# Quantum Zeno effect in cavity quantum electrodynamics: Experimental proposal with nonideal cavities and detectors

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We propose an experiment for the observation of the quantum Zeno effect (QZE) in a bipartite system. The setup involves two microwave cavities and a "tunneling" photon, which is observed by the passage of Rydberg atoms. Our proposal allows for the consideration of two types of measurements, namely, sequential observations of the atomic state and its inclusive measurement. In the present system the two processes are shown to lead to the same result in the ideal case. We consider realistic atom-field interaction times, cavity dissipation, and limited detection efficiency. Analytical expressions for the "tunneling" probability are obtained exhibiting a competition between the environment induced exponential decay and the characteristic $t^2$ (for short times) dependence of the QZE. We show that for sufficiently small dissipation constants the effect can be observed with current experimental facilities.

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## I. INTRODUCTION

The success of physical theories is intimately connected to its potentiality to describe existing empirical data and to predict new, yet to be observed, phenomena [1]. However, the interpretation of empirical data is not completely independent of the proposed theory. Therefore in natural sciences the measurement process plays a double role: it is at the same time a testing tool of theories and also a physical process in itself, subjected to theoretical analysis. In the quantum domain theoretical descriptions of the measurement process are a matter of innumerous discussions.

In 1932, in his famous treatise [2], von Neumann proposed a quantum measurement theory, which became quickly well known. An initial premise of this theory is the postulate that the measurement of a given observable always yields one of the eigenvalues of this observable and, after the measurement, the system collapses to the corresponding eigenvector. This working hypothesis is known as "projection postulate" and is responsible for several counterintuitive aspects of the theory. It has led to the formulation of several paradoxes.

The "quantum Zeno paradox" was presented in a mathematically rigorous fashion in 1977 by Misra and Sudarshan [3]. In this formulation the authors show that a sequence of projective measurements on a system inhibits its time evolution. The paradoxical character of this conclusion becomes explicit when one continuously observes the state of an unstable particle. When the quantum Zeno effect (QZE) was first formulated, it has been associated to two factors: an initially quadratic time decay and the projection postulate.

In the 1990s, after the realization of the pioneer experiment [4] on the effect, which showed the inhibition of transitions between quantum states by means of frequent observations, the QZE became the center of fervorous debates [5,6]. The role attributed to the projection postulate was at the center of the discussions. New approaches have been proposed [5,7] and the strong association between the QZE and the projection postulate was no longer a necessary ingredient. Nowadays the literature on the subject is vast, especially on the theoretical side. In order to give an idea of the broad range of questions raised by the QZE we quote a (very limited) set of examples. Essentially formal structures have been developed to study general properties of the QZE [8]; its relation with quantum jumps is considered in [9]; temperature effects on the visibility of the QZE are discussed in [10] and in Ref. [11] the QZE is shown to be compatible with a measuring process which produces a random phase on the measured state of multilevel atoms. There are also many proposals to use quantum Zeno (and Zeno-type) effect as a strategy for state preservation and control [12–14].

In the present contribution we consider the experimental setup proposed in [15], which consists of two microwave cavities coupled by a waveguide. We devise therewith two conceptually different schemes. In the first one a single photon initially in cavity $A$ may tunnel through the waveguide to cavity $B$. A resonant atomic probe is sent through cavity $B$ and the final atomic state is detected by field ionization detectors. According to this scheme the probability of $N$ successful events (the atom remains in the ground state) increases as $N$ increases (for perfect detectors). In our second proposal the atomic probes are sent through cavity $B$, interact with it but are not measured, i.e., an inclusive experiment. The transition of the excitation from $A$ to $B$ is due to the periodic interactions of the atoms with the mode in cavity $B$. The same final probability is obtained for both schemes in the ideal case: being resonant with the photon, the atom takes it away in case the transition has occurred and leaves the system in a stationary state (the vacuum in this case) making the repopulation process impossible. In the experiment by Itano *et al.* [4] repopulation occurs in a natural way [16] since the laser pulses populate an intermediate metastable state. The type of measurement in Ref. [4] corresponds to an inclusive measurement (like in our second scheme). The result, however, is different from ours in what concerns the finite $N$ dependence.

Our first scheme is in line with Ref. [17] (see also [18]), where on page 3, Sec. V, the importance of sequential measurements in connection to the QZE is discussed. To our knowledge, one of the novel aspects of the present contribution is the fact that the experimental proposal involves a bipartite system, where entanglement plays a very crucial role in the resulting QZE. Moreover, several limiting aspects of a realistic observation of the effect are taken into account in detail. For example, the finite quality factor of the cavities is shown to yield an exponential decay, which competes with the $t^2$ tunneling probability characteristic of the QZE for short times. Recently a nonexponential decay has been indeed observed [19,20] in the tunneling of trapped ions.

Another result from the present investigation is the study of the experimental limitations on $N$, the number of probe atoms. We present two limits for $N$, one for each experimental proposal. In the first one $N$ is limited by the detectors inefficiency (the analysis of dissipative effects is not necessary in this proposal since they are irrelevant when compared to the limitations imposed by the detection process). In the second proposal $N$ is limited by dissipation since the time for the realization of the measurement depends on $N$. All our results are analytical and the physics becomes transparent.

This contribution is divided as follows: In Sec. II, we describe the main elements of the proposed experiment and their interaction. In Section III, the QZE is investigated in the situation where several atoms interact with one cavity mode and next with ionization detectors. In Sec. IV, we show that these measurements of several atomic states are not essential for the QZE; the effects of finite atom-field interaction times and of field dissipation are also studied in this section. In Sec. V we draw the conclusions.

## II. MODEL FOR AN EXPERIMENT

Let us consider two cavity modes coupled by a conducting wire (waveguide), as proposed in [15]. The Hamiltonian for the system is given by

$$H_{AB} = \hbar\omega a^\dagger a + \hbar\omega b^\dagger b + \hbar g(a^\dagger b + b^\dagger a), \tag{1}$$

where $a^\dagger$ ($a$) and $b^\dagger$ ($b$) are creation (annihilation) operators for modes $M_A$ and $M_B$, $\omega$ their frequency and $g$ a coupling constant [15]. The situation we shall consider concerning the electromagnetic degree of freedom will always involve the following initial state:

$$\rho_F(0) = |1_A,0_B\rangle\langle 1_A,0_B| = |1,0\rangle\langle 1,0|,$$

where the ket (bra) $|n,m\rangle$ ($\langle n,m|$) refers to $n$ excitations in mode $M_A$ and $m$ excitations in mode $M_B$. The evolution of this state according to Eq. (1) in a time interval $T$ is given by

$$\rho_F(T) = |c_1(T)|^2|1,0\rangle\langle 1,0| + |c_2(T)|^2|0,1\rangle\langle 0,1| + [c_1(T)c_2^*(T)|1,0\rangle\langle 0,1| + \text{H.c.}], \tag{2}$$

where $c_1(T)=\cos(gT)$, $c_2(T)=\sin(gT)$, and H.c. stands for Hermitian conjugate. Thus, due to the coupling between the cavities, a photon initially in cavity $A$ may be found at time $T$ in cavity $B$ with probability $|c_2(T)|^2$. At $T=\pi/2g$ the photon has performed a complete transition from mode $M_A$ to mode $M_B$: $\rho_C(T)=|0,1\rangle\langle 0,1|$.

In order to experimentally verify the occurrence of this transition, one can measure the number of photons in cavity $B$: if the value found is zero we know for sure that the transition did not occur. This may be realized by sending an effectively two level atom [21] in its lowest state through cavity $B$. The atom prepared in its lowest state works as a probe for the field state. In order to realize this "two level atom" one uses a Rydberg atom whose relevant transition may be tuned to the field *quanta* $\hbar\omega$. We denote by $|e\rangle$ ($|g\rangle$) the higher (lower) energy atomic state. This tuning may be effected by using a quadratic Stark effect, as in Ref. [22]. The control of the atom-field interaction time may be performed by this method with a precision of 1 $\mu$s. Since this time is small compared to the other relevant times in the experiment, we will not consider imperfections in the atom-field interaction time. The interaction of the atom with the field mode in cavity $B$ may be described by the Jaynes-Cummings model, which gives $\tau_\pi=\pi/\Omega_0$, where $\Omega_0$ is the vacuum Rabi frequency, for the $\pi$ pulse time, the time in which one excitation moves from mode $M_B$ to the atom. If the atom-field coupling is much stronger than the coupling between modes $M_A$ and $M_B$, $\tau_\pi$ may be disregarded [32], and we may write the density operator for the system composed of the atom and the field modes, after the atom-field interaction, as

$$\rho_{AF}(T) = |c_1(T)|^2|1,0,g\rangle\langle 1,0,g| + |c_2(T)|^2|0,0,e\rangle\langle 0,0,e| + [c_1(T)c_2^*(T)|1,0,g\rangle\langle 0,0,e| + \text{H.c.}]. \tag{3}$$

Since the atom-field state is maximally entangled, to measure the atomic level in an ionization detector is equivalent to measuring the number of photons in each cavity before the atom-field interaction.

## III. SEQUENTIAL OBSERVATIONS

In this section we will consider the measurement of the atomic state by ionization detectors $D_e$ and $D_g$ constructed in such a way as to ionize the atom in states $|e\rangle$ and $|g\rangle$, respectively.

### A. Perfect detectors

If one has perfect detectors, each atom sent through cavity $B$ will produce a click either in $D_e$ or $D_g$. Thus the probability $p_{1,0}$ that a photon initially in mode $M_A$ did not reach cavity $B$ is equal to the probability $p_{\text{click }D_g}$ of one click in detector $D_g$ [$p_{1,0}=p_{\text{click }D_g}=|c_1(T)|^2$].

If we send $N$ atoms, one at each time $t=iT_0/N$ ($i=1$ to $N$), during the fixed time interval $T_0=\pi/2g$, we can, in principle, monitor the photon transition from mode $M_A$ to mode $M_B$. The temporal evolution of the system under such conditions consists of $N$ steps composed of a free evolution during a time interval $\tau_{A,B}=T_0/N$, followed by an atom-field interaction, when the atom and the mode $M_B$ perform a $\pi$ Rabi pulse (regarded as instantaneous).

If in one of these steps we observe one click in $D_e$, we must conclude that the photon was found in cavity $B$. As may be seen in Eq. (3), after this click the field state becomes $\rho_F=|0,0\rangle\langle 0,0|$, and all the subsequent atoms will be detected in the $|g\rangle$ state.

If no clicks in $D_e$ are observed, the state of the field collapses to the initial state at the end of each step; therefore, the evolution of the system is composed of $N$ identical steps. The probability of $N$ clicks in $D_g$ is

$$p^{(N)}_{\text{click } D_g}=[|c_1(\tau_{AB})|^2]^N, \tag{4}$$

which is equal to the probability $p^{(N)}_{1,0}$ that the photon is still in cavity $A$ at time $T_0$, after the interaction between the field and the $N$ atoms. If we consider the limit $N\to\infty$,

$$\lim_{N\to\infty} p^{(N)}_{1,0}=\lim_{N\to\infty} p^{(N)}_{\text{click } D_g}=1. \tag{5}$$

The Zeno effect becomes explicit: the continuous measuring of the number of photons in cavity $B$ inhibits the transition of the photon from cavity $A$ to cavity $B$.

### B. Inefficient detectors

In order to take the limited efficiency of the detectors into account we need a model for the detection process. In what follows we consider a schematic model for the atom-detector interaction [23]:

$$H_D=\hbar\epsilon_g|g\rangle\langle g|+\hbar\epsilon_e|e\rangle\langle e|+\hbar\int dk\epsilon_k|k\rangle\langle k| + \hbar v_g\int dk(|g\rangle\langle k|+|k\rangle\langle g|)+\hbar v_e\int dk(|e\rangle\langle k|+|k\rangle\langle e|), \tag{6}$$

where $|e\rangle$ and $|g\rangle$ represent the same atomic levels as in previous sections, and the set $\{|k\rangle\}$ concerns the continuum of atomic levels related to the ionization of the atom. We next consider several possibilities.

#### 1. Only detector $D_g$ is present

This case corresponds to the Hamiltonian (6) with $v_e=0$. For the calculation of the probability that $D_g$ (the inefficient detector) clicks for $N$ atoms, we include the description of the atom-detector interaction for all the steps. As a click in $D_g$ collapses the state of the field to the initial state, this evolution is also composed by $N$ identical steps, and the probability of $N$ clicks can be written as

$$P^{(N)}_{\text{click } D_g}=[|c_1(\tau_{AB})|^2p_g]^N, \tag{7}$$

where $p_g$ is the efficiency of the detector $D_g$ as defined in [23]

$$p_g=\int dk\left|\int d\mu\langle\psi_\mu|g\rangle|\langle k|\psi_\mu\rangle e^{-i\epsilon_\mu\tau_g}\right|^2. \tag{8}$$

The sets $\{|\psi^g_\mu\rangle\}$ and $\{\epsilon^g_\mu\}$ correspond to eigenvectors and eigenvalues of $H_D$ with $v_e=0$. In the limit $p_g=1$ one recovers the result of the previous section,

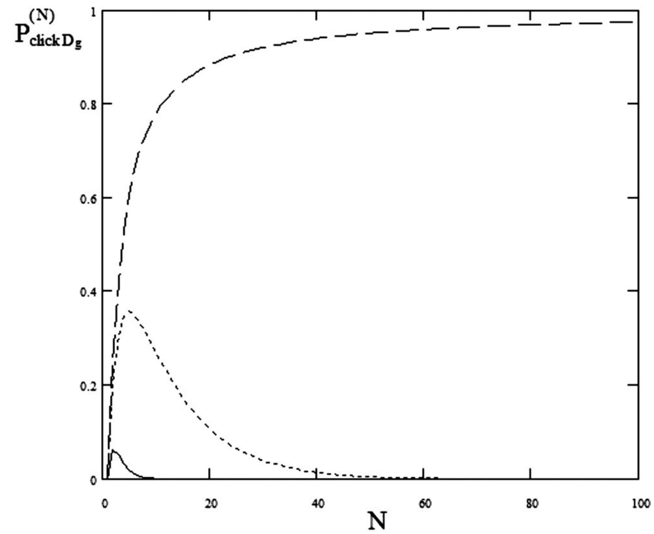


FIG. 1. Probability of consecutive clicks in $D_g$ as a function of $N$, for $T=\frac{\pi}{2g}$ and different values of $p_g$: $p_g=1$ (dashed), $p_g=0,9$ (dotted) and $p_g=0,5$ (continuous).

$$\lim_{N\to\infty} P^{(N)}_{\text{click } D_g}=1.$$

The effect of having an inefficient measurement, i.e., having a detection efficiency $p_g<1$, will change this scenario. This is illustrated in Fig. 1, where we plot the probability of $N$ consecutive clicks in $D_g$ as a function of $N$ for different values of $p_g$. In this case the limit $N\to\infty$ yields

$$\lim_{N\to\infty} P^{(N)}_{\text{click } D_g}=0. \tag{9}$$

This does not mean that the Zeno effect is not present. Given the detector's inefficiency one cannot associate the effect to the statistics of $D_g$ clicks: no click in $D_g$ does not necessarily mean that the photon in fact decayed from cavity $A$ to $B$. The intrinsic detection inefficiency limits the experimental visibility of the Zeno effect in the present experimental scheme.

#### 2. Only detector $D_e$ is present

Another possibility of investigating the limited detection efficiency in the same experimental scheme consists in having only detector $D_e$ present. This corresponds to the Hamiltonian (6) with $v_g=0$. Note that in this case one click in $D_e$ projects the cavity state to $|0{,}0\rangle\langle 0{,}0|$; thus, in order to observe the effect we must study sequences of events that do not give rise to any click in $D_e$. After each step of such a sequence the atomic state is projected into the subspace spanned by the discrete levels $(|e\rangle,|g\rangle)$ [23]. Therefore, the probability of $N$ consecutive no clicks in $D_e$ may be computed as

$$P^{(N)}_{\bar{n}\text{ click } D_e}=[|c_1(\tau_{AB})|^2]^N+|c_2(\tau_{AB})|^2(1-p_e)\times\left(\sum_{k=1}^{N}|c_1(\tau_{AB})|^{k-1}\right), \tag{10}$$

where $p_e$, the efficiency of the detector, is given by

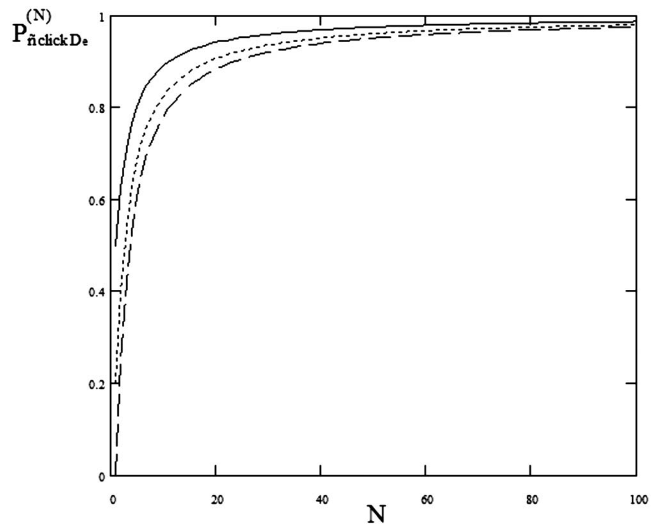


FIG. 2. Probability of consecutive no clicks in $D_e$ as a function of $N$, for $T=\frac{\pi}{2g}$ and different values of $p_e$: $p_e=1$ (dashed), $p_e=0,8$ (dotted), and $p_e=0,5$ (continuous).

$$p_e = \int dk \left| \int d\mu \langle \psi^e_\mu | e \rangle | \langle k | \psi^e_\mu \rangle e^{-i\epsilon^e_\mu \tau_e} \right|^2 .$$

The sets $\{|\psi^e_\mu\rangle\}$ and $\{\epsilon^e_\mu\}$ correspond to eigenvectors and eigenvalues of $H_D$ with $v_g=0$. In the limit $N\to\infty$,

$$\lim_{N\to\infty} P^{(N)}_{\tilde{n}\text{ click }D_e} = 1.$$

In Fig. 2 we show the probability of $N$ consecutive no clicks in $D_e$ for different values of $p_e$. For $p_e=1$ the curve is the same as the one for $p_g=1$, since no clicks in a perfect $D_e$ is equivalent to clicks in a perfect $D_g$. For inefficient detectors, the probability of $N$ consecutive no clicks must be larger than this probability for perfect detectors. This is illustrated in Fig. 2, where the curves representing smaller $p_e$ tend to reach the asymptotic value 1 faster as $N\to\infty$. Note that for inefficient detectors no click in $D_e$ does not necessarily mean that the photon is for sure in cavity $A$: the monitoring of the photon transition is not perfect. However, the asymptotic behavior of $P^{(N)}_{\tilde{n}\text{ click }D_e}$, tending to 1 for any value of $p_e$, is most certainly a consequence of the Zeno effect.

## IV. NO INTERMEDIATE MEASUREMENTS

In the experimental setups discussed in the previous sections the photon transition was monitored by $N$ probe atoms and a macroscopic signal was generated. We were interested in the probability of occurrence of selected sequences, namely, $N$ consecutive clicks in $D_g$ or $N$ consecutive no clicks in $D_e$, which would be associated to the permanence of the photon in cavity $A$. Obviously, a complete correlation cannot be achieved due to the inefficiency of the detectors.

Pascazio and Namiki propose in Ref. [7] a dynamical approach to QZE and show the essential role of the *generalized spectral decomposition*. They propose that QZE occurs even in the absence of intermediate measures, which explains Itano results in [4]. For the system composed by two coupled cavity modes, the generalized spectral decomposition is brought about by the interaction between the two level probe atom and the cavity $B$ mode. As we will see, the classical signals generated by the ionization detectors in each step (intermediate measures) are not necessary for inhibiting the photon transition and, accordingly, with the approach in [7], are not essential for the characterization of the QZE.

The idea now is to send atoms through cavity $B$, also in $T_0/N$ intervals, and not to measure the outcome of the atom-cavity interaction each time. After $N$ such interactions one atom is sent through cavity $A$ and measured by a detector $D_e$.

As in the previous schemes, the first step of the evolution starts with the atom-fields state given by

$$\rho_{AF}(0) = |1,0,g\rangle\langle 1,0,g|, \tag{11}$$

which evolves to

$$\rho_{AF}(\tau_{A,B}) = |c_1(\tau_{AB})|^2 |1,0,g\rangle\langle 1,0,g| + |c_2(\tau_{AB})|^2 |0,1,g\rangle\langle 0,1,g| + [c_1(\tau_{AB}) c_2^*(\tau_{AB}) |1,0,g\rangle\langle 0,1,g| + \text{H.c.}], \tag{12}$$

and then to

$$\rho_{AF}(\tau_{A,B}) = |c_1(\tau_{AB})|^2 |1,0,g\rangle\langle 1,0,g| + |c_2(\tau_{AB})|^2 |0,0,e\rangle\langle 0,0,e| + [c_1(\tau_{AB}) c_2^*(\tau_{AB}) |1,0,g\rangle\langle 0,0,e| + \text{H.c.}]. \tag{13}$$

Since this atom is not measured, the field state must be represented in the end of the step by

$$\rho_F(\tau_{AB}) = \text{Tr}_A\{\rho_{AF}(\tau_{A,B})\} \tag{14}$$

$$= |c_1(\tau_{AB})|^2 |1,0\rangle\langle 1,0| + |c_2(\tau_{AB})|^2 |0,0\rangle\langle 0,0|, \tag{15}$$

where $\text{Tr}_A$ is the trace over the variables of the atom, and accounts for the lack of information about the atomic state.

In order to calculate the final state of the following steps, we must observe that only the part of $\rho_A$ related to $|1,0\rangle\langle 1,0|$ changes with time, in a way that may be described by

$$|1,0\rangle\langle 1,0| \to |c_1(\tau_{AB})|^2 |1,0\rangle\langle 1,0| + |c_2(\tau_{AB})|^2 |0,0\rangle\langle 0,0|. \tag{16}$$

Thus, it is easy to see that the state operator for the fields in the cavities, after the interaction of $M_B$ with $N$ atoms, can be written as

$$\rho_F(T_0) = [|c_1(\tau_{AB})|^2]^N |1,0\rangle\langle 1,0| + |c_2(\tau_{AB})|^2 \sum_{k=1}^{N} [|c_1(\tau_{AB})|^2]^{k-1} |0,0\rangle\langle 0,0|. \tag{17}$$

The probability that the photon transition from cavity $A$ to cavity $B$ has not occurred is

$$p^{(N)}_{1,0} = [|c_1(\tau_{AB})|^2]^N, \tag{18}$$

and, in the limit $N\to\infty$,

$$\lim_{N\to\infty} p^{(N)}_{1,0} = 1. \tag{19}$$

This, according to the dynamical approach in [7], characterizes the Zeno effect. The measurement of this probability can

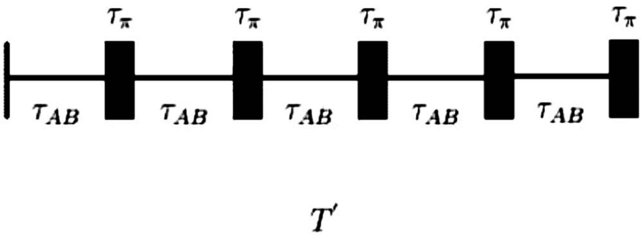


FIG. 3. Sketch of the total time of one experimental sequence.

be done by using one probe atom prepared in the $|g\rangle$ state and sent through cavity $A$ immediately after the interaction of $M_B$ with the $N$th atom. If this probe atom and mode $M_A$ perform a $\pi$ Rabi pulse, the atom-fields state will be given by

$$\rho_{AF}(T_0) = [|c_1(\tau_{AB})|^2]^N|0,0,e\rangle\langle 0,0,e| + |c_2(\tau_{AB})|^2\sum_{k=1}^{N}[|c_1(\tau_{AB})|^2]^{k-1}|0,0,g\rangle\langle 0,0,g|, \tag{20}$$

and measuring the energy level of the atom with an ionization detector tells us about the field state. The inefficiency of the detector enters just as a multiplicative factor in the data.

### A. Finite interaction times and lossy cavities

The problems related to the inefficiency of the ionization detectors, which imposed important limitations for the observation of the Zeno effect in the proposals of Sec. III, have been overcome by the experimental proposal of the present section. However, there are other limitations if a realistic experiment is to be performed. Firstly the cavity is not perfect and dissipation and decoherence will also affect the visibility of the effect. And secondly, the interaction time is finite. We consider all these effects in the present section.

Figure 3 sketches the time evolution, divided in $N$ steps, each one composed of two parts: no atom is present and the cavities are coupled (clear zones), and the atom interacts with mode $M_B$ during a $\pi$ Rabi pulse (dark zones). Each clear zone corresponds to the time interval $\tau_{AB}=T_0/N$, where $T_0$ is, as in previous sections, the time during which a photon passes from cavity $A$ to cavity $B$ if no atom is present: $T_0=\pi/2g$. Since our goal here is to study the inhibition (due to intermediate interactions) of such a photon transition, the cavities will be uncoupled during the atom-field interactions, in order to keep the total interaction time between modes $M_A$ and $M_B$ fixed in $T_0$ [33]. For the rubidium atoms used in the experiment [24], the $\pi$ Rabi pulse time is $\tau_\pi \simeq 10^{-5}$ s, and the increase in the number of probe atoms $N$ may turn the total time of atom-field interactions $N\tau_\pi$ quantitatively important. In order to take this time into account, we must consider

$$T_0' = T_0 + N\tau_\pi$$

as the total time of one experimental sequence.

Let us start by modeling the environment as a large set of harmonic oscillators linearly coupled to the system of interest (modes $M_A$ and $M_B$) [25]. This model has been used to calculate the time evolution of two microwave modes constructed in a single cavity, and the theoretical results showed good agreement with experimental ones [26]. In Ref. [27] it is shown that, for identical cavities and zero temperature, the model leads to the master equation

$$\begin{aligned}\frac{d}{dt}\rho_F(t) &= k[2a\rho_F(t)a^\dagger - \rho_F(t)a^\dagger a - a^\dagger a\rho_F(t)] - i\omega[a^\dagger a,\rho_F(t)]\\ &\quad + k[2b\rho_F(t)b^\dagger - \rho_F(t)b^\dagger b - b^\dagger b\rho_F(t)]\\ &\quad - i\omega[b^\dagger b,\rho_F(t)] - ig[b^\dagger a + a^\dagger b,\rho_F(t)],\end{aligned} \tag{21}$$

where $\omega$ is the frequency of the modes of interest, $g$ is their coupling constant, and $k$ gives the decay rate of the cavities; cross decay rates and shifts in $\omega$ and $g$, which tend to be small [28], were disregarded. Using this master equation, we calculate the time evolution of the state

$$\rho_F(0) = |1_A,0_B\rangle\langle 1_A,0_B| = |1,0\rangle\langle 1,0| \tag{22}$$

as

$$\begin{aligned}\rho_F(t) &= [f_1(t)|1,0\rangle + l_2(t)|0,1\rangle](\mathrm{H.c.})\\ &\quad + [1 - |f_1(t)|^2 - |l_2(t)|^2]|0,0\rangle\langle 0,0|,\end{aligned} \tag{23}$$

where

$$\begin{aligned}f_1(t) &= \exp[-(k+i\omega)t]\cosh[-igt],\\ l_2(t) &= \exp[-(k+i\omega)t]\sinh[-igt].\end{aligned} \tag{24}$$

The probability of finding the photon in cavity $A$, in this case, is given by

$$|f_1(t)|^2 = e^{-2kt}\cos^2(gt). \tag{25}$$

If the field state has evolved from $t=0$ to $t=\tau_{AB}$ in the manner described above, and at time $t=\tau_{AB}$ an atom prepared in the $|g\rangle$ state begins its interaction with mode $M_B$, the state of the whole system will be given by

$$\begin{aligned}\rho_{AF}(\tau_{AB}) &= [f_1(\tau_{AB})|1,0,g\rangle + l_2(\tau_{AB})|0,1,g\rangle](\mathrm{H.c.})\\ &\quad + [1 - |f_1(\tau_{AB})|^2 - |l_2(\tau_{AB})|^2]|0,0,g\rangle\langle 0,0,g|.\end{aligned} \tag{26}$$

During the atom-field interaction, the field modes evolve independently, since they are uncoupled. The evolution of state (26) is described by the master equation

$$\begin{aligned}\frac{d}{dt}\rho_{AF}(t) &= k[2a\rho_{AF}(t)a^\dagger - \rho_{AF}(t)a^\dagger a - a^\dagger a\rho_{AF}(t)]\\ &\quad + i\omega[a^\dagger a,\rho_{AF}(t)] + k[2b\rho_{AF}(t)b^\dagger - \rho_{AF}(t)b^\dagger b\\ &\quad - b^\dagger b\rho_{AF}(t)] - i\frac{\Omega_0}{2}[b^\dagger\sigma_- + b\sigma_+,\rho_{AF}(t)],\end{aligned} \tag{27}$$

where $\Omega_0$ is vacuum Rabi frequency, and $\sigma_-=\sigma_+^\dagger=|g\rangle\langle e|$. The first line of Eq. (27) describes the dissipation of mode $M_A$; the second line describes the interaction of the atom with mode $M_B$ according to the dissipative Jaynes-Cummings model [29]. In previous calculations, $\tau_\pi$ was the time spent by an atom to absorb the excitation of mode $M_B$. Here, $\tau_\pi$ plays an analogous role, and will be defined as

$$\tau_\pi = \frac{1}{\sqrt{\Omega_0^2 - k^2}} \arccos\left(\frac{2k^2 - \Omega_0^2}{\Omega_0^2}\right). \tag{28}$$

This time, which depends not only on the vacuum Rabi frequency, but also on the dissipation constant, is the time for a complete transfer of the excitation of mode $M_B$ to the atom or to the environment. This definition coincides with the previous one if no dissipation is considered ($k=0$). Using master equation (27) to describe the evolution of the system from $t=\tau_{AB}$ to $t=\tau_{AB}+\tau_\pi$, we get

$$\begin{aligned}\rho_{AF}(\tau_{AB}+\tau_\pi) = {} & |f_1(\tau_{AB})|^2 e^{-2k\tau_\pi}|1,0,g\rangle\langle 1,0,g| \\ & + |l_2(\tau_{AB})|^2 e^{-k\tau_\pi}|0,0,e\rangle\langle 0,0,e| \\ & + [1 - |f_1(\tau_{AB})|^2 e^{-2k\tau_\pi} \\ & - |l_2(\tau_{AB})|^2 e^{-k\tau_\pi}]|0,0,g\rangle\langle 0,0,g|. \end{aligned} \tag{29}$$

The state of the fields after the interaction with the first atom is obtained by taking the trace over the atomic variables as follows:

$$\begin{aligned}\rho_F(\tau_{AB}+\tau_\pi) &= \mathrm{Tr}_A\{\rho_{AF}(\tau_{AB}+\tau_\pi)\} \\ &= |f_1(\tau_{AB})|^2 e^{-2k\tau_\pi}|1,0\rangle\langle 1,0| \\ &\quad + [1 - |f_1(\tau_{AB})|^2 e^{-2k\tau_\pi}]|0,0\rangle\langle 0,0|.\end{aligned}$$

Observing that the part of the density operator associated to $|0,0\rangle\langle 0,0|$ does not change with time, it is easy to calculate the probability to find the photon in cavity $A$ after the interaction with $N$ atoms as follows:

$$p_{1,0}^{(N)} = [|f_1(\tau_{AB})|^2 e^{-2k\tau_\pi}]^N = e^{-2k(T_0+N\tau_\pi)}\left[\cos^2\left(\frac{gT_0}{N}\right)\right]^N. \tag{30}$$

This equation makes explicit the effect of $N$ intermediate interactions over two kinds of temporal dependencies. The term $\left[\cos^2\left(\frac{gT_0}{N}\right)\right]^N$ represents no transition of the photon from cavity $A$ to cavity $B$. It grows when $N$ increases, tending to 1 when $N\to\infty$. The term $e^{-2k(T_0+N\tau_\pi)}$, related to the probability that the photon has not decayed to the environment, decreases to zero when $N\to\infty$. Of course this decrease is due to the enhancement of the total time in which the field is exposed to the environment, not being related to any kind of anti-Zeno effect. Since the dynamics of dissipation is exponential, it is not affected by intermediate measurements. The role played by the finite interaction time $\tau_\pi$ is also made explicit and will become quantitatively important as $N\to\infty$.

In order to observe the dependence of $p_{1,0}^{(N)}$ on $N$, an atom prepared in the $|g\rangle$ state is sent into cavity $A$ just after the interaction of the $N$th atom with mode $M_B$. The atom then performs a $\pi$ Rabi pulse, and passes through a $D_e$ detector. If the efficiency of $D_e$ is $p_e$, the probability of a click will be given by

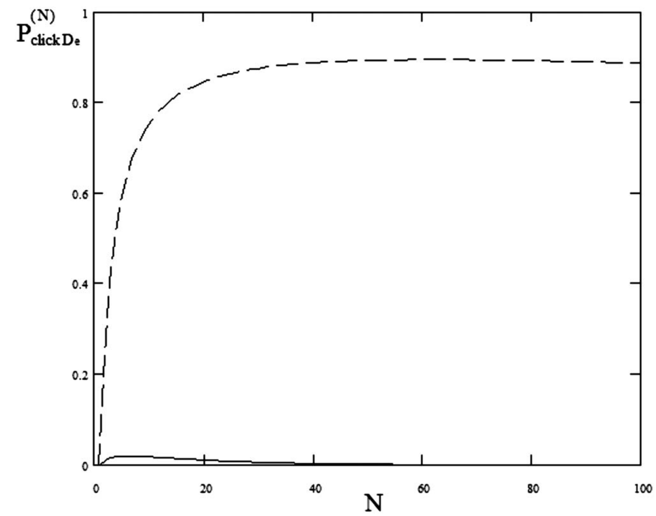


FIG. 4. Probability of one click in $D_e$ as a function of $N$, for $T=\frac{\pi}{2g}$, $\Omega_0=10^5$ s$^{-1}$, $p_e=1$, $g=10^3$ s$^{-1}$, and different values of $k$: $k=10^3$ s$^{-1}$ (continuous) and $k=10$ s$^{-1}$ (dashed).

$$p_{D_e\text{ click}}^{(N)} = p_e e^{-2k(T_0+N\tau_\pi)}\left[\cos^2\left(\frac{gT_0}{N}\right)\right]^N. \tag{31}$$

This is the empirical quantity to be measured in the present proposal.

There will be no problems associated with the efficiency $p_e$, since it enters just as a multiplicative factor that does not depend on $N$. However, the term $e^{-2k(T_0+N\tau_\pi)}$ *depends* on $N$, and may prevent the observation of the Zeno effect if the decay constant $k$ is not small enough. In Fig. 4, we may observe the competition between the tendencies of $p_{D_e\text{ click}}^{(N)}$ when $N$ grows: the increasing one, due to the Zeno effect, and the decreasing one, due to dissipation. In the continuous curve $k=10^3$ s$^{-1}$, corresponding to the cavities used in several experiments [22]. In this case it would be very difficult to observe the Zeno effect, since dissipation dominates even for small values of $N$. For the dashed curve $k=10$ s$^{-1}$; this value corresponds to the cavity described in [30,31], and makes the observation of the Zeno effect possible.

## V. CONCLUSION

We proposed two experimental arrangements to observe the QZE in cavity QED.

Differences between sequential measurements and inclusive measurements have been reported [16–18]. In the present proposal we obtain the same result in both cases (disregarding experimental imperfections, Eqs. (7), (10), and (31) give the same result, even for finite $N$), differently from the results reported in [4].

Also as to the matter of exponential versus $t^2$ characteristic times, we can say that we get an exponential decay inevitably due to environment effects superimposed to the usual expression for the time dependence of the probability for observing the effect as follows:

$$p_{D_e\ \text{click}}^{(N)} = p_e e^{-2k(T_0+N\tau_\pi)}\left[\cos^2\left(\frac{gT_0}{N}\right)\right]^N. \qquad (32)$$

This is the main result of the present contribution. It explicates, within the context of the present model, the role played on the visibility of the QZE by a realistic apparatus and realistic detectors. We hope this result may encourage the experimental realization of the present proposal.

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[32] In Sec. IV we will consider the effects of $\tau_\pi$, taking its value from experimental data.

[33] We must put a piezoelectric element acting in the waveguide, in order to interrupt the cavities coupling during atom-field interaction.