Imbert-Fedorov shift at metamaterial interfaces

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We analyze the transverse shift (Imbert-Fedorov shift) of a focused beam upon reflection at stratified (doubly dispersive) metamaterials. In deriving analytical expressions for the reflection of a focused beam at such an interface this shift can be quantitatively forecast. Solely based on symmetry considerations of the vectorial reflection coefficient we analyze potential geometries where such a shift occurs. Contrary to the common belief, this shift is observable for a linearly polarized beam at total internal reflection. Furthermore, we predict a giant Imbert-Fedorov shift if light is partially reflected at impedance-matched media.

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I. INTRODUCTION

Recent progress in the field of metamaterials encourage researchers to revisit well-known optical effects but taking into account strong dispersion of both the permittivity and the permeability. Veselago systematically explored such properties first, e.g., he showed that the Doppler effect and the Vavilov-Cherenkov effect are reversed [1]. Such work was driven by the desire to understand the peculiarities of light propagation in such media, to propose effects that prove experimentally the dispersion in both material parameters as well as to render potential applications. Among these effects, the Goos-Hänchen shift attracted considerable interest. It describes the longitudinal shift, along the beam axis, of a reflected beam at total internal reflection (TIR) at the interface between two homogeneous half-spaces or, more generally, at the interface of a homogeneous and an inhomogeneous half-space (arbitrarily stratified medium) [2]. If the geometry of the stratified medium is chosen appropriately, this shift can become giant if resonances are involved [3]. Besides this longitudinal shift a transverse one can be observed. It was predicted theoretically and proven experimentally [4–8]. This transverse shift is known as the Imbert-Fedorov shift (IFS).

The longitudinal and/or transverse shifts of beams are traditionally explained by using two different models. The first is called the Artmann model. It describes the shifts in terms of the phase of the reflected beam. Predictions based on that model are correct if the phase of the reflected beam varies linearly in the reciprocal ($\vec{k}$) space where the illuminating beam has a nonzero amplitude. The second model is called the energy-flux method [7–11]. There, the shifts are explained on the basis of energy conservation arguments. In an initial formulation, as given by Renard, the beam shifts are described in terms of the energy flux parallel to the surface of the evanescent waves in the totally reflecting medium. Therefore, the model is usually referred to as the Renard model [10]. However, since its introduction a discrepancy was encountered in comparison to the Artmann model [12–15]. Yasumoto *et al.* have proven that this is due to nonjustified simplifications in the Renard model [15]. It could be shown that the beam shifts do not only depend on the energy flux of the evanescent waves in the totally reflecting medium, but also on the interference terms in the energy flux between the incident and reflected fields. For the important example of total internal reflection at the interface of two half-spaces, it is easy to show that the sign of the energy fluxes predicted by the Renard model gives the correct sign for the Goos-Hänchen shift, whereas the modulus deviates as compared to the Artmann model. Near the TIR angle both models yield identical results. The differences increase with increasing angle of incidence. For predictions of the beam shift upon reflection at layered medium the Renard model fails even qualitatively. But without the simplification made by Renard, the energy-flux method loses its attractiveness because the fields and the energy fluxes in both half-spaces must be calculated, a rather cumbersome procedure. Particularly, with respect to the IFS that relies on the vectorial nature of light, an illustrative explanation within the Renard model cannot be given anymore.

Therefore, a more sophisticated strategy is in need to calculate the IFS. Such an advantageous quantitative prediction is supported by qualitative considerations. This will permit one to derive criteria for an overall observability of the IFS. This qualitative analysis will be performed upon considering the symmetry of the phase of the reflected beam for cardinal geometries. In general, the phase of the reflected field in the reciprocal ($\vec{k}$) space is symmetric around the principal propagation direction. The gradient of a symmetric function is antisymmetric and hence vanishes when averaged. Thus, no IFS will be observed. As a consequence, one must elaborate on cases only where the phase of the reflected beam is nonsymmetric.

In this work, we shall discuss the origin of the IFS considering the symmetries of the reflected beam's phase. Furthermore we shall develop a computational scheme which allows for calculating the IFS upon tracking the center of gravity of the reflected beam. We shall prove that for IFS to occur circularly or elliptically polarized light is not required as usually assumed [6–8] but can also appear for linearly polarized light. Furthermore, and contrary to the common belief, we outline that the angle of incidence has not to exceed the TIR angle to observe the IFS.

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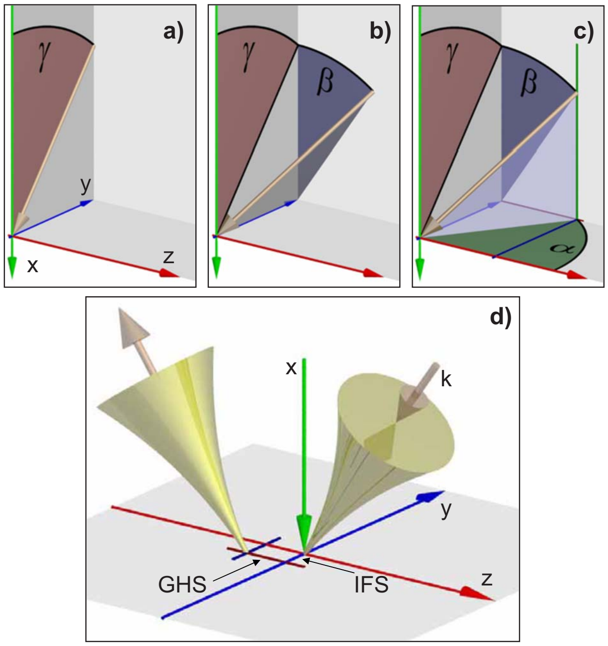

FIG. 1. (Color online) Sketches of the geometry under consideration. (a) Rotation of the plane wave represented by the bronze $\vec{k}$ arrow around the z axis by an angle γ. (b) Subsequent rotation of the plane wave from (a) around the y axis by an angle β. (c) Definition of the angle α by projection of $\vec{k}$ onto the y-z plane. (d) Incident and reflected beams with $\vec{k}$ indicating the main propagation direction. Medium 1 is situated in the half-space $x<0$, medium 2 in $x\geq 0$.

The shift is analyzed for metamaterials where both material parameters are dispersive. There the ratio of impedances of both half-spaces was found to be the relevant parameter that dictates the strength of the IFS. The present work applies to both positive and negative refractive index media.

II. BASIC CONSIDERATIONS

We consider a finite beam that propagates in the positive x direction (see Fig. 1),

$$\vec{E}_I(\vec{r},t) = \int_{-\infty}^{\infty} [\vec{E}_I(\vec{k})e^{i\vec{k}\cdot\vec{r}}]e^{-i\omega t}d\vec{k}.$$

It illuminates an interface located at $x=0$ under an arbitrary angle of incidence. Its principal propagation direction is characterized by an angle β_0 with respect to the x-y plane and by an angle γ_0 with respect to the x-z plane. The interface divides the space into two half-spaces. Both are assumed to be homogeneous and isotropic. They are characterized by the respective permittivity $\varepsilon_{1/2}$ and permeability $\mu_{1/2}$, respectively. The interface is infinite in the y-z plane. Hence the system is invariant under translation for a fixed value of x and rotationally symmetric around the x axis. However, it is not required that the second half-space must be homogeneous.

The following explanation applies likewise for arbitrarily layered rotational symmetric media. Without loss of generality we choose the coordinate system such that the principal propagation direction of the inclined beam is within the x-z plane, i.e., $\gamma_0=0$ and $k_{y_0}=0$. Nevertheless, for most plane waves of the finite beam spectrum $k_y\neq 0$ holds in general. Each plane wave is inclined relative to the x-y plane by an angle β and to the x-z plane by an angle γ. To ensure the arbitrary propagation direction of the incident beam, each plane wave used to represent this beam in terms of a Fourier expansion is first rotated around the z axis and subsequently rotated around the y axis by the appropriate angles. This ensures that for the principal propagation direction $k_{y_0}=0$ holds. Total internal reflection occurs if β_0 is larger than the critical angle. Two more angles δ and κ are introduced for the description of the polarization of the beam on the Poincaré sphere. We assume that all plane waves used to represent the beam exhibit the same polarization. All possible polarization states can be expressed within the space $\{\kappa\in[0,\pi)\}\times\{\delta\in[0,2\pi)\}$. The polarization is specified by considering a plane wave at normal incidence relative to the surface by

$$\vec{E} = E_0\begin{pmatrix} 0 \\ \sin\kappa \\ e^{i\delta}\cos\kappa \end{pmatrix} = E_0\begin{pmatrix} 0 \\ e_y \\ e_z \end{pmatrix}. \quad (1)$$

Here the electric field vector $\vec{E}$ has the amplitude E_0 and the phase between e_y and e_z is fixed by δ. A temporal dependency of $e^{-i\omega t}$ and the spatial variation $e^{i\vec{k}\cdot\vec{r}}$ of each plane wave is omitted for the reason of brevity but assumed throughout the paper. Rotating this plane wave by the angles γ and β in the aforementioned manner using the rotation matrices

$$D_y(\beta) = \begin{pmatrix} \cos(\beta) & 0 & \sin(\beta) \\ 0 & 1 & 0 \\ -\sin(\beta) & 0 & \cos(\beta) \end{pmatrix}$$

and

$$D_z(\gamma) = \begin{pmatrix} \cos(\gamma) & \sin(\gamma) & 0 \\ -\sin(\gamma) & \cos(\gamma) & 0 \\ 0 & 0 & 1 \end{pmatrix}$$

yields

$$D_y(\beta)D_z(\gamma)\vec{E} = E_0\begin{pmatrix} e_z\sin(\beta) + e_y\cos(\beta)\sin(\gamma) \\ e_y\cos(\gamma) \\ e_z\cos(\beta) - e_y\sin(\beta)\sin(\gamma) \end{pmatrix} = \begin{pmatrix} E_{I,x} \\ E_{I,y} \\ E_{I,z} \end{pmatrix} = \vec{E}_I. \quad (2)$$

This field vector provides the electrical field of an arbitrarily polarized plane wave under an arbitrary angle of incidence. Its wave vector is given by

$$D_y(\beta)D_z(\gamma)\vec{k} = D_y(\beta)D_z(\gamma)\begin{pmatrix} k \\ 0 \\ 0 \end{pmatrix} = \begin{pmatrix} k\cos(\beta)\cos(\gamma) \\ -k\sin(\gamma) \\ -k\cos(\gamma)\sin(\beta) \end{pmatrix} = \begin{pmatrix} k_{I,x} \\ k_{I,y} \\ k_{I,z} \end{pmatrix}. \quad (3)$$

Obviously, this field cannot be separated into TE- and TM-polarized components. This makes the computation of the shift rather cumbersome. To allow for such decomposition, we rotate the coordinate system of each plane wave by an angle α within the y-z plane. All wave-vector components in this new coordinate system are denoted by primes. Hence, particularly $k_y'=0$ holds now for each plane wave. Every single plane wave is rotated by an angle α,

$$\alpha = \arctan\left(-\frac{k_{I,y}}{k_{I,z}}\right) = -\arctan\left(\frac{\tan(\gamma)}{\sin(\beta)}\right) \quad (4)$$

around the x axis. Clearly the angles α and γ have the same symmetry for a fixed value of β. The wave vector is then given by using the rotation matrix

$$D_x(\alpha) = \begin{pmatrix} 1 & 0 & 0 \\ 0 & \cos(\alpha) & \sin(\alpha) \\ 0 & -\sin(\alpha) & \cos(\alpha) \end{pmatrix}$$

as

$$D_x(\alpha)\vec{k} = \begin{pmatrix} k_{I,x} \\ 0 \\ k_{I,z}\cos(\alpha) - k_{I,y}\sin(\alpha) \end{pmatrix} = \begin{pmatrix} k_x' \\ 0 \\ k_z' \end{pmatrix}.$$

The electric field vector is given in an analogous manner. In this primed system the fields can be decomposed into TE and TM components and their reflection and transmission coefficients can be calculated separately. Then, for convenience, the complex reflection coefficients can be expressed in terms of a matrix $\hat{r}$ whose diagonal elements are given by the vector $\mathrm{diag}(\hat{r})=\{r_{\mathrm{TM}}, r_{\mathrm{TE}}, -r_{\mathrm{TM}}\}$. Expressions for the reflection coefficients r_{TE} and r_{TM} can be readily found in the literature [16]. After having calculated the reflection coefficients all vectors must be transformed back by rotation with $-\alpha$,

$$\vec{E}_R = D_x(-\alpha)\hat{r}D_x(\alpha)\vec{E}_I = \begin{pmatrix} E_x r_{\mathrm{TM}} \\ E_y r_{\mathrm{TE}}\cos^2(\alpha) - E_y r_{\mathrm{TM}}\sin^2(\alpha) + E_z(r_{\mathrm{TE}} + r_{\mathrm{TM}})\cos(\alpha)\sin(\alpha) \\ -E_z r_{\mathrm{TM}}\cos^2(\alpha) + E_z r_{\mathrm{TE}}\sin^2(\alpha) + E_y(r_{\mathrm{TE}} + r_{\mathrm{TM}})\cos(\alpha)\sin(\alpha) \end{pmatrix}. \quad (5)$$

The wave vectors of incident, reflected, and transmitted fields are in the same plane. The transmitted field is given in an analogous manner. The wave vector and the incident field are not affected by the latter rotation of the coordinate system that yields $k_y'=0$,

$$D_x(-\alpha)D_x(\alpha)\vec{k}_I = \vec{k}_I, \quad (6)$$

$$D_x(-\alpha)D_x(\alpha)\vec{E}_I = \vec{E}_I. \quad (7)$$

Obviously the field components $\{E_x, E_y, E_z\}$ depend on β and γ, the angles that describe each plane wave used in the decomposition.

Inserting all parameters into the reflected field leads in the general case to complicated expressions that do not allow for further insights into the physical response of the system. It is therefore instructive to investigate first particular cases that allow for a simplification of the equation.

By assuming that $e_y=0$, then

$$D_y(\beta)D_z(\gamma)\vec{E} = E_0\begin{pmatrix} e_z\sin(\beta) \\ 0 \\ e_z\cos(\beta) \end{pmatrix} = \begin{pmatrix} E_x \\ 0 \\ E_z \end{pmatrix}. \quad (8)$$

Now at a first glance the field seems to be TM polarized. However, this is wrong, because $\vec{k}$ has a k_y component. Therefore, already in this case all field components mix up. The reflected field is given by

$$\vec{E}_R = \begin{pmatrix} E_x r_{\mathrm{TM}} \\ E_z(r_{\mathrm{TE}} + r_{\mathrm{TM}})\cos(\alpha)\sin(\alpha) \\ -E_z r_{\mathrm{TM}}\cos^2(\alpha) + E_z r_{\mathrm{TE}}\sin^2(\alpha) \end{pmatrix} = E_0 e_z \begin{pmatrix} r_{\mathrm{TM}}\sin(\beta) \\ (r_{\mathrm{TE}} + r_{\mathrm{TM}})\cos(\alpha)\sin(\alpha)\cos(\beta) \\ [-r_{\mathrm{TM}}\cos^2(\alpha) + r_{\mathrm{TE}}\sin^2(\alpha)]\cos(\beta) \end{pmatrix}. \quad (9)$$

Analogously to the GHS, the IFS can be understood in terms of the phase of the reflection coefficients $r_{\mathrm{TE,TM}}$. For the IFS and the chosen geometry (finite beam with a principal propagation direction in the x-z plane) an optional transverse shift occurs normal to this plane. The phase of each Fourier component depends on α for a fixed β. To deduce whether a transverse shift occurs, it is necessary to investigate the symmetry of each component separately. As outlined before, a shift will occur if the phase of the reflection coefficient is asymmetric around $\alpha=0$. To be precise, symmetric angular distributions are not of interest, as it is physically clear that a symmetric complex angular spectrum will give a symmetric field distribution. Hence, the analysis of the symmetry of this phase is sufficient to verify the existence of the IFS.

Obviously, the reflection coefficient for fixed ω depends only on k_x (respectively, $k_y^2+k_z^2$), so the phase is symmetric around $\gamma=0$ and correspondingly $\alpha=0$. Hence the x and z components of $\vec{E}_R$ are symmetric functions in α, whereas the y component is antisymmetric. The phase is a function of the ratio between imaginary and real parts of the field, i.e., since both parts have the same symmetry, the gradient of the phase is antisymmetric. Clearly for antisymmetric gradient the overall IFS is vanishing.

For a second example we assume $e_z=0$, then

$$D_y(\beta)D_z(\gamma)\vec{E}=E_0\begin{pmatrix}\cos(\beta)\sin(\gamma)\\ \cos(\gamma)\\ -\sin(\beta)\sin(\gamma)\end{pmatrix}=\begin{pmatrix}E_x\\ E_y\\ E_z\end{pmatrix}. \quad (10)$$

The reflected field is given by Eq. (5). The x component depends only on r_{TM} and hence does not contribute to an IFS. E_x never contributes to the IFS. It is sufficient to investigate the y and z components of the field. For a fixed β, the functions of α and γ are antisymmetric and therefore the z component of the incident field is antisymmetric. The y component of the reflected field is then a symmetric function of γ, whereas the reflected z component is antisymmetric. An IFS in such a configuration does not occur.

Analogous statements can be made in the special case of $r_{\mathrm{TE}}=r_{\mathrm{TM}}$, i.e., normal incidence or $\varepsilon=\mu$. For linear polarization, i.e., $\delta=n\pi$, the IFS then vanishes for all κ. Independent of the particular configuration an IFS exists in the general TIR case. This is obvious because the y and z components are losing their symmetries and become asymmetric functions in γ and α, respectively. Then the phase of each component depends on γ and their gradients are asymmetric. Particularly, the IFS exists also for linear polarization if $r_{\mathrm{TE}}\neq r_{\mathrm{TM}}$. It is again sufficient to consider the y component of the reflected field only.

For the general case of arbitrary polarization one obtains

$$\begin{aligned}E_{y,R}=&[r_{\mathrm{TE}}\cos^2(\alpha)-r_{\mathrm{TM}}\sin^2(\alpha)]e_y\cos(\gamma)\\&-(r_{\mathrm{TE}}+r_{\mathrm{TM}})\cos(\alpha)\sin(\alpha)e_y\sin(\beta)\sin(\gamma)\\&+(r_{\mathrm{TE}}+r_{\mathrm{TM}})\cos(\alpha)\sin(\alpha)e_z\cos(\beta).\end{aligned} \quad (11)$$

For fixed β, $E_{y,R}$ is an asymmetric function in α. This implies that the phase is asymmetric too and therefore its mean gradient is not vanishing. An IFS will be observable. The asymmetry in $E_{y,R}$ can be shown by analyzing the terms separately for their symmetries. The first and second summands are symmetric and will be denoted by $S=S'+iS''$. The third summand is antisymmetric and is denoted by $A=A'+iA''$. For the phase of the reflected field we obtain

$$\Phi(\alpha,\gamma)=\arctan\left(\frac{S''+A''}{S'+A'}\right)\neq\arctan\left(\frac{S''-A''}{S'-A'}\right)=\Phi(-\alpha,-\gamma). \quad (12)$$

Hence the mean gradient will not vanish. Analogous conclusions can be drawn for the z component of the reflected field. But care is required; asymmetry of a function does not guarantee an IFS. It is important that this asymmetry affects the real and imaginary parts of the component in different ways.

Let us consider another special case: a beam focused along the z axis. For fixed β the expression for the reflected field cannot be simplified. The missing z dependency yields no observable GHS. If neither e_y nor e_z are vanishing an IFS is observable without a GHS.

Upon considering the IFS for this special case, we proceed in analyzing configurations beyond linearly polarized light. Although qualitatively all subsequent results will hold for generally elliptically polarized light, we restrict ourselves to the case where both y and z field amplitudes have the same magnitude and assume $\kappa=\pi/4$, hence $e_y=1$ and $e_z=e^{i\delta}$. For the y component of the reflected field we therefore obtain

$$\begin{aligned}E_{y,R}&=E_0[r_{\mathrm{TE}}\cos^2(\alpha)-r_{\mathrm{TM}}\sin^2(\alpha)]\cos(\gamma)\\&\quad+(r_{\mathrm{TE}}+r_{\mathrm{TM}})\cos(\alpha)\sin(\alpha)[e^{i\delta}\cos(\beta)-\sin(\beta)\sin(\gamma)]\\&=E_0\{[r_{\mathrm{TE}}\cos^2(\alpha)-r_{\mathrm{TM}}\sin^2(\alpha)]\cos(\gamma)\\&\quad-(r_{\mathrm{TE}}+r_{\mathrm{TM}})\cos(\alpha)\sin(\alpha)\sin(\beta)\sin(\gamma)\}\\&\quad+E_0[e^{i\delta}(r_{\mathrm{TE}}+r_{\mathrm{TM}})\cos(\alpha)\sin(\alpha)\cos(\beta)].\end{aligned} \quad (13)$$

Similar expressions are obtained for $E_{z,R}$. This expression can be decomposed into a symmetric and an antisymmetric part. As it can be shown the phase gradient is then asymmetric and its average does not vanish evoking an IFS. As the phase of the reflected field depends on the polarization, it may potentially become asymmetric even for a real valued $r_{\mathrm{TE/TM}}$ coefficient. Particularly this implies that the IFS may exist at partial reflection of the light at the interface if $\delta\neq n\pi$, e.g., total internal reflection is not a strict requirement to observe the IFS.

Before numerically investigating IFS, some remarks are in order. For calculating the IFS every plane wave component of the reflected field must be investigated separately. In the general case of elliptically polarized light the incident components have already asymmetric phases, so their center of gravity does not automatically coincide with the beam's center. The IFS for different components has different strengths. The IFS of the entire beam is then given by the sum of the shifts of all components weighted by their strength. Due to the vectorial nature of light it is not possible to obtain a simple, analytical expression of the IFS and it is necessary to calculate the IFS numerically. This can be performed by superimposing all components of the angular spectrum of the incident field and a subsequent determination of its center of gravity.

Although the described effort in computing the shift seems to be tremendous, we believe that the proposed approach has certain advantages. This holds especially if the approach is compared to the classical models, which could have also been used. We did not analyze the IFS by the simplified Renard model, because this model is not appropriate for exact quantitative analysis, as outlined already in the Introduction. The correct implementation of this model, on the other hand, which does not resort to the usual simplifications, is much more difficult to implement than the proposed rigorous numerical calculation. Concerning the Artmann model it must be stated at first that the consideration of the linear gradient of the reflected phase only is difficult to regard as a model at all. It is rather a simplification of the situation by analyzing the variation of the phase in reflection up to first order in a Taylor expansion. Nevertheless, the phase gradient that causes the IFS cannot be approximated linearly due to its symmetry properties, as outlined before. In consequence, the Artmann model seems to be likewise inappropriate for the present purpose of calculating the IFS. To avoid the inherent problems of these classical methods, we employ in the present work the outlined analytical and rigorous description of the reflected beam and the subsequent numerical evaluation of the shift.

III. NUMERICAL RESULTS

In order to calculate the IFS the angular spectrum of the incident beam was chosen to be Gaussian

$$E_0 \propto \exp\left(-\frac{(\beta-\beta_0)^2}{w_0^2}\right)\exp\left(-\frac{\gamma^2}{w_0^2}\right), \tag{14}$$

where w_0 defines the spectral width of the beam in degrees. The parameters used to describe a particular incident beam will be given in the legends of the figures. Such a spectral representation by angles approximates an inclined Gaussian beam much better than the commonly used representation by wave numbers. Note that no analytical expression exists for the angular spectrum in wave numbers of an inclined Gaussian beam. Nevertheless other forms of beams are also possible since the results are affected only minor which can be shown numerically.

Equation (14) can be motivated by considering a Gaussian beam impinging normally to the structure. The spectrum is then given by

$$\vec{E}_I(k_y,k_z) = \vec{E}_0 \exp\left(-\frac{k_y^2+k_z^2}{w^2}\right).$$

The associated field distribution is then Gaussian at the interface. For small angles β and γ the trigonometric function describing $\vec{k}_I$ can be approximated as

$$\vec{k} = k_0\begin{pmatrix}\cos(\beta)\cos(\gamma)\\ -\sin(\gamma)\\ -\cos(\gamma)\sin(\beta)\end{pmatrix} \approx k_0\begin{pmatrix}1\\ -\gamma\\ -\beta\end{pmatrix}.$$

This yields

$$\vec{E}_I(k_y,k_z) \approx \vec{E}(\beta,\gamma) = \vec{E}_0 \exp\left(-\frac{\beta^2+\gamma^2}{w_0^2}\right).$$

The spectrum for an inclined Gaussian beam is then given by Eq. (14). An analytic expression for $\vec{E}(k_y,k_z)$ is not available but it can be shown numerically that the spectrum of the rotated Gaussian beam is of this form. If one considers highly divergent beams this approximation is of course not valid anymore. Nevertheless, the angular spectrum still represents a strongly localized beam with approximately Gaussian shape. The particular shape is of no importance.

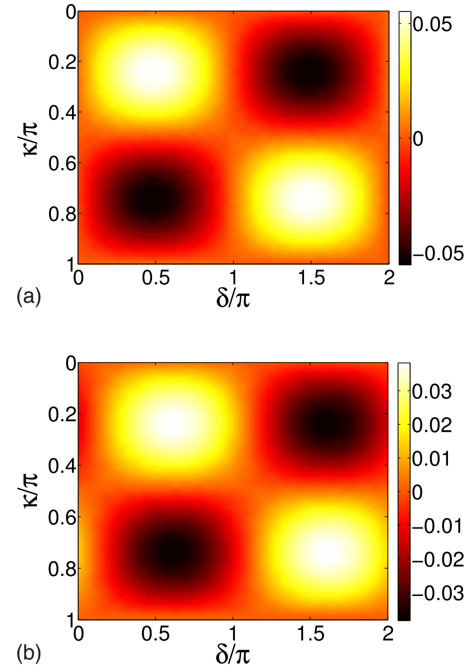

FIG. 2. (Color online) IFS (in units of λ_0) vs the polarization parameters κ and δ for total internal reflection. (a) At the interface between two impedance-matched media (medium 1, $\varepsilon_1=4$, $\mu_1=4$; medium 2, air). The incident beam is characterized by $\beta \in (45°,65°)$, $\beta_0=55°$, $\gamma \in (-10°,10°)$, $w_0^2=10$. (b) At the interface between two impedance-mismatched media (medium 1, $\varepsilon_1=12$, $\mu_1=1$; medium 2, air). The incident beam here is characterized by $\beta \in (45°,85°)$, $\beta_0=65°$, $\gamma \in (-20°,20°)$, $w_0^2=50$.

A. Total internal reflection

For total internal reflection both contributions to the phase provided by the polarization and by the reflection coefficient contribute to the shift. As we have shown above, the shift is therefore not vanishing for linear polarization in general. For an interface between two media with the same impedance, we see from Fig. 2 that the shift vanishes for linear polarization ($\delta=n\pi$) and reaches a maximum for circular polarization ($\delta=\pi/2+n\pi$). It must be mentioned, that the material parameters assumed in the calculations ($\varepsilon_1=4$ and $\mu_1=4$) do not occur in naturally available media but can be achieved by the use of metamaterials [17–19]. With the use of such artificially designed media, the range of possible material parameters for both the permittivity and the permeability is significantly enlarged. On the other hand, if both media do not have the same impedance, a shift is also observed for linear polarization, as also shown in Fig. 2. The dependency on δ is sinusoidal and therefore the maximum is not at circular polarization.

The dependency on κ is also sinusoidal and vanishes at multiples of $n\pi/2$ and reaches a maximum at multiples of $\pi/2+n\pi/4$. In both figures the IFS is plotted over the whole Poincaré sphere surface. In all of the cases of total internal reflection we have investigated, the maximum of the shift is

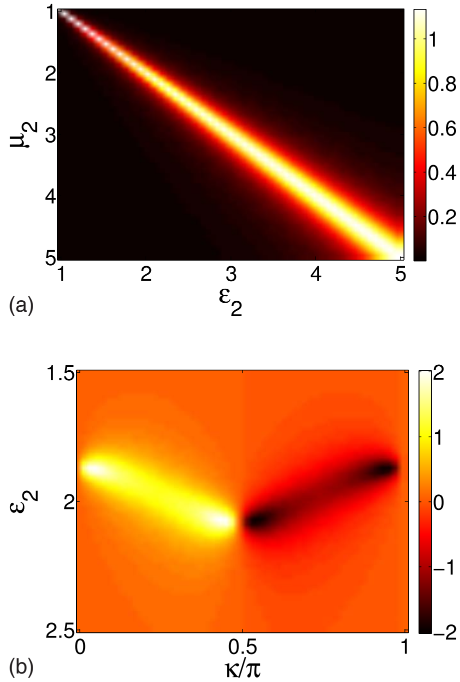

FIG. 3. (Color online) IFS (in units of λ_0) for partial reflection of an incident beam characterized by $\beta \in (5°,25°)$, $\beta_0=15°$, $\gamma \in (-10°,10°)$, $w_0^2=10$. Medium 1 is assumed to be air. (a) IFS vs material parameters of medium 2. The incident beam was circularly polarized ($\delta=\pi/2$, $\kappa=\pi/4$) to maximize IFS. (b) IFS as a function of ε_2 and the polarization state varied by κ with $\delta=\pi/2$; $\mu_2=2$.

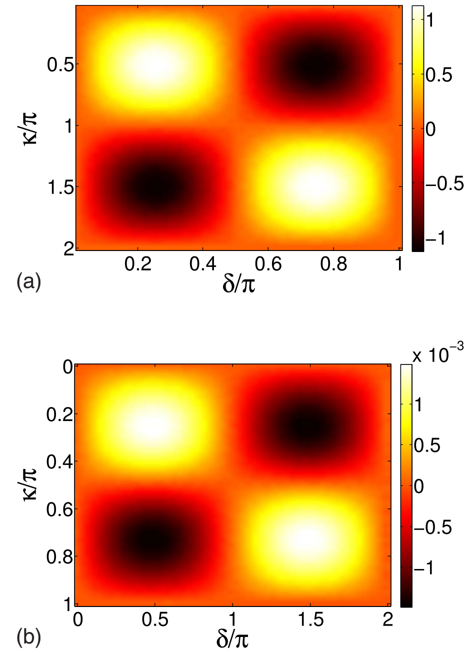

FIG. 4. (Color online) IFS (in units of λ_0) vs the polarization parameters for partial reflection. The beam is characterized by $\beta \in (5°,25°)$, $\beta_0=15°$, $\gamma \in (-10°,10°)$, $w_0^2=10$. Medium 1 is air. (a) Medium 2 is impedance matched, $\varepsilon_2=2$, $\mu_2=2$. The IFS depends sinusoidally on both δ and κ. (b) Medium 2 is impedance mismatched, $\varepsilon_2=4$ and $\mu_2=1$. Again the shift depends sinusoidally on δ but is only antisymmetric in κ because it is a continuous function of κ.

of the order of $0.01-0.1\lambda_0$, where λ_0 corresponds to the beam wavelength in vacuum. Note that the IFS weakly depends on the parameters of the incident beam and is only significantly affected by variation of the impedances. The numerical apertures of all beams used in the calculations were chosen to be quite large for two reasons. The asymmetry of the reflected phase is more important for highly divergent beams and second to increase the ratio between shift and beam diameter.

For completeness and for verifying the applicability of our approach, we compared numerical results obtained with this method with experimental results as presented in the literature [20]. The IFS was calculated for all polarization states on the surface of the Poincaré sphere for the incident angles as given in Ref. [20]. Furthermore, we also assumed a refractive index of $n_1=\sqrt{\varepsilon_1}=1.506$ for the incidence medium. Qualitatively, the results for this configuration were identical to results shown in Fig. 2(b). Particularly, the dependency of the IFS on the polarization states is in good agreement with the experimental data and the shift is not vanishing for linearly polarized light. Overall the experimental data as reported in Ref. [20] could be well predicted by the present approach. Upon numerical evaluation we achieved a maximum value of the IFS of 0.1826 μm for an angle of incidence of $\beta_0=49°$, 0.1056 μm for $\beta_0=64°$ and 0.0479 μm for $\beta_0=78°$. The assumed beam parameters in the simulation are $\beta \in [\beta_0-0.2°,\beta_0+0.2°]$, $\gamma \in [-0.2°,0.2°]$, and $w_0^2=5e-3$. The difference between the numerically calculated and the experimental shifts is approximately a factor of 2. We believe that this difference results only from the unknown beam parameters, which must be assumed in the simulation. Particularly, the important divergence angle was not given in Ref. [20]. Nevertheless, the ratio between the shifts for different angles is in good agreement and we believe that our method yields the exact experimental data if the beam parameters would have been known.

B. Partial reflection

As we have shown above, an IFS occurs only for $\delta \neq 0$ as the reflection coefficients $R_{TE/TM}$ are real valued at a single interface. On the contrary, the shift gets maximal for circular polarization but it will strongly depend on κ for specific materials. For convenience medium 1 is chosen to be air in all cases from now on. It can be seen from Fig. 3 that the shift depends critically on the impedance, i.e., for media with the same impedance the shift is maximal and independent of the modulus of ε and μ.

However, this does not imply that the shift reaches its global maximum for impedance-matched interfaces as can be seen from Fig. 3. For slightly differing impedances the shift depends strongly on κ and can be as large as $2\lambda_0$, i.e., comparable to the beam waist. The IFS is then very large but the reflectance is only of the order of 10^{-4} for $\varepsilon \approx \mu$. For larger impedance mismatch the shift is very small (about $10^{-3}\lambda_0$).

But in all cases the IFS exhibits a sinusoidal dependency on δ and vanishes for linear polarization and multiples of

$\pi/2$ for κ. This is verified by the results given in Fig. 4. In general the results of our numerical simulations agree with the theoretical predictions particularly for the polarization dependency and the experimental measurements provided in [20].

IV. CONCLUSIONS

In this paper we have elaborated an appropriate description to evaluate the Imbert-Fedorov shift of doubly dispersive media by the analysis of the phase behavior of the spectral components (plane waves) forming a finite beam. The shift is shown to be of fully vectorial nature and thus cannot be understood in scalar approximation. In the total internal reflection regime we proved, in contrast to the commonly assumed polarization dependency of the shift, its existence even for linear polarization. For partial reflection, we have shown that the impedance mismatch between the interfaces is of crucial importance for the magnitude of the shift. For impedance-matched interfaces the shift can become as large as the beam width.

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