Backward surface electromagnetic waves in semi-infinite one-dimensional photonic crystals containing left-handed materials

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We study the electromagnetic surface waves localized at an interface separating a homogeneous dielectric medium and a semi-infinite one-dimensional photonic crystal made of alternative left-handed metamaterial and right-handed material. An analytical direct matching procedure within the Kronig-Penney model was applied to analyze the dispersion properties of the localized surface states. We show that the presence of metamaterial in the photonic crystal structure can support the surface waves with a backward energy flow and allows a flexible control of dispersion properties of the surface modes. The surface states can be either forward or backward waves depending on the physical parameters of the photonic crystal, physical parameters of the cap layer, the position of the surface plane, and incident angle of the incoming beam.

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I. INTRODUCTION

The left-handed metamaterial (LHM) with simultaneously negative effective dielectric permittivity and effective magnetic permeability has recently attracted much attention due to its unique physical properties and novel applications of these materials [1,2] and triggered the debates on the application of the left-handed slab as so-called "superlenses" [3,4]. These materials can support an electromagnetic wave where the phase propagation is antiparallel to the direction of energy flow. Their properties were first considered theoretically by Veselago [5] during the 1960s but they have only been fabricated recently [6,7]. They are predicted to exhibit many unusual properties such as refraction at a negative angle, an inverse Doppler shift, and a backwards oriented Chernekov radiation cone [5] negative giant Goos-Hanchen effect [8]. All these phenomena are rooted in the fact that the phase velocity of light in LHM is opposite to the velocity of energy flow, i.e., the Poynting vector and wave vector are antiparallel so that the wave vector, the electric field, and the magnetic field form a left-handed system.

Interfaces between different physical media can support a special type of localized waves as surface waves or surface modes, where the wave vector becomes complex causing the wave to exponentially decay away from the surface. Surface states have been studied in many different fields of physics, including optics [9,10] where such waves are confined to an interface between periodic and homogeneous dielectric media. In optics, the periodic structures have to be manufactured artificially in order to manipulate dispersion properties of light in a similar way as the properties of electrons are controlled in crystals. Such periodic dielectric structures are known as photonic crystals (PC). An analogy between solid-state physics and optics suggests that surface electromagnetic waves should exist at the interfaces of photonic crystals, and indeed they were predicted theoretically [9,10] and observed experimentally [11]. Such surface waves have some advantages. First, surface states supported by PCs can exist in virtually any optical frequency regime due to the scaling nature of dielectric PCs. Second, the low dielectric loss in the structures can lead to sharp resonant coupling between the incoming light and the surface states [12].

Band structure of one-dimensional (1D) photonic crystals containing alternative layers of left- and right-handed materials have been reported by Bria *et al.* [13]. Recently it has been shown that the interface separating a one-dimensional conventional PC and a homogeneous left-handed material can support backward Tamm states [14]. The existence of these modes depends only on the presence of homogeneous left-handed materials. In this paper we have demonstrated that the presence of metamaterials in the PC structure can support backward surface modes localized at the interface with right or left homogeneous medium. In this case, there is more possibility to control the dispersion properties of surface modes. In our model we study an electromagnetic surface wave guided by an interface between right-handed metamaterial (RHM) and a semi-infinite one-dimensional photonic crystal consisting of alternate LHM and RHM layers which we refer to as L-R PC throughout this paper. We assume that the terminating layer of the periodic structure has the width the same or different from the width of other layers of the structure. We study the effect of the width and type of this termination layer on surface states and explore a possibility to control the dispersion properties of surface waves by adjusting termination layer thickness. We also show that the presence of the LHM layers allows for a flexible control of the dispersion properties of surface waves and can support the *unusual* type of surface wave with a *backward* energy flow.

In our calculations, the dielectric permittivity and magnetic permeability are, in general, assumed to take constant values. Although these parameters in LHMs are in general frequency dependent, our results can be used to design spe-

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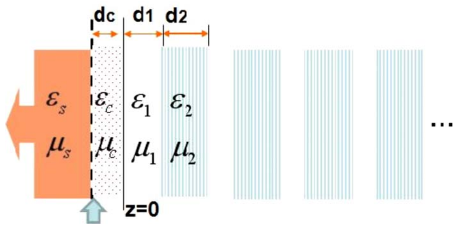

FIG. 1. (Color online) Schematic representation of the problem. The dashed line is the position of the surface plane.

cific metamaterials that would lead to a typical behavior around a given frequency.

The paper is organized as follows. In Sec. II, we applied the direct matching procedure for a solution of localized surface modes in 1D L-R PCs. In Sec. III, the dispersion of surface modes and dependence of these modes on the position surface plane and physical parameters of the cap layer is investigated. Finally, Sec. IV concludes with brief comments.

II. BASIC EQUATIONS

In order to obtain the surface states in 1D PCs containing alternative layers of left and right-handed materials with a cap layer, the direct matching procedure within the Kronig-Penney model has been used [15]. Geometry of our problem is sketched in Fig. 1. As shown in Fig. 1, each unit cell is composed of two layers, which are stacked along the z-axis direction where d_i, ε_i, and μ_i are the thickness, the dielectric permittivity, and the magnetic permeability of the ith layer, respectively. d_c, ε_c, μ_c; ε_s, μ_s are the physical parameters of the cap layer (which can be LHM or RHM) and homogeneous semi-infinite medium, respectively. Due to translational invariance in the XOY plane, the parallel wave vector, $k_\parallel$, is a conservative quantity in all domains of the crystal.

In a one-dimensional periodic structure the propagating waves have the form of *Bloch modes*, for which the electric field amplitudes satisfy the periodicity condition

$$E(z+d,x) = E(z,0)\exp(iK_B z + ik_x x), \tag{1}$$

where $d=d_1+d_2$ is the period of the structure. Here K_B is the Bloch wave number which defines the wave transmission across the layers, and its dependence on the wave vector component perpendicular to the layers k_z can be found explicitly for two-layered periodic structures via the dispersion relation of the PC [13],

$$\cos(K_B d) = \cosh(k_{1z}d_1)\cosh(k_{2z}d_2) + \frac{1}{2}\left(\frac{F_1}{F_2}+\frac{F_2}{F_1}\right)\sinh(k_{1z}d_1)\sinh(k_{2z}d_2) = B(\omega), \tag{2}$$

where the perpendicular wave vector component in each medium is given by $k_{iz}=[k_\parallel^2-\varepsilon_i\mu_i(\omega/c)^2]^{1/2}$, $F_i=k_{iz}/\mu_i$ ($i=1,2$), $k_\parallel^2=k_x^2+k_y^2$, ω is the angular frequency, and c is the light speed in vacuum. Only TE modes have been considered in this paper and the TM modes can be considered in a similar way.

It is well known that when any periodic system is limited, K_B should be complex,

$$K_B = i\eta + \frac{n\pi}{d}, \quad \eta > 0, \quad n = 0, \pm 1, \pm 2, \ldots . \tag{3}$$

It is straightforward to write the electromagnetic field of TE polarization under the form

$$E_y^s = A_s e^{k_s z} e^{i(k_\parallel x - \omega t)}, \quad -\infty < z \leqslant -d_c,$$

$$E_y^c = (A_c \sinh k_c z + B_c \cosh k_c z) e^{i(k_\parallel x - \omega t)}, \quad -d_c \leqslant z \leqslant 0,$$

$$E_y^{PC} = C_1(\sinh k_1 z + \gamma \cosh k_1 z) e^{i(k_\parallel x - \omega t)}, \quad 0 \leqslant z \leqslant d_1, \tag{4}$$

where

$$\gamma = \frac{\sinh(k_{1z}d_1) + \frac{F_1}{F_2} e^{iK_B d}\sinh(k_{2z}d_2)}{e^{iK_B d}\cosh(k_{2z}d_2) - \cosh(k_{1z}d_1)},$$

$k_s=[k_\parallel^2-\varepsilon_s\mu_s(\omega/c)^2]^{1/2}$, and $k_c=[k_\parallel^2-\varepsilon_c\mu_c(\omega/c)^2]^{1/2}$. Here, s and c indexes represent the homogeneous semi-infinite media and cap layer, respectively. It must be noted that in the TE (TM) modes electric (magnetic) field is parallel to the interfaces of layers and is perpendicular to the wave vector which is indicated as y direction in this paper. To obtain the explicit form of E_y^{PC} any three equations from the set of four boundary conditions, i.e., continuity of the tangential components of the electric and magnetic fields, at the interfaces of $z=d_1$ and $z=d_1+d_2$ have been used. It is easy to obtain the surface modes dispersion relation using the boundary conditions at $z=0$ and $z=-d_c$:

$$\frac{F_1}{\gamma}\cosh k_c d_c - F_s \cosh k_c d_c - F_c \sinh k_c d_c + \frac{F_1 F_s}{F_c \gamma}\sinh k_c d_c = 0, \tag{5}$$

where $F_s=k_s/\mu_s$ and $F_c=k_c/\mu_c$. In the absence of the cap layer ($d_c=0$), this equation reduces to $\frac{F_1}{\gamma}-F_s=0$, see Ref. [16].

The surface modes decay exponentially along the normal direction away from the surface into both the photonic crystal and the homogeneous background. By eliminating η between Eqs. (2) and (5) and using Eq. (3), we get the following relation:

$$S(\omega) = B(\omega) \mp [B^2(\omega)-1]^{1/2}, \quad S(\omega) = (-1)^n e^{-\eta d}, \tag{6}$$

where $-$ and $+$ correspond to even and odd n, respectively [17]. By applying this procedure, some spurious solutions are introduced. This can be identified and should be rejected by applying existence condition, $e^{-\eta d}<1$ [18].

III. RESULTS AND DISCUSSION

First, we studied the behavior of the surface states of a photonic crystal without a cap layer ($d_c=0$). We choose the following parameters for the structure: $d_1=d/3$, $\varepsilon_1=5$, $\mu_1=1$; $d_2=2d/3$, $\varepsilon_2=-1.25$, and $\mu_2=-1$. Band structures and

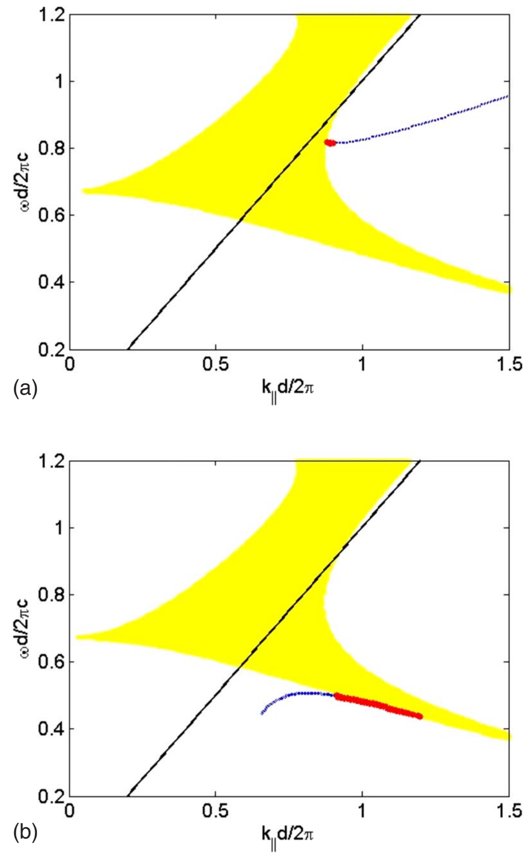

FIG. 2. (Color online) Calculated bulk and surface modes for TE polarization in a semi-infinite 1D L-R PC. The physical parameters used in (a) are $d_1=d/3$, $\varepsilon_1=5$, $\mu_1=1$; $d_2=2d/3$, $\varepsilon_2=-1.25$, and $\mu_2=-1$ and in (b) are $d_1=2d/3$, $\varepsilon_1=-1.25$, $\mu_1=-1$; $d_2=d/3$, $\varepsilon_2=5$, $\mu_2=1$. By these choices the average index of refraction in the PC, $\langle n\rangle=(n_1d_1+n_2d_2)/d$, is zero. In both cases $d_c=0$. The gray and white regions are pass and forbidden bands, respectively. The bold and thin curves in the gaps show the backward and forward modes, respectively. The straight bold line shows the light line of homogeneous medium with $\varepsilon_s=\mu_s=1$.

dispersion properties of this case are displayed in Fig. 2(a). We can see from this figure that the dispersion of surface states is positive, which is similar to the surface modes in conventional PC, but, in contrast to the conventional PC, these states are far from band edges and consequently the surface modes become more localized.

In the next structure we choose the following parameters: $d_1=2d/3$, $\varepsilon_1=-1.25$, $\mu_1=-1$; $d_2=d/3$, $\varepsilon_2=5$, $\mu_2=1$, indeed we interchanged the positions of two layers of PC so that the first layer is a LHM. We can see from Fig. 2(b) and the narrow range of $k_\parallel$ in Fig. 2(a) that there is an *unusual* effect, i.e., negative slope of the dispersion curve. It must be noted that the slope of the dispersion curve for the conventional PC is always positive, while it may be negative for L-R PC due to the presence of LHM. There are modes with negative and positive group velocities which are termed as *backward* and *forward*, respectively. In the forward wave, the direction of the total energy flow coincides with the propagation direction, while in the backward wave the energy flow is backward with respect to the wave vector.

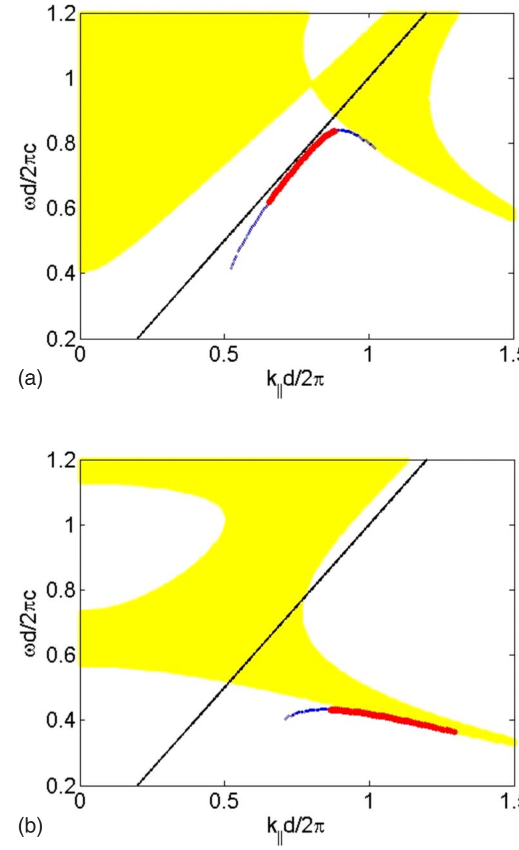

FIG. 3. (Color online) All parameters are the same as Fig. 2(b) except ε_2 is 3 in (a) and 6 in (b).

It is very interesting to identify the type of the surface modes (forward and backward). The surface waves are forward or backward when the total energy flux is positive or negative, respectively [19]. The energy flow is described by the Poynting vector, which defines the energy density flux averaged over the period $T=2\pi/\omega$, and can be written in the form

$$\vec{S}=\frac{c}{8\pi}\,\mathrm{Re}[\vec{E}\times\vec{H}^*], \tag{7}$$

where $\vec{E}$, $\vec{H}$ are the electric field and magnetic field of a surface wave, respectively, and the asterisk stands for the complex conjugation. The energy flux in the RH and LH media is an integral of the Poynting vector over the corresponding spatial regions. Our calculations show that almost all of the surface modes in Fig. 2(a) have a positive energy flux and are forward surface modes exceptionally near the band edges, but in the Fig. 2(b), there are many surface modes that have negative total energy flow and are backward. The backward modes are shown as bold curves.

It is interesting to note that our model allows a flexible control of the dispersion properties of surface modes by varying the physical and the structural parameters of the L-R PC. For example, Fig. 3 shows that by changing the dielectric permittivity of the right-handed layer in PC, we observe a dramatic change in the dispersion curves of the surface modes. Also, it is worth noting that we can observe the de-

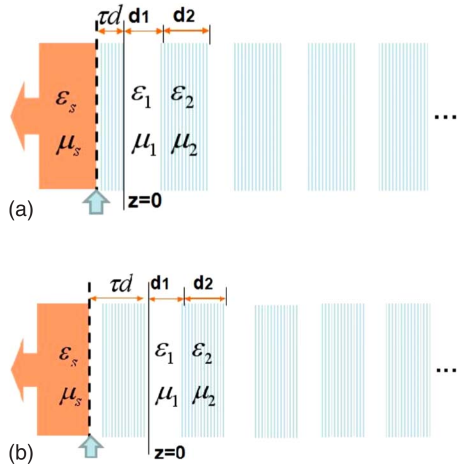

FIG. 4. (Color online) Variation of the position of the surface plane, marked as a dashed line and an arrow. The physical parameters PC layers are $d_1=2d/3$, $\varepsilon_1=-1.25$, $\mu_1=-1$; $d_2=d/3$, $\varepsilon_2=5$, $\mu_2=1$. (a) The terminated layer is right-handed material and $0\leqslant\tau\leqslant d_2/d$. (b) The terminated layer is left-handed material and $d_2/d\leqslant\tau\leqslant 1$.

generacy of surface modes (two surface modes with the same frequency and different $k_{\parallel}$) in Figs. 2 and 3.

Second, in order to show the effect of the position of the surface plane on the surface cell, we introduce the cut parameter, τ, as indicated in Fig. 4. Dependence of typical surface mode eigenfrequency on the cut parameter has been investigated in Fig. 5 for a given parallel component of wave vector, $k_{\parallel}$. One can see from this figure how the termination of PC determined the type (forward or backward) and frequency position of surface modes in the band structure. Fig. 5 also shows that there are regions of truncations where surface modes did not take place. Calculations show that for parameters of this structure, the backward surface modes can be observed only when $0\leqslant\tau\leqslant\frac{1}{3}$ (see Fig. 5).

Thus by choosing an appropriate truncation, we have backward or forward modes with suitable distance from the band edge and so suitable localization. We are also interested in the study of the dependence of surface modes on the physical parameters of the cap layer. To do this, first we consider the PC case studied in which the first layer of PC is left-handed. The type of cap layer is right-handed and the values of electric permittivity and magnetic permeability of the cap layer are the same as layer 2, but the thickness of the cap layer is regarded as a variable parameter. Position variation of surface modes and the types of them are shown in Fig. 6(a). As can be seen, surface modes are backward near the band edge and change to forward as the thickness of the cap layer increases. Also in some thicknesses there are two surface modes; one is backward and the other is forward.

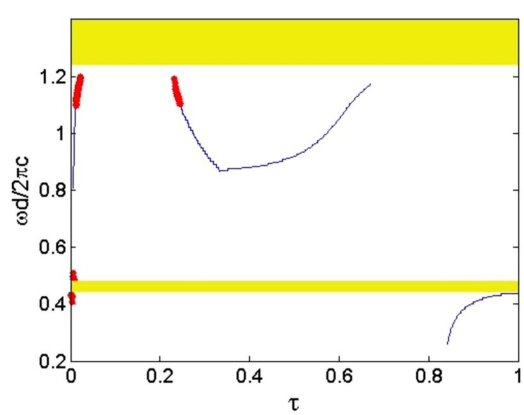

FIG. 5. (Color online) Variation of typical surface mode eigenfrequency versus cut parameter for a typical value of $k_{\parallel}=1.2$. The bold line in the figure is backward modes and the other are forward modes.

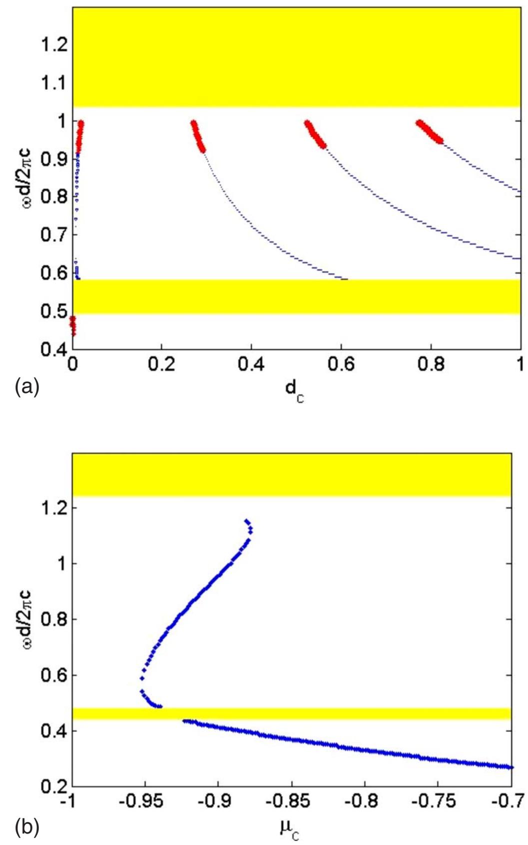

FIG. 6. (Color online) Variation of typical surface mode (a) versus d_c for given values of $k_{\parallel}=1$, $\varepsilon_c=\varepsilon_2$, and $\mu_c=\mu_2$ and (b) versus μ_c for given values of $k_{\parallel}=1.2$, $d_c=0.5d$, and $\varepsilon_c=\varepsilon_2$.

Calculations show that the position of surface states depend strongly on μ_c rather than ε_c. To clarify this dependence, we consider the PC case studied in Fig. 2(a), where the first layer of the PC is right-handed and the cap layer is left-handed. The dependence of eigenfrequencies as a function of μ_c is displayed in Fig. 6(b).

As seen from this figure, the surface modes are sensitive to variations of magnetic permeability, especially in the upper band gap, near a value of $\mu_c=-0.93$.

IV. CONCLUSION

We have studied electromagnetic surface waves guided by an interface between a homogeneous medium and a semi-infinite one-dimensional photonic crystal containing left-handed metamaterial. We have shown that in the presence of the left-handed metamaterial in photonic crystal, the surface modes can be either forward or backward while for conventional structures the surface modes are always forward. Also, the dependence of the surface modes on the cut parameter and physical parameters of the cap layer has been studied. Calculations show that the surface modes occur in the definite cut parameter.

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