonic band structure, the charge qubits collectively behave as spin waves in the low excitation limit, which modify the band-gap structure to slow and stop the microwave propagation. The conceptual exploration here suggests an electromagnetically controlled quantum device based on the on-chip circuit QED for the coherent manipulation of photons, such as the dynamic creation of laserlike output from the waveguide by pumping the artificial atoms for population inversion.

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I. INTRODUCTION

Recent experiments with on-chip all optical setups [1–4] have displayed slow light phenomenon similar to that due to the electromagnetically induced transparency (EIT) [5,6]. Here, a physical mechanism is presented using a model with a coupled resonator optical waveguide, which behaves as a photonic crystal with a band-gap spectrum. The coupling of each resonator to some external cavities can shift the resonant spectral line and compress the bandwidth, and thus stop or store the propagating light pulses [1–3].

Motivated by this progress, both in experimental and theoretical aspects, we propose and study a hybrid structure with a cavity waveguide interacting with two-level artificial atoms. Here, the control mechanism for coherent transmission of microwave photons in the waveguide is to utilize the collective excitations of the atoms, which can be described as quasispin waves [7] in the low excitation limit. As illustrated in Fig. 1, we suggest a co-planar on-chip setup based on the superconducting circuit QED [8–16]: each cavity in our proposed setup has been experimentally implemented as a superconducting transmission line resonator; the spatially distributed artificial atoms are the biased Cooper pair boxes (charge qubits), which play the same role as the external cavity for controlling light in the waveguide of the on-chip all optical experiment [1–4].

Due to its engineered photonic band structure, such a cavity array, coupled to quasispin wave excitations, can result in much richer quantum coherent phenomena. We show that the quasispin waves of charge qubits can controllably affect the engineered photonic band structure so that the modified dispersion relation results in a slow (and even zero) group velocity of photons propagating along the waveguide of coupled line resonators. The spin-wave excitation can also drive a coherent state of photons in the coupled resonator waveguide. Some pumping methods can make a population inversion to produce a laserlike output above the threshold.

This paper is organized as follows. In Sec. II, we present our hybrid system: an array of coupled-line resonators based on a superconducting circuit. Each resonator is coupled to a biased Cooper pair box (CPB). The array of the coupled-line resonator exhibits a band structure. In Sec. III, in the large-N and low-excitation limits, the hybrid system is modeled as two coupled boson models, with one for the photonic band and the other for the spin wave of N CPBs. Here, the slow light phenomenon is found by controlling the detuning and the coupling strength. The dynamic creation of the laserlike output from the array of the coupled-line resonator is proposed in Secs. IV and V. This is because two kinds of bosons couple linearly with each other and the population inversion of the artificial atoms is pumped. Section VI presents our conclusions.

II. COUPLED CAVITY QED BASED ON SUPERCONDUCTING CIRCUIT

Now we consider a superconducting quantum circuit including N CPBs and an array of coupled-line resonators. Re-

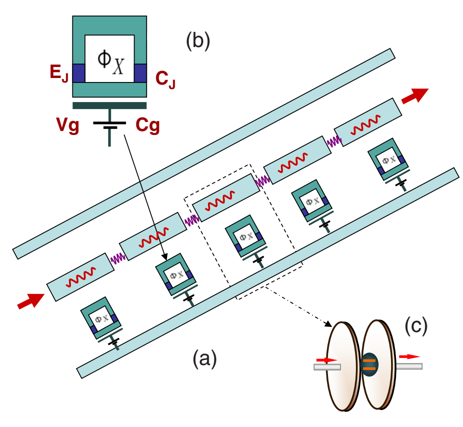

FIG. 1. (Color online) Configuration of the setup for controlling light propagation in a coupled-line-resonator waveguide (a) by coupling to charge qubits (b). The coupled (line resonators)–(charge qubits) system can behave similarly to the cavity QED for a single atom interacting with a single-mode cavity (c).

cently, most experiments have demonstrated the possibility to fabricate a superconducting qubit array [17–19]. As shown in Fig. 1, the array of coupled-line resonators is constructed by cutting the superconductor transmission line into N segments; and the N CPBs are made of N dc SQUIDs (superconducting quantum interference devices) which consist of two tunnel junctions.

With a proper biased voltage V_g, each CPB behaves as a two-level system (charge qubit) [20]. Typically, the model Hamiltonian for a charge qubit can be written as in Refs. [8,21],

$$H = \frac{1}{2}B_z\sigma_z - \frac{1}{2}B_x\sigma_x \tag{1}$$

with the level spacing of the charge qubit

$$B_z = 4E_C(2n_g - 1) \tag{2}$$

and effective Josephson energy

$$B_x = 2E_J \cos\left(\pi\frac{\Phi_x}{\Phi_0}\right). \tag{3}$$

The quasispin operators σ_z and σ_x are defined in the charge qubit basis ($|0\rangle$ and $|1\rangle$), where 0 and 1 represent excess Cooper pairs on the superconducting island, respectively. Here, E_C and E_J represent the charge energy and the Josephson energy, respectively. Through $n_g = C_g V_g/2e$, we can tune the degenerate point of the charge qubit by controlling the gate voltage V_g applied on the gate capacitance C_g. In addition, Φ_x represents the magnetic flux through the SQUID loop induced by the externally applied magnetic field and $\Phi_0 = h/2e$ is the quantum flux.

Based on the above results, we can write the Hamiltonian including N CPBs as

$$H_A = \frac{\omega_A}{2}\sum_j (|e_j\rangle\langle e_j| - |g_j\rangle\langle g_j|), \tag{4}$$

where the eigenfrequency $\omega_A = \sqrt{B_z^2 + B_x^2}$. Based on recent experiments, the charge energy can be taken as $E_C = 29.5$ GHz, while the Josephson energy $E_J = 8$ GHz. For simplicity, all qubits are assumed to be identical and biased off the degenerate point. The energy eigenstates

$$|e_j\rangle = \cos\left(\frac{\theta}{2}\right)|0_j\rangle - \sin\left(\frac{\theta}{2}\right)|1_j\rangle \tag{5}$$

and

$$|g_j\rangle = \sin\left(\frac{\theta}{2}\right)|0_j\rangle + \cos\left(\frac{\theta}{2}\right)|1_j\rangle \tag{6}$$

are the superpositions of charge eigenstates $|0_j\rangle$ and $|1_j\rangle$, and $\theta = \arctan(B_x/B_z)$.

In our setup, each qubit is placed at the position of the antinode of the standing wave field in each transmission line resonator. The London equation provides the vanishing boundary conditions for the quantized electromagnetic field at the two ends of each line resonator. Thus the quantized magnetic field vanishes at those antinodes and the qubits are only coupled to the electric component. Here, the gate voltage applied to the capacitance of the jth Cooper pair box is given by $V_x^j = V_g + V_q^j$, where V_q^j stands for the quantized part of the voltage. Let only the $n=2$ mode be activated. Then the gate voltage is quantized as [13,14]

$$V_q^j = (\hat{a}_j + \hat{a}_j^\dagger)\sqrt{\frac{\omega}{LC}} \tag{7}$$

at its maximum while the magnetic field vanishes. Here, we only assumed a single mode of the quantized electric field of frequency ω with creation (annihilation) operator $\hat{a}_j^\dagger$ ($\hat{a}_j$). L is the length of the line resonators and C is the capacitance per unit length.

There exists the coupling J between two neighbor line resonators through the dielectric material. Then the Hamiltonian of the coupled line resonators waveguide is

$$H_C = \omega\sum_j \hat{a}_j^\dagger\hat{a}_j + J\sum_j (\hat{a}_j^\dagger\hat{a}_{j+1} + \hat{a}_{j+1}^\dagger\hat{a}_j), \tag{8}$$

where J depends on the coupling mechanism, e.g., the magnetic penetration depth.

Following recent experiments [13], we can take the frequency of the line resonators $\omega = 10.0$ GHz, the length of line resonators $L = 1.0$ cm, and the capacitance per unit length $C = 0.13$ fF/μm. We now make the rotating wave approximation to write down the interaction Hamiltonian [13,14]

$$H_I = g\sum_{j=1}^{N} (\hat{a}_j|e_j\rangle\langle g_j| + \text{H.c.}), \tag{9}$$

between the qubits and the fields, where

$$g = e\sin\theta\frac{C_g}{C_\Sigma}\sqrt{\frac{\omega}{LC}} \tag{10}$$

is the coupling strength between the resonator and the artificial two-level atom, and C_Σ is the sum of the gate capacitance and the capacitance of the tunnel junction.

In practical experiments, the coupling constants g and J should depend on the position of the qubit. For simplicity, in this paper we take a uniform g and J. Theoretically, the fluctuations of the coupling constants are assumed to be innocuous and do not change the results of this paper, qualitatively. The above model has been used to demonstrate the photon blocked effect [22–24], which leads to the Mott insulating effect for the polaritons formed by dressing atoms with a cavity field.

Although our proposed configuration setup is an extension of the single cavity proposed in [9,10,13,14] to N coupled cavities, there are significant difference between them. In experiments [12,13], to demonstrate the typical cavity QED character of a superconducting quantum circuit for quantum computing, only a single cavity is used rather than a coupled cavity array. A single cavity possesses discrete photon modes with large frequency spacings, but an array of a coupled-line resonator possesses a band-gap spectrum and can transmit a wave packet of light. A single cavity coupled to many charge qubits in a similar superconducting circuit has been modeled [25] to probe the dynamic behavior of quantum phase transition [26]. Obviously, different from the setup in [13,14], our setup uses many coupled transmission line resonators to realize an electromagnetically controlled quantum device for coherent control of photon transmission.

III. TWO-MODE BOSON MODEL FOR THE SPIN-WAVE DRESSED PHOTONIC BAND

Now we consider the low-excitation limit that a few charge qubits are populated in their excited state. The crucial issue here is to use the collective operator [7]

$$\hat{B}_k^\dagger = N^{-1/2}\sum_{j=1}^{N} \exp(ik\ell j)|e_j\rangle\langle g_j| \tag{11}$$

and its conjugate $\hat{B}_k=(\hat{B}_k^\dagger)^\dagger$ to describe the spin wave excitation of the charge qubit array, where $k=2\pi n/\ell N$ with $n=0,1,\ldots,N-1$. In the large N limit with low excitations, i.e., $\langle \Sigma_k \hat{B}_k^\dagger \hat{B}_k\rangle \ll N$, these collective excitations behave as bosons, since the usual bosonic commutation relation $[\hat{B}_k,\hat{B}_{k'}^\dagger]=\delta_{kk'}$ can be approached when $N\to\infty$.

The Fourier transformation $\hat{a}_k=\Sigma_j \exp(ik\ell j)\hat{a}_j/\sqrt{N}$ diagonalizes the coupled resonator Hamiltonian as

$$H_C = \sum_k \Omega_k \hat{a}_k^\dagger \hat{a}_k$$

to give a dispersion relation with band structure

$$\Omega_k = \omega + 2J\cos(k\ell), \tag{12}$$

where ℓ is the site distance. Then the normal modes of the hybrid system with Hamiltonian $H=\Sigma_k H_k$ are characterized by $\hat{a}_k$ and $\hat{b}_k=\lim_{N\to\infty}\hat{B}_k$. The evolution of each mode is governed by the following Hamiltonian:

$$\begin{aligned} H_k &= \Omega_k \hat{a}_k^\dagger \hat{a}_k + g(\hat{a}_k \hat{b}_k^\dagger + \text{H.c.}) + \omega_A \hat{b}_k^\dagger \hat{b}_k \\ &= \Omega_{Dk}\hat{N}_k + \epsilon_k(\hat{P}_k^\dagger \hat{P}_k - \hat{Q}_k^\dagger \hat{Q}_k). \end{aligned} \tag{13}$$

Here, we have introduced the polariton operators [7]

$$\hat{P}_k = \cos\theta_k \hat{a}_k + \sin\theta_k \hat{b}_k; \tag{14}$$

$$\hat{Q}_k = \sin\theta_k \hat{a}_k - \cos\theta_k \hat{b}_k, \tag{15}$$

which are linear combinations of the quantized electromagnetic field operators and atomic collective excitation operators of quasispin waves. The mixing angle θ is determined by

$$\tan\theta_k = 2g/(\Omega_k - \omega_A). \tag{16}$$

The total excitation number for the kth mode is given by

$$\hat{N}_k = \hat{P}_k^\dagger \hat{P}_k + \hat{Q}_k^\dagger \hat{Q}_k = \hat{a}_k^\dagger \hat{a}_k + \hat{b}_k^\dagger \hat{b}_k. \tag{17}$$

The dispersion relation for the photonic band is obtained as

$$\epsilon_{\pm k} = \Omega_{Dk} \pm \varepsilon_k, \tag{18}$$

where

$$\Omega_{Dk} = \frac{1}{2}(\Omega_k + \omega_A),$$

$$\varepsilon_k = \frac{1}{2}\sqrt{(\Omega_k - \omega_A)^2 + 4g^2}. \tag{19}$$

Since $\hat{N}_k$ commutes with H_k, the number of excitations $\hat{N}_k$ is conserved, while the number of different type excitations $\hat{a}_k^\dagger \hat{a}_k$ and $\hat{b}_k^\dagger \hat{b}_k$ are mutually convertible by adjusting the coupling strength g and the detuning $\delta=\omega-\omega_A$ between the artificial atom and the resonator. Due to the coupling between artificial atoms and the resonators, the origin band structure for the array of coupled-line resonators is split into two in the one excitation subspace. Obviously there exists a gap between two bands for nonvanishing g.

Notice that the band structure (19) is only available in the low excitation limit $\langle \Sigma_k \hat{b}_k^\dagger \hat{b}_k\rangle \ll N$. In the following, we focus on the single excitation subspace. For $\delta>0$, the corresponding dressed spectrum $\epsilon_{\pm k}=\Omega_{Dk}\pm\epsilon_k$ shows a bandwidth narrowing effect on the coupled line resonators due to its couplings to the charge qubits. This is because the bandwidth $W_-=|\epsilon_{-k=0}-\epsilon_{-k=\pi}|$ of the low-band changes from $2J$ to $2J-\Delta$, where

$$\Delta = A(J) - A(-J) \tag{20}$$

and

$$A(J) = \sqrt{\left(\frac{\delta}{2}+J\right)^2 + g^2}. \tag{21}$$

Meanwhile, the bandwidth $W_+=|\epsilon_{+k=0}-\epsilon_{+k=\pi}|$ of the upper band changes from $2J$ to $2J+\Delta$. Obviously when $\delta=0$, the bandwidths W_- and W_+ of the two bands are equal to $2J$; when $\delta>0$ ($\delta<0$), W_- (W_+) is narrower than $2J$ while W_+ (W_-) is wider than $2J$. Then the wave packet of light in the lower band can be adjusted by δ. The couplings also shift the central spectral line from ω and ω_A to $\epsilon_{\pm k=\pi/2}$, respectively. This just recovers the same result obtained in Refs. [1–4].

Then we can obtain the group velocities $v_\pm(k)=d\epsilon_{\pm k}/dk$ for the lower and the upper bands as

$$v_\pm = \text{Re}\left\{J\ell\sin(k\ell)\left[1 \pm \frac{\Lambda + 2J\cos(k\ell)}{\sqrt{[\Lambda + 2J\cos(k\ell)]^2 + 4g^2}}\right]\right\}, \tag{22}$$

where $\Lambda=\delta+i\kappa$. Here, $\kappa=\eta-\gamma$ is the difference between the damping constant η of each cavity and the decay rate γ of the qubit, where η is caused by the cavity loss and $\gamma=T_1^{-1}+T_2^{-1}$: in experiments, the energy relaxation time T_1 of a superconducting qubit is 0.1 to a few microseconds, and the dephasing time T_2 between the ground and the excited state is a few dozens of nanoseconds. It can be seen that the group velocity becomes independent of loss when $\eta=\gamma$. At the band center $k=\pi/(2\ell)$, the group velocity for the lower band becomes

$$v_-\left(-\frac{\pi}{2\ell}\right) = \text{Re}\left\{J\ell\left(1 - \frac{\Lambda}{\sqrt{\Lambda^2 + 4g^2}}\right)\right\}. \tag{23}$$

In the large detuning case, i.e., $\delta \gg 2|g|$, the group velocity reaches its minimum

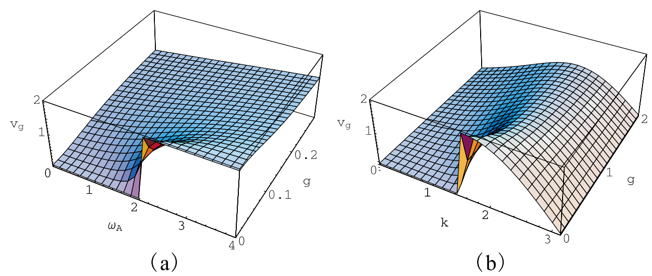

FIG. 2. (Color online) (a) The group velocity v_g of the lowest band as a function of ω_A and coupling strength g at $k=\pi/(2\ell)$ for $\eta=\gamma$, and $\omega=J$. It can be seen that when $\delta \gg 2g$, the group velocity reaches its minimum $v_g(-\pi/(2\ell))\approx 0$; when $\delta \ll -2g$, the group velocity reaches its maximum $v_g(-\pi/(2\ell))\approx 2J\ell$. (b) The group velocity of the lowest band as a function of k and coupling strength g in the case of resonance. Here, g and ω_A are in units of J, and k is in units of the lattice constant ℓ.

$$v_-(-\pi/(2\ell)) = \mathrm{Re}[J\ell g^2/\Lambda^2] \approx 0 \tag{24}$$

corresponding to the compressed lower bandwidth. In the case of $\delta \ll -2|g|$, the group velocity for the lower photonic band reaches its maximum,

$$v_-(-\pi/(2\ell)) \approx 2J\ell, \tag{25}$$

and the lower band has a large bandwidth, which accommodates the entire pulse bandwidth. In Fig. 2(a) we have plotted the group velocity of the lowest band as a function of the level spacing ω_A of the two-level artificial atom and the coupling strength g at $k=-\pi/(2\ell)$. It can be seen that when a large detuning occurs, the group velocity is minimum at large blue detuning and is maximum at large red detuning. Hence for a microwave pulse as a superposition of many k states, its distribution in the k space can be entirely contained in the energy band by setting $\delta \ll -2|g|$. Therefore the microwave pulse can be stopped by adiabatically tuning the detuning from $\delta \ll -2|g|$ to $\delta \gg 2|g|$.

In the near-resonance case $\delta \sim 0$ with strong couplings, the group velocity in the lower band

$$v_- = \mathrm{Re}\left\{ J\ell \sin(k\ell)\left[\frac{g}{J\cos(k\ell)+i\kappa}\right]^2\right\}$$

is reduced to zero approximately within the range of k satisfying $J\cos(k\ell) \gg g$. For mode k, which satisfies $J\cos(k\ell) \ll -g$, the corresponding group velocity becomes $v_- \approx 2J\ell \sin(k\ell)$. For mode k, which satisfies $-g \ll J\cos(k\ell) \ll g$, the group velocity becomes $v_- \approx J\ell \sin(k\ell)$, which is illustrated in Fig. 2(b). It tells us that when a microwave pulse inputs into such a coupled-line-resonator waveguide, some components will be stopped completely by adjusting the coupling strength g, while others still pass through. So the microwave pulse is distorted. Hence in the case of resonance, one cannot obtain the whole information that the wave packet carries.

IV. COHERENT OUTPUT OF SLOW LIGHT

The above boson model describes the linear couplings between two kinds of boson modes. If one can prepare the state of the charge qubit array with population inversion through coherent pumping, the dynamic evolution will drive the coupled-line-resonator mode to output a laserlike light in a coherent state. Such a pumping mechanism with superconducting qubits has been explored most recently for superconducting flux qubits [27].

Now we assume the charge qubits are prepared coherently in excited states. Then the initial state of the spatially distributed atomic ensemble is a k-mode coherent state

$$|\alpha_k\rangle = D(\alpha_k)|G\rangle \equiv D(\alpha_k)|g_1\rangle \otimes |g_2\rangle \otimes \cdots \otimes |g_N\rangle \tag{26}$$

where the displacement operator

$$D(\alpha_k) = \exp(\alpha_k \hat{b}_k^\dagger - \alpha_k^* \hat{b}_k) \tag{27}$$

describes a superposition of n-quasiparticle excitation states $|n_k\rangle$ of mode k. Here, the one quasiparticle excitation characterizes the spatially distributed qubit populations with definite phases, a quasispin wave in the charge qubit array. Let the total system initially start with a coherent state of atomic ensemble and the vacuum state of the coupled-line-resonator array, i.e., $|\psi(0)\rangle = |0\rangle \otimes |\alpha_k\rangle$. Here, $|0\rangle$ means no photon contained in the coupled-line-resonator array. After time t, the initial state is evolved into $|\psi(t)\rangle = U(t)|\psi(0)\rangle$.

In order to find an explicit expression for $|\psi(t)\rangle$, we formally rewrite the quantum state at time t as

$$|\psi(t)\rangle = U(t)D(\alpha_k)U^{-1}(t)U(t)|G\rangle|0\rangle, \tag{28}$$

where $U(t)=\exp(it\Sigma_k H_k)$ is the time evolution operator. Because the number of excitations in the total system is conserved and $|G\rangle|0\rangle$ is the ground state of the system corresponding to the zero eigenvalue, the quantum state defined by Eq. (28) becomes

$$|\psi(t)\rangle = U(t)D(\alpha_k)U^{-1}(t)|G\rangle|0\rangle, \tag{29}$$

which is completely determined by the time dependent displacement operator

$$D(t) \equiv U(t)D(\alpha_k)U^{-1}(t) = \exp[\hat{A}(-t)], \tag{30}$$

where $\hat{A}(-t) = U(t)\hat{A}U^{-1}(t)$.

With respect to the polariton operators $\hat{P}$ and $\hat{Q}$, the initial coherent state $|\alpha_k\rangle$ of the kth mode can be rewritten based on the displacement operator $D(\alpha_k)=\exp[\hat{A}(0)]$, where

$$\hat{A}(0) = \alpha_k \hat{P}_k^\dagger \sin\theta_k - \alpha_k \hat{Q}_k^\dagger \cos\theta_k - \mathrm{H.c.} \tag{31}$$

is an anti-Hermitian operator. Since polaritons are the eigenexcitation of the total system, their creation operators $\hat{P}_k^\dagger$ and $\hat{Q}_k^\dagger$ evolve according to the eigenfrequencies, i.e.,

$$U(t)\hat{P}_k^\dagger U^{-1}(t) = \hat{P}_k^\dagger e^{-i(\Omega_{Dk}+\varepsilon_k)t},$$

$$U(t)\hat{Q}_k^\dagger U^{-1}(t) = \hat{Q}_k^\dagger e^{-i(\Omega_{Dk}-\varepsilon_k)t}.$$

Accordingly, the operator $\hat{A}(t)$ can be explicitly obtained as

$$\hat{A}(t) = \alpha_k P_k^\dagger e^{-i(\Omega_{Dk}+\varepsilon_k)t} \sin\theta_k + \alpha_k^* Q_k e^{i(\Omega_{Dk}-\varepsilon_k)t} \cos\theta_k - \text{H.c.} \tag{32}$$

Transformed back to the original representation with operators $\hat{a}_k$ and $\hat{b}_k$, the displacement operator $D(t)$ becomes the product of two displacement operators, i.e.,

$$D(t) = D[\alpha_k(t)]D[\beta_k(t)], \tag{33}$$

where the factor

$$D[\alpha_k(t)] = \exp[\alpha_k(t)b_k^\dagger - \alpha_k^*(t)b_k] \tag{34}$$

acts on the atomic excitation state while

$$D[\beta_k(t)] = \exp[\beta_k(t)a_k^\dagger - \beta_k^*(t)a_k] \tag{35}$$

acts on the state space of the coupled-line-resonator array.

Therefore the state $|\psi(t)\rangle$ can be factorized as $|\psi(t)\rangle = |\beta_k(t)\rangle \otimes |\alpha_k(t)\rangle$ with the time-dependent amplitudes

$$\alpha_k(t) = \alpha_k e^{-i\Omega_{Dk}t}(e^{i\varepsilon_k t}\cos^2\theta_k + e^{-i\epsilon_k t}\sin^2\theta_k), \tag{36a}$$

$$\beta_k(t) = -ie^{-i\Omega_{Dk}t}\alpha_k \sin(\varepsilon_k t)\sin(2\theta_k), \tag{36b}$$

Here the coherent state $|\beta_k(t)\rangle$ localizes in the k mode, and it actually is a spatially multimode coherent state; the periodically modulated amplitudes $\beta_j(t)$ mean a quasiclassical wave packet of photons with distribution $P_j = |\beta_j(t)|^2$. The above argument means that the initial coherent input of the atomic excitation can result in a coherent output in the photonic mode, which is characterized by a coherent state $|\beta_k(t)\rangle$. This just displays a laserlike behavior for the photons output from the coupled-line-resonator waveguide.

V. LASERLIKE PROCESS WITH POPULATION INVERSION

The above intuitive discussion shows an obvious laser behavior, but we need the population inversion implemented by some coherent pumping. We also need to consider the threshold condition for the lasing in the coupled-line-resonator waveguide. To this end we study the coherent radiation in the coupled-line-resonator waveguide stimulated by the artificial atoms with a collective coherent excitation.

Ignoring the fluctuations due to the couplings to the thermal bath, the dynamic variable of this system obeys the following equations:

$$\partial_t \hat{a}_k = -i(\Omega_k - i\eta)\hat{a}_k - ig\hat{B}_k, \tag{37a}$$

$$\partial_t \hat{B}_k = -i(\omega_A - i\gamma)\hat{B}_k + i\frac{g}{N}\sum_{k'} \hat{S}_{k'-k}\hat{a}_{k'}, \tag{37b}$$

$$\partial_t \hat{S}_0 = \Gamma(Nd_0 - \hat{S}_0) - i2g\sum_k (\hat{B}_k^\dagger \hat{a}_k - \hat{a}_k^\dagger \hat{B}_k), \tag{37c}$$

where

$$\hat{S}_{k'-k} = \sum_{j=0}^{N-1} e^{i\ell(k-k')j}\sigma_j^z$$

and $\hat{S}_0$ is defined by $k' = k$. By neglecting the scattering from k to k' due to large N, one can write down a system of laserlike equations [28],

$$\partial_t \hat{a}_k = -i(\Omega_k - i\eta)\hat{a}_k - ig\hat{b}_k,$$

$$\partial_t \hat{b}_k = -i(\omega_A - i\gamma)\hat{b}_k + ig\hat{n}\hat{a}_k/N,$$

$$\partial_t \hat{n} = \Gamma(Nd_0 - \hat{n}) - i2g\sum_k (\hat{b}_k^\dagger \hat{a}_k - \hat{a}_k^\dagger \hat{b}_k), \tag{38}$$

where d_0 is the input rate for equilibrium inversion. The term $\Gamma(Nd_0 - \hat{n})$ in Eq. (38) is phenomenologically introduced to characterize the role of pumping by some population inversion for the excitation number $\hat{n} = \Sigma \hat{b}_k^\dagger \hat{b}_k$; Γ is the relaxation time to equilibrium.

Next, we eliminate the fast time dependence in Eq. (38) by the substitutions

$$\hat{a}_k = \hat{\tilde{a}}_k \exp(-i\Omega_k t) \tag{39}$$

and

$$\hat{b}_k = \hat{\tilde{b}}_k \exp(-i\Omega_k t). \tag{40}$$

We lock the cavity mode in the coupled-line resonator in a resonance frequency $\Omega_k \approx \omega_A$, that is, $\exp[i(\omega_A - \Omega_k)]$ varies slowly. When the relaxation time of charge-qubits is much smaller than the relaxation time of the cavity as well as that of the population inversion, i.e., $\gamma \gg \Gamma \gg \eta$, we can adiabatically eliminate $\hat{b}_k$ and $\hat{n}$ by setting the corresponding time derivatives to zero in the obtained equations about $\hat{\tilde{a}}_k$, $\hat{\tilde{b}}_k$, and $\hat{n}$ from Eq. (38). Finally we obtain the equation of motion for $\hat{\tilde{a}}_k$,

$$\partial_t \hat{\tilde{a}}_k = (d_0 g^2 L - \eta)\hat{\tilde{a}}_k - \frac{4\gamma d_0 g^4 L|L|^2}{N\Gamma}\hat{\tilde{a}}_k^\dagger \hat{\tilde{a}}_k \hat{\tilde{a}}_k, \tag{41}$$

where the Lorentz distribution

$$L = \frac{1}{\gamma + i(\omega_A - \Omega_k)} \tag{42}$$

shapes the spectra of coherent output. It is exactly the laser equation [28] defining a threshold $d_0 = \eta/(g^2 L)$ for the laserlike output in the coupled-line-resonator waveguide by the coherent pumping with an input rate d_0.

This dynamical lasing behavior can be described by the nonlinear equation

$$\dot{x} = -\sigma x - \lambda x^3 \tag{43}$$

about the order parameters $x = \langle \alpha | \hat{\tilde{a}}_k | \alpha \rangle$. Here, $|\alpha\rangle$ is a coherent state with real number α; the coefficient of the linear term of Eq. (43)

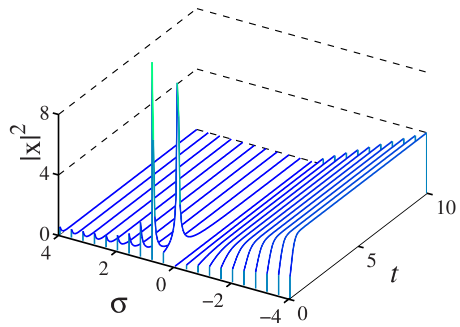

FIG. 3. (Color online) The probability amplitude $|x|^2$ of the output photon as a function of σ and time. When $\sigma<0$, there is a laserlike output depicted by the stable solution for $t\rightarrow\infty$. t is in units of seconds "s" and σ is in units of s^{-1}.

$$\sigma = \eta - d_0 g^2 L \tag{44}$$

describes the amplification effect of the light field when $\sigma<0$; the coefficient

$$\lambda = \frac{4\gamma d_0 g^4 L|L|^2}{N\Gamma} \tag{45}$$

represents the nonlinearity of the effective theory obtained by averaging the atomic excitations. The nonlinear coefficient competes with the parameter σ to realize a lasing "phase transition." As illustrated in Fig. 3, the solutions of the nonlinear equation obviously possess a critical behavior near the threshold $\sigma=0$. The solution of Eq. (43) gives

$$|x|^2 = [c \exp(2\sigma t) - \lambda/\sigma]^{-1} \tag{46}$$

for $\sigma\neq 0$; and

$$|x|^2 = (2\lambda t + c)^{-1} \tag{47}$$

for $\sigma=0$, where c is a constant determined by the initial state. The stable solutions that

$$|x|^2 = -\sigma/\lambda \tag{48}$$

for $\sigma<0$ (above the threshold) and $|x|^2=0$ for $\sigma>0$ (below the threshold) mean a laserlike output of the coupled-line resonators. Since the expectation value of $\hat{\tilde{a}}_k$ does not vanish, we can understand the above laserlike effect as a pumping-induced symmetry breaking.

Together with the intuitive argument in the last section, the analysis in this section definitely shows the existence of a threshold for laserlike behavior in the present artificial system. However, to reduce the threshold by overcoming decoherence, including dissipation and dephasing, is still a challenge to implement a laser of slow light in practical hybrid systems.

VI. CONCLUDING REMARKS

In conclusion, we have conceptually proposed an electromagnetically controllable quantum device based on superconducting circuit QED for the coherent manipulation of photons. It can realize a laserlike slow microwave output from a coupled-line-resonator waveguide by controlling each cavity connected to a charge qubit. Our studies are motivated by the all-optical experiment for stopping light in a coupled resonator waveguide, which is coupled to other cavities, but here we show that some results for the coupled resonator waveguide can be obtained by replacing the coupling cavities with a more practical system, a spatially distributed array of charge qubits. Finally, we would like to mention that, without the limitation of the superconducting system, the present analysis represents a universal setup for the coherent manipulation for light or microwave propagations in some all-optical or electromagnetic-optical system.

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