Theory of spatial mode competition in a fiber amplifier

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A theory is developed of monochromatic wave field amplification in a waveguide array based on expansion of the wave field in terms of guided array modes. The equations for the expansion coefficients include cross-modal gain, which completely changes the behavior of the amplified wave field. Analysis of the two-mode amplification reveals unusual features in its characteristics. Instead of unlimited growth of both modes for incoherent fields, one of the modes grows with no limit and suppresses the lower-power mode. Effects associated with the cross-modal gain are illustrated analytically on a system of two thin parallel planar waveguides. Conditions are found where the mode with lower gain can become the dominant one at the output of the amplifier.

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I. INTRODUCTION

Wave field propagation over an array of parallel waveguides has up to now been the subject of numerous theoretical and experimental studies, mostly for optoelectronic devices. In particular, nonlinear refraction of materials can lead to a wide variety of nontrivial physical phenomena in waveguide arrays, thus making them very appealing for research. Lattice solitons [1] and nonlinear optical switches [2] are among such phenomena. An array of nonlinear waveguides in a multicore fiber (MCF) laser [3] is another example very important for laser design. The MCF laser construction is an array of single-mode cores doped by rare-earth-metal ions in a common pump cladding. Such a design allows one to enlarge the combined mode area, thus diminishing the fiber length. Single-mode cores can be arranged also in the form of active defects in photonic crystal fiber lasers [4]. The MCF design has an advantage over single-core large-mode area constructions only if the fields in the cores are phase locked. Phase locking can be provided by an external optics or by diffractive exchange of fields between the cores.

The purpose of this paper is to analyze the mode competition in a multicore fiber amplifier. This problem seems to be important far outside the fiber laser theory because it can be considered as a general dynamic problem for the Schrödinger equation with a complex-valued nonlinear potential. The nearest quantum mechanical analogy for optical wave field propagation is wave function temporal variation in an array of potential wells.

The method commonly used in describing guided field evolution in passive systems is the coupled-mode theory [5] (CMT), in which the wave field is presented as a sum of individual waveguide modes with the unknown coefficients being a function of the propagation distance. These coefficients satisfy equations that are derived by the perturbation method. CMT is intuitively evident and can be easily extended to new physical effects [6]. The CMT for a pair of nonlinear waveguides was analyzed in [7,8]. This theory was generalized by Hardy and Streifer [9], who introduced nonidentical waveguides and vector field effects. Phase self-synchronization at high intensity ($\sim 1\ \mathrm{GW/cm^2}$) and interaction of wave fields with different polarizations and frequencies in systems of two and three evanescently coupled waveguides were topics of earlier work, which was summarized in [2].

Recently [10], the CMT approach was implemented for analysis of dynamic stability for MCF configurations with a separate multichannel coupler. This work was inspired by experiments [11]. Using a model coupling matrix, Peles *et al.* [10] explained the robustness of the phase synchronization in such systems provided special asymmetry is introduced into the construction.

A MCF laser with seven-core hexagonal structure has exhibited experimentally [12] the far-field patterns typical for phase-locked operation at power levels more than 100 W. An analogous 19-core fiber amplifier [13] has achieved 20dB gain at near to diffraction limit beam quality. A calculation based on the CMT predicts [14] that the observed self-synchronization stems from the intensity dependence of the refractive index of a rare-earth-metal-doped fiber. However, direct numerical modeling based on a three-dimensional (3D) beam propagation method [15] (BPM) has clearly shown [16] that the gain nonuniformity in the system is the major factor responsible for the effect, while the role of refractive index nonlinearity is subsidiary.

An important feature of the MCF amplifier is that the field distribution in a transverse plane is strictly determined by the index profile, with distortions induced by gain being negligible. This fact supports an approach based on wave field expansion over passive structure modes, which can be found easily by a standard solver. In particular, such modes for the seven-core fiber design were found in [17] by a finite-element solver.

Comparison between results of 3D BPM calculations [16] and of modal analysis revealed [18] a seeming contradiction: an in-phase mode according to the mode solver possesses the lowest gain, while according to the 3D BPM approach the wave field converges to this mode, if the differences in phases of the launched beams are within a few tenths of a radian. To resolve this contradiction, the theory of light am-

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plification in a system of parallel waveguides with saturable gain should be reconsidered.

Mode competition for gain has been studied since the early times of laser research. Differing spatial profiles of modal intensities result in the gain spatial hole-burning effect, which, in turn, induces instability of single-mode lasing [19,20]. Earlier studies concentrated on the competition of optical modes with different wavelengths. An analysis of oscillation dynamics in a two-mode laser cavity was performed in [21,22]. It is traditionally believed that this dynamics is described satisfactorily by taking into account the effect of the modal gain saturation by the intensity of another mode (so-called cross-saturation; it has been studied mainly for counterpropagating modes of ring lasers [23]).

It is shown below that the major factor governing competition of the modes in the multicore amplifier is the cross-gain [24], which is defined as the product of the mode field profiles with the gain distribution integrated over the fiber cross-sectional area. General equations governing the wave field propagation over the MCF amplifier are formulated in Sec. II. The mode suppression mechanism in two coupled waveguides is discussed in Sec. III. A system of two δ-function type waveguides is considered in Sec. IV, where optical modes are found, and Sec. V, where simultaneous amplification of two modes in this system is analyzed. Amplifier characteristics are compared for cases of two incoherent and two coherent modes. A weak mode suppression mechanism by a strong one is identified for two-δ-function-type waveguides. The conclusion is made that this mechanism associated with cross-gain, which has remained unnoticed until recently, is expected to exist in any system of active waveguides.

II. BASIC EQUATIONS

The built-in index profile of a multicore fiber amplifier is a stepwise function with higher-index regions, called cores, each guiding a single mode. To approach the goal of having a large mode area, small values ($\sim 10^{-3}$) of the core-cladding index difference Δn are used in experiments. In addition, we do not consider birefringence effects. It is then possible [25] to use the scalar approximation instead of solving Maxwell's equations for the electromagnetic field in the fiber. The wave field can be characterized in this approximation by a scalar function $\psi(x,y)\exp(i\beta z - i\omega t)$, where z is the propagation distance, $\omega = kc$ is the oscillation frequency, k is the vacuum wave number, c is the speed of light, and β is the propagation constant. The function ψ could be any transverse component of the electrical or magnetic field, obeying the 2D Helmholtz equation, which formally coincides with the stationary Schrödinger equation in quantum mechanics:

$$\mathcal{H}\psi = -\beta^2\psi, \quad \mathcal{H}\psi = -\left(\frac{\partial^2\psi}{\partial x^2} + \frac{\partial^2\psi}{\partial y^2}\right) + V\psi, \tag{1}$$

where $\mathcal{H}$ is the Hamiltonian, β^2 plays the role of energy, x and y are the transverse spatial variables, the potential function $V = -k^2 n^2(x,y)$, and n is the refractive index. Equation (1) is solved with corresponding conditions at the core-cladding boundary and at infinity. This problem has a continuous spectrum of the radiating modes ψ_Q and a discrete spectrum of the guided modes ψ_j, so any field injected into the MCF can be presented as the sum over the spectrum of ψ_j plus the integral over the spectrum of ψ_Q with some coefficients [25]. The field amplitudes of guided modes $\psi_j(x,y)$ are real-valued functions for no-loss fiber. Additionally, the guided and radiating modes are mutually orthogonal to each other.

The wave field $\Psi(x,y,z)$ in an active fiber with gain $g(x,y,\Psi)$ satisfies the 3D Helmholtz equation

$$\frac{\partial^2\Psi}{\partial z^2} + \frac{\partial^2\Psi}{\partial x^2} + \frac{\partial^2\Psi}{\partial y^2} + (k^2 n^2 - ikn_0 g)\Psi = 0, \tag{2}$$

where n_0 is the cladding index. Thus, in contrast to quantum mechanics, the potential in laser optics is a complex-valued function $V + ikn_0 g(x,y,\Psi)$.

Since we are interested in the behavior of guided modes in the amplifier, it is convenient to take the modes of Eq. (1) as a basis for expansion of the wave field:

$$\Psi(x,y,z) = \sum_j c_j(z)\psi_j e^{i\beta_j z} + \int c_Q(z)\psi_Q e^{i\beta_j z} dQ. \tag{3}$$

The nonuniform gain leads to an interaction between guided and radiating (leaky) modes, thus producing additional losses for the guided modes due to the radiating modes carrying away the energy. However, transformation of guided modes into leaky ones is a negligible effect for typical parameters of fiber amplifiers, as was confirmed by 3D BPM calculations [26]. From the other side, leaky modes experience a gain much lower than do the guided modes because the cladding area is much larger than the area of active cores. For these reasons we can substitute expansion (3) into (2) and neglect the radiating modes. The gain in the cores is usually of the order of 10^{-1} cm^{-1}, while a typical spectral separation between the propagation constants of guided modes is ~ 10 cm^{-1}. Thus, variation of the guided mode amplitudes c_j due to amplification is a slow process, and it is possible to use the approximation in which the terms $d^2 c_j / dz^2$ are neglected. Then the following system of equations for the expansion coefficients c_j can be derived by multiplying the resulting equation by $\psi_j \exp(-i\beta_j z)$ and integrating over the fiber aperture:

$$\frac{dc_j}{dz} = \frac{1}{2} c_j g_{jj} + \frac{1}{2}\sum_{l \neq j} g_{jl} c_l e^{i(\beta_l - \beta_j)z}, \tag{4}$$

where the summation is made over N guided modes. The optical modes introduced are normalized for convenience as $\iint \psi_j^2 dx\, dy = 1$. The difference of propagation constants is a small parameter in comparison with kn_0, so the factors $\beta_j/(kn_0)$ in (4) have been replaced by 1. The applicability of this approximation was verified by 3D modeling of the MCF amplifier. The matrix elements g_{jl} describe an interaction between the wave field and the gain:

$$g_{jl} = \int\int g(x,y)\psi_j\psi_l dx\, dy, \tag{5}$$

where integration is made over the fiber aperture. The coefficients g_{jj} are the modal gains of the jth mode, while the coefficients g_{jl} in the sum in (4) describe the cross-modal gain effect.

The system (4) of complex equations can serve as the basis for analyzing the mode competition in a wide variety of MCF amplifiers. To give an idea of the features possessed by the theory based on the equation system (4), let us consider the simplest situation when a waveguide array supports only two guided modes. In this case, system (4) can be reduced to three ordinary equations for real-valued functions,

$$\frac{dP_1}{dz} = g_{11}P_1 + \sqrt{P_1P_2}g_{12}\cos\phi, \tag{6a}$$

$$\frac{dP_2}{dz} = g_{22}P_2 + \sqrt{P_1P_2}g_{12}\cos\phi, \tag{6b}$$

$$\frac{d\phi}{dz} = \delta\beta - \frac{P_1+P_2}{\sqrt{P_1P_2}}g_{12}\sin\phi. \tag{6c}$$

Here the modal powers P_j, $j=1,2$, are introduced by the expression $c_j=\sqrt{P_j}\exp(i\phi_j-i\beta_j z)$, and $\phi=\phi_2-\phi_1$ is the phase difference between the modal fields, $\delta\beta=\beta_2-\beta_1$. P_j is the fraction of the wave field power carried by the jth mode. Equation (6c) describes the behavior of the modal phase difference. Since gain coefficients are much smaller than $\delta\beta$, this phase difference grows along the propagation axis almost linearly, $d\phi/dz\approx\delta\beta$. In this approximation, the problem is reduced to solving Eqs. (6a) and (6b) only.

Spatial beating between two modes leads to oscillatory modulation of the total field intensity along the axis with a characteristic spatial frequency of a few tens of cm^{-1}. This modulation induces through the gain saturation similar oscillations in the material gain. The resulting nonuniform gain profiles in different cores correlate with the modal powers. In this case we have a tangled interaction between the wave field and transverse and longitudinal gain nonuniformities. Our purpose is to make this interaction clearer by using the simplest waveguide configuration.

It should be noted that the dynamical equations for the two-mode laser [21] have the same sort of mathematical structure as Eqs. (6a)–(6c). However, the physical content of the problem studied is essentially different: Basov *et al.* [21] analyzed the stability of two-mode regimes in a two-wavelength laser, while we study nonlinear effects in an amplifier with continuous wave (cw) input signal.

III. TWO-MODE COMPETITION IN THE MCF

To complete the system of Eqs. (6a)–(6c) it is necessary to specify how the material gain coefficient depends on the total intensity. Generally, the kinetics of population inversion varies from one type of laser to another. The common feature for all cw systems is so-called gain saturation, i.e., reduction of inversion and gain induced by stimulated emission. This effect can be qualitatively well described by the simplest formula $g=g_0/(1+I/I_{\mathrm{sat}})$, where $g_0(x,y)$ is the small signal gain, I is the wave field intensity, and I_{sat} is the saturation intensity. We will normalize the intensity to the saturation intensity value.

Even with such a simple model for gain saturation and taking the mode phase difference as $\phi\approx\delta\beta z$, the system (6a)–(6c) is still rather difficult to analyze. The point is that the g_{jl} terms defined by (5) are complicated functions of $P_{1,2}$, which cannot be found explicitly for realistic waveguide structures. Nevertheless, some general properties of solutions to (6a)–(6c) can be identified.

An important distinction of these equations is the identity of the second terms in the right-hand sides of Eqs. (6a) and (6b). For this reason we can deduce from Eqs. (6a) and (6b) for the evolution of the mode power ratio

$$\frac{d}{dz}\left(\frac{P_2}{P_1}\right) = \frac{P_2}{P_1}\left(g_{22}-g_{11}+g_{12}\cos\phi\frac{P_1-P_2}{\sqrt{P_1P_2}}\right). \tag{7}$$

If $g_{12}\cos\phi$ is negative, then the last term in parentheses on the right-hand side of (7) supports the trend of the power ratio to grow when this ratio is greater than 1.

The dimensionless total wave field intensity can be expressed in the form $I=P_1\psi_1^2+P_2\psi_2^2+2\sqrt{P_1P_2}\psi_1\psi_2\cos\phi$. The cross-gain variation as a function of z can be understood from the expression

$$g_{12}\cos\phi = \int\int \frac{g_0\psi_1\psi_2\cos\phi\, dx\, dy}{1+P_1\psi_1^2+P_2\psi_2^2+2\sqrt{P_1P_2}\cos\phi},$$

which is obtained from (5). The mode amplitude product necessarily changes sign within the cores to provide orthogonality of modes. It is seen that the integrand generally takes on a negative value with larger absolute magnitude when $\psi_1\psi_2\cos\phi<0$. Therefore the quantity $g_{12}\cos\phi$ is preferably negative. It can be rigorously proved that $g_{12}\cos\phi$ is nonpositive for a system of two parallel waveguides possessing mirror symmetry $\Delta n(x,y)=\Delta n(-x,y)$ and supporting two guided modes. One of the modes is symmetric ($j=1$) and another mode is antisymmetric ($j=2$). Taking into account the symmetry properties, the cross-gain coefficient can be expressed as

$$g_{12}\cos\phi = -4\sqrt{P_1P_2}\cos^2\phi\int\int_{S_1}\frac{g_0\psi_1^2\psi_2^2}{C_\psi}dx\, dy, \tag{8}$$

where the integration is made over one of the waveguides, and the saturation factor C_ψ is

$$C_\psi = (1+P_1\psi_1^2+P_2\psi_2^2)^2 - 4P_1P_2\psi_1^2\psi_2\cos^2\phi.$$

It is evident from (8) that the term $g_{12}\cos\phi$ in this case is always nonpositive. It turns to zero at $P_1=0$ or $P_2=0$.

Since the cross-gain term $g_{12}\cos\phi$ diminishes the amount of energy extracted by stimulated emission equally for both modes, the increase of the mode power is favored for the mode with higher power. This is a rather important conclusion, radically differing from the intuitive suggestion that the mode possessing higher saturated gain is dominant at the output of a sufficiently long amplifier. It is shown below that the mode can possess higher modal gain throughout the entire amplifier but the part of the power carried by this mode in the total power diminishes in the course of amplification.

The idea of using the saturated modal gains [27] stems from consideration of the incoherent fields of competing modes. Such a situation can be thought of as the competition of two signals launched in the fiber amplifier from independent sources at the same frequency. The wave field intensity is expressed in the incoherent case as $I = P_1\psi_1^2 + P_2\psi_2^2$, and the equations for the two-mode evolution are

$$\frac{dP_1}{dz} = g_{11}P_1, \quad \frac{dP_2}{dz} = g_{22}P_2, \tag{9}$$

where g_{11} and g_{22} are the modal gains of modes 1 and 2, respectively. These equations include so-called cross-saturation of gain associated with the fact that both modes saturate gain. In the limit of weak saturation, Eqs. (9) are reduced to a form closely resembling the known dynamic equations for a two-mode ring laser ([28]):

$$\dot{X} = 2X(\alpha_1 - \beta_1 X - \theta_{12}Y),$$

$$\dot{Y} = 2Y(\alpha_2 - \beta_2 Y - \theta_{21}X), \tag{10}$$

where X and Y are the dimensionless intensities of modes 1 and 2, α_j and β_j, $j=1,2$, are the above-threshold small signal gains and self-saturation coefficients, respectively, and θ_{12} and θ_{21} are the cross-saturation coefficients. If there is a difference in the modal gains, then one mode may suppress the growth of the other. If the modal gains are equal, both modes lase the same power for the case of an inhomogeneously broadened ($\theta_{12}\theta_{21} \le \beta_1\beta_2$) ring laser [28]. For a homogeneously broadened ($\theta_{12}\theta_{21} > \beta_1\beta_2$) ring laser [29] Eqs. (10) predict that, of two modes starting at $t=0$, the mode with higher power completely suppresses the second mode. In practice, the operation regime of a laser in the last case is random due to spontaneous emission effects [23,30].

Actually, the analogy between the weak-saturation limit of (9) and the laser equations (10) is valid only for low-power wave fields, i.e., for lasers starting from small signals.

IV. SYSTEM OF TWO ULTRATHIN PLANAR WAVEGUIDES

The delta function [$\delta(x)$] is a favorite potential well in quantum mechanics. In application to optics, the δ-function well is the mathematical limit of a high-contrast thin planar waveguide with arbitrarily small waveguide width d and a high refractive index difference Δn, characterized by a single parameter $d\Delta n$. In this limit, the waveguide supports a single mode, the wave field of which is almost constant within the core and extends far outside the core. It should be mentioned that the core-cladding difference is restricted by the condition $\Delta n \ll 1$ for applicability of the scalar model (1). Usage of the δ function as a model waveguide allows one to study waveguide arrays analytically [31].

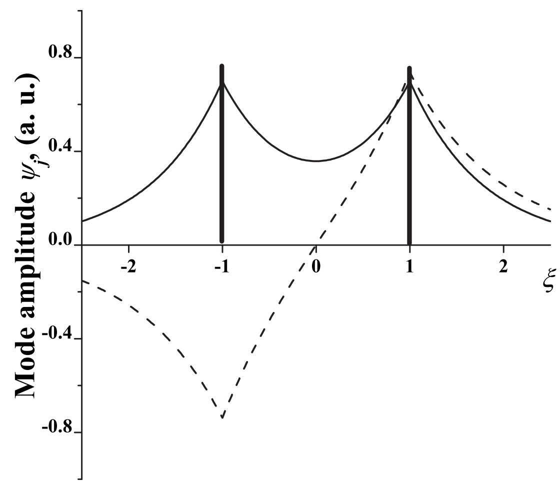

FIG. 1. Schematic of the system of two parallel ultrathin waveguides. The profiles of the symmetric (solid line) and antisymmetric (dashed line) modes correspond to $\kappa' = 1.2$.

Figure 1 shows the system of two ultrathin waveguides situated at locations $x = \pm a$. For this particular system, it is convenient to introduce a dimensionless transverse coordinate $\xi = x/a$. Equation (1) for guided modes of the waveguide array reads

$$\frac{d^2\psi}{d\xi^2} + \{-\alpha^2 + 2\kappa[\delta(\xi-1) + \delta(\xi+1)]\}\psi = 0, \tag{11}$$

where $\kappa = k^2 a d n_0 \Delta n$ is the δ-function amplitude, α is the wave field attenuation rate outside the waveguides, and α^2 is an eigenvalue characteristic for a given mode. As is well known [32], such a system supports the two guided modes shown in Fig. 1, provided the condition $2\kappa \ge 1$ is satisfied. The fundamental mode is symmetric $\psi_S(\xi)$ (in-phase mode), and the second mode is antisymmetric, $\psi_A(\xi)$ (out-of-phase mode). The amplitudes of these modes attenuate exponentially at $\xi \to \pm\infty$ with corresponding rates α_S and α_A, while in the space between the waveguides $\psi_S \sim \cosh(\alpha_S \xi)$, and $\psi_A \sim \sinh(\alpha_A \xi)$. The attenuation rates satisfy the transcendental equations

$$\alpha_S = \kappa(1 + e^{-2\alpha_S}), \quad \alpha_A = \kappa(1 - e^{-2\alpha_A}). \tag{12}$$

The modal shift of the propagation constant $\delta\beta_j = \beta_j - kn_0$ is expressed as $\delta\beta_j = \alpha_j^2/(4L_R)$, where $L_R = kn_0 a^2/2$ is the Rayleigh length. Gain in the waveguides can be included in the model by adding an imaginary part into κ, $\kappa = \kappa' + i\kappa''$, where $\kappa'' = g\kappa'/(2k\Delta n)$. The modal gains in the small-signal limit read

$$G_S = \frac{2\alpha_S'\kappa''}{L_R}\left(1 + \frac{\sinh 2\alpha_S' + (\sin 2\alpha_S'')\alpha_S'/\alpha_S''}{\sinh 2\alpha_S' + \cos 2\alpha_S''}\right)^{-1},$$

$$G_A = \frac{2\alpha_A'\kappa''}{L_R}\left(1 + \frac{\sinh 2\alpha_A' - (\sin 2\alpha_A'')\alpha_A'/\alpha_A''}{\sinh 2\alpha_A' - \cos 2\alpha_A''}\right)^{-1},$$

where α_j'' and α_j'' are the real and imaginary parts of the corresponding parameters satisfying (12). For the symmetric

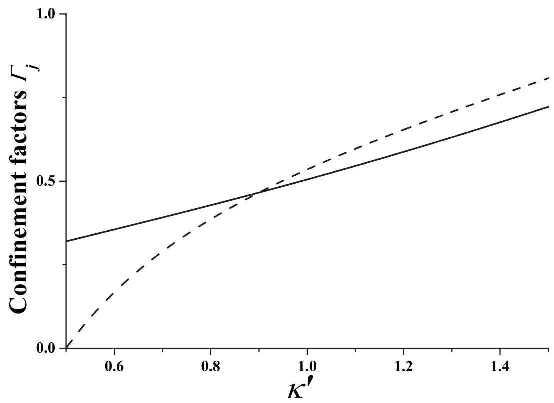

FIG. 2. Confinement factors for the symmetric (solid line) and antisymmetric (dashed) modes vs the coupling strength parameter κ'. The quantities are dimensionless.

system under consideration, it is convenient to introduce a confinement factor of the mode, Γ_j, as being proportional to the overlap of the mode intensity and gain in *one* waveguide. For ultrathin waveguides the confinement factor is equal to the squared amplitude of the mode in the waveguide. The modal gain is expressed as $G_j=2\Gamma_j gd/a$. As long as $\kappa' \gg \kappa''$ ($2k\Delta n \gg g$), the expressions for modal gains and confinement factors can be simplified. The confinement factors of the symmetric and antisymmetric modes can be expressed as

$$\Gamma_S = \frac{\kappa'}{2}\frac{[1+\exp(-2\alpha_S')]^2}{1+2\kappa'\exp(-2\alpha_S')},$$

$$\Gamma_A = \frac{\kappa'}{2}\frac{[1-\exp(-2\alpha_A'))^2}{1-2\kappa'\exp(-2\alpha_A')}.$$

α_j ($j=S,A$) satisfy Eqs. (12) with κ replaced by κ'. In the limit of weak coupling between waveguides ($\kappa' \gg 1$), the antisymmetric mode has a higher confinement factor than the symmetric one:

$$\Gamma_S \approx \frac{\kappa'}{2}[1+2\kappa'\exp(-2\alpha_S')]^{-1},$$

$$\Gamma_A \approx \frac{\kappa'}{2}[1-2\kappa'\exp(-2\alpha_A')]^{-1}.$$

As coupling between waveguides increases, the antisymmetric mode tends to transform into a leaky mode ($\alpha_A \to 0$), and its modal gain diminishes proportionally to α_A, while the symmetric mode gain remains of finite value. Thus, there is a critical distance between the waveguides at which both gains are equalized. The confinement factors are shown in Fig. 2 as functions of κ'. The curves intersect at $\kappa'_{\mathrm{cr}}=0.900\,126\ldots$. This value corresponds to $a \approx 3.9\,\mu$m for a system of two waveguides with $\Delta n = 2\times 10^{-3}$, $d=2\,\mu$m, $n_0=1.456$, and the radiation wavelength $2\pi/k=1\,\mu$m. The Rayleigh length L_R in this case is $70\,\mu$m.

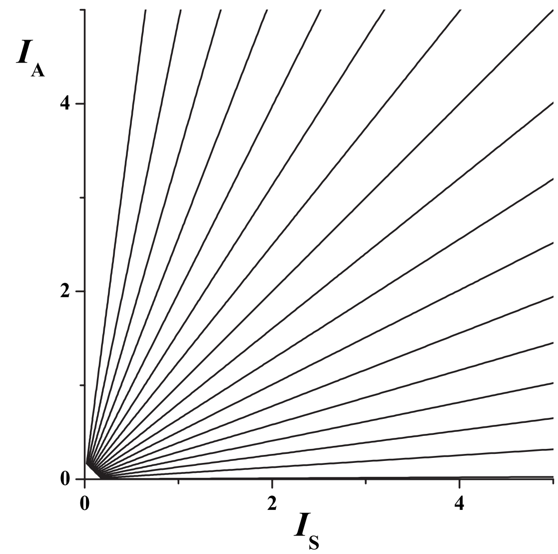

FIG. 3. Mode intensities in the waveguide normalized to I_{sat} for different values of initial power ratio for the incoherent wave fields of the modes $\Gamma_S=\Gamma_A$.

V. WAVE FIELD AMPLIFICATION IN A SYSTEM OF TWO ULTRATHIN WAVEGUIDES

It is instructive to analyze wave field amplification in two δ-function-type coupled waveguides, taking as a reference case the standard equations applicable for description of two incoherent modes with gain cross-saturation taken into account. For definiteness, the simplest gain saturation model is adopted, $g=g_0/(1+I/I_{\mathrm{sat}})$, and the field intensity in the following is measured in I_{sat} units.

A. Incoherent wave fields

For the incoherent fields of two modes in the waveguide system shown in Fig. 1, the equations for the modal powers (9) read

$$\frac{dP_S}{d\zeta} = \frac{2\kappa_0'' P_S \Gamma_S}{1+P_S\Gamma_S+P_A\Gamma_A},$$

$$\frac{dP_A}{d\zeta} = \frac{2\kappa_0'' P_A \Gamma_A}{1+P_S\Gamma_S+P_A\Gamma_A},$$

where $\zeta=z/L_R$ and $\kappa_0''=g_0(L_R d/a)$. This system of equations can be easily integrated:

$$\ln P_S/P_{S0} + \Gamma_S(P_S-P_{S0}) + \Gamma_S(P_A-P_{A0}) = 2\Gamma_S\kappa_0''\zeta,$$

where $P_A/P_{A0}=(P_S/P_{S0})^\gamma$, $\gamma=\Gamma_A/\Gamma_S$, and P_{S0} and P_{A0} are the mode powers at the amplifier entrance. The asymptotic ($\zeta\to\infty$) behavior of the modes depends on the value of γ, which is equal to the ratio of modal gains. That is, at $\gamma=1$ both modes grow with equal rates. This case is illustrated in Fig. 3, which shows the diagram in dimensionless variables $I_S=P_S\Gamma_S$ and $I_A=P_A\Gamma_A$. It is clearly seen that the proportion P_A/P_S remains constant at any distance. If the modes have different overlaps with the gain, the mode with higher confinement factor wins, and its power increases linearly with length, while the power of the second mode grows as a fractional power of the length. This means that the powers of both modes increase with no limit. This result does not depend on the initial proportion of the powers of the modes, if the amplification is large enough.

B. Coherent wave fields

If the wave fields of two competing modes are coherent, the gain coefficients entering into the system of Eqs. (6a)–(6c) can be found explicitly from Eqs. (5) and (8) as

$$g_{jj} = 2\kappa_0''\Gamma_j(1 + P_S\Gamma_S + P_A\Gamma_A)/(CL_R),$$

$$g_{AS} = -4\kappa_0''\Gamma_A\Gamma_S\sqrt{P_AP_S}\cos^2\phi/(CL_R); \qquad (13)$$

here $j=A,S$ and the denominator contains

$$C = (1 + P_S\Gamma_S + P_A\Gamma_A)^2 - 4\Gamma_A\Gamma_S P_A P_S \cos^2\phi.$$

In the specific system under consideration, the ratio of the modal gains of the two modes is independent of the field in the amplifier:

$$g_{SS}/g_{AA} = \Gamma_S/\Gamma_A.$$

The reason is that both the modes are equally distributed over the two waveguides and the mode profiles inside the waveguides are considered to be identical due to the small waveguide thickness. Equations (6a)–(6c) read

$$\frac{dP_S}{d\zeta} = \frac{2\kappa_0''P_S\Gamma_S}{C}(1 + P_S\Gamma_S - P_A\Gamma_A\cos 2\phi), \qquad (14a)$$

$$\frac{dP_A}{d\zeta} = \frac{2\kappa_0''P_A\Gamma_A}{C}(1 + P_A\Gamma_A - P_S\Gamma_S\cos 2\phi), \qquad (14b)$$

$$\frac{d\phi}{d\zeta} \approx L_R\delta\beta. \qquad (14c)$$

Direct integration of (14a)–(14c) was performed using a MATHCAD software solver based on the fourth-order Runge-Kutta method. The calculations were made for the cases (a) $\kappa'=0.900\,126$ ($\Gamma_A=\Gamma_S$), and (b) $\kappa'=1.2$ ($\Gamma_A=1.112\Gamma_S$). For calculations we took $\kappa_0''=kn_0g_0da/2=0.001$. This value can be achieved at the small signal gain $g_0\approx 0.21\text{cm}^{-1}$ for $a=5.2\mu\text{m}$ and the other parameters of the construction as at the end of Sec. VI, so that $\kappa'=1.2$. The spectral separation of the modes in this case is $\delta\beta=0.277L_R^{-1}\approx 22\text{cm}^{-1}$. The results of the calculations are presented in Fig. 4. For equal confinement factors $\Gamma_A=\Gamma_S$, the behavior of the curves on the diagram in Fig. 4(a) showing I_A as a function of I_S for different initial conditions is in contrast with their behavior in the case of incoherent modes (Fig. 3). In the latter case, both modes increase in power so that the proportion of the powers is constant. In the former case, there is the line $I_S=I_A$ that is unstable in Lyapunov's sense: an infinitesimal deviation from this trajectory results in further divergence, with the curves approaching asymptotically either a vertical or a horizontal line depending on the direction of the initial deviation. This

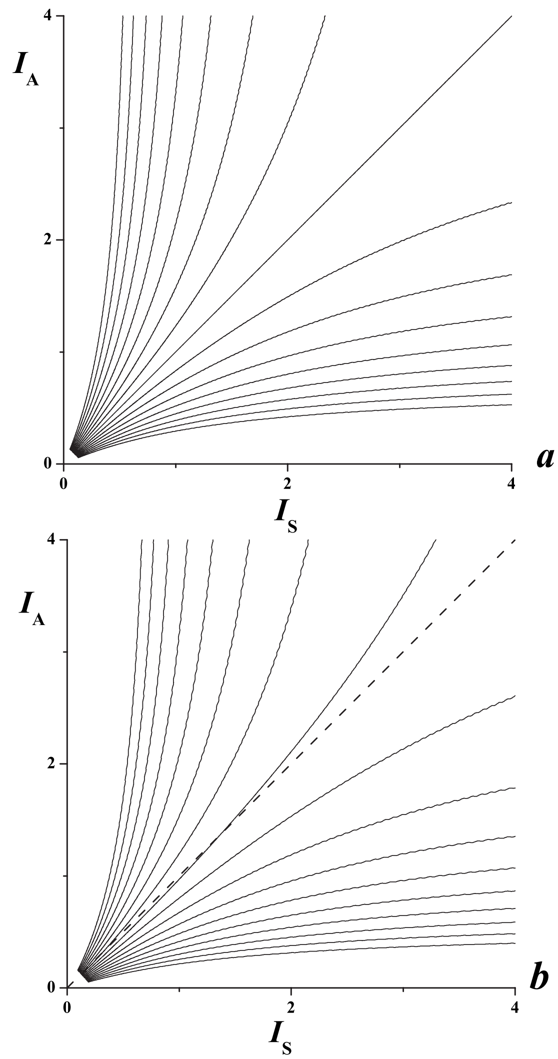

FIG. 4. Mode intensities in the waveguide normalized to I_{sat} for different values of initial power ratio: (a) $\Gamma_S=\Gamma_A$; (b) $\Gamma_A=1.112\Gamma_S$. Dashed line in (b) is the bisector $I_A=I_S$.

behavior corresponds to dominance of one mode, the power of which grows linearly with the length, while the power carried by the other mode is stabilized at a certain level. This behavior is similar to that observed in Lamb's model [28] for a two-mode laser in the case when the gain cross-saturation coefficient is greater than the self-saturation coefficient.

If the confinement factors are different [see Fig. 4(b)], the diagram on the whole is nearly the same. The straight line $I_S=I_A$ in the diagram $I_A(I_S)$ in Fig. 4(a) transforms to a curve [not shown in Fig. 4(b)], the shape of which can be found numerically. This curve is a separatrix dividing the (I_S,I_A) plane into two parts. In the upper part all curves approach vertical asymptotes (the asymmetric mode dominates), while in the lower part all curves approach horizontal asymptotes (the symmetric mode dominates).

To get better insight into the mechanism leading to depression of one of the modes in two-mode amplification, let us illustrate the general arguments adduced in Sec. III by the results of analysis of the specific construction under consideration. Figure 5 illustrates the behavior of the terms in the

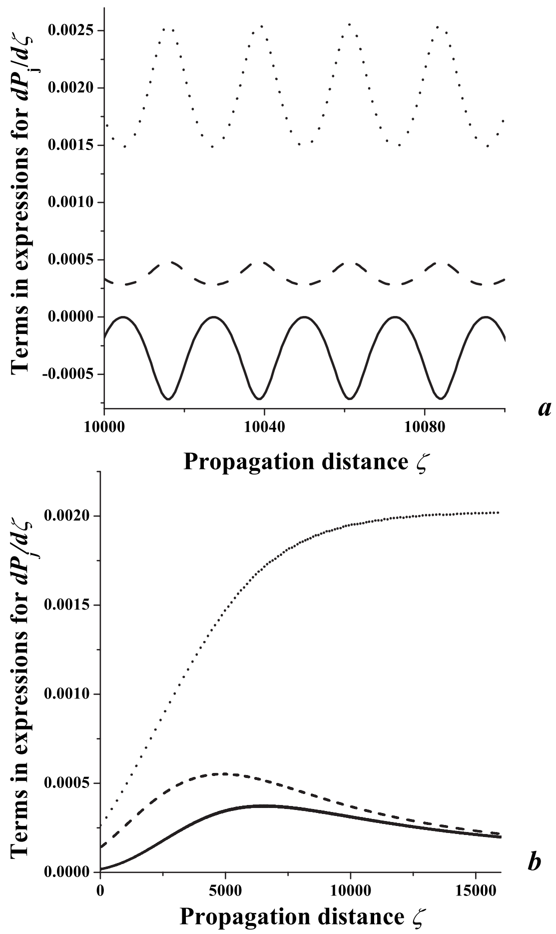

FIG. 5. Dimensionless terms $g_{AS}\cos\phi\sqrt{P_A P_S}L_R$ (solid line), $g_{AA}P_A L_R$ (dashed line), and $g_{SS}P_S L_R$ (dotted line) as functions of propagation distance normalized to L_R: (a) axial oscillations shown within a short propagation interval; (b) values averaged over oscillations; $g_{AS}\cos\phi\sqrt{P_A P_S}L_R$ (solid line) is taken with minus sign. P_{S0}=0.27 and P_{A0}=0.13; Γ_A=1.112Γ_S.

right-hand sides of Eqs. (6a)–(6c) $g_{jj}P_j$ (j=A,S) and $\sqrt{P_A P_S}g_{AS}\cos\phi$ with modal gain and cross-gain coefficients defined by (13). P_{S0}=0.27 and P_{A0}=0.13. Small-scale oscillations of these terms associated with mode beating are shown in Fig. 5(a). It is seen that the power increase $g_{AA}P_A$ in the asymmetric mode is lower than that of the symmetric mode even though its modal gain is higher. The cross-gain component $\sqrt{P_A P_S}g_{AS}\cos\phi$ is nonpositive and antiphase to the power increments associated with modal stimulated emission. The same terms averaged over oscillations are shown in Fig. 5(b). For convenience of presentation, the cross-gain term is multiplied by (−1). It is clearly seen in Fig. 5(b) that the decrease in emitted power caused by the cross-gain term tends to equilibrate the term $g_{AA}P_A$ at a sufficiently long amplification length. In dimensional variables, the Rayleigh length at the parameters taken is 123μm. This phenomenon leads to stabilization of the antisymmetric mode power on the level reached at that moment.

The trend to domination of one of the modes in simultaneous amplification of two coherent modes can be illustrated by the behavior of the mode power ratio P_A/P_S. The proportion P_A/P_S calculated for κ'=1.2 is shown in Fig. 6 as a function of propagation distance for various input proportions at constant total input power $P_{A0}+P_{S0}$=0.4. It is seen that increasing P_{A0}/P_{S0} results in a change of amplification regime from dominance of the symmetric mode to dominance of the asymmetric one. It is worth noting that the sign of the derivative at ζ=0 of the proportion P_A/P_S cannot serve as a criterion for the change of the amplification regime. As long as P_{A0}/P_{S0} grows, the curve appears, for which growth at small distances changes to decrease at longer distances. Figure 6 proves that there exists a critical value of P_{A0}/P_{S0}, which separates the regimes of dominance of the symmetric or asymmetric mode. Actually, the critical value of P_{A0}/P_{S0} is a function of the total power and depends parametrically on the confinement factor values.

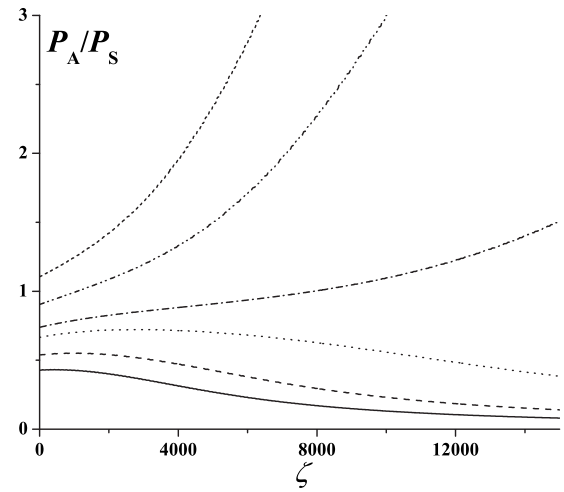

FIG. 6. Proportion of the modal powers for varied inputs at constant total input power as a function of the amplifier length. The quantities are dimensionless, Γ_A=1.112Γ_S.

A series of calculations allows us to find the critical fraction of the symmetric (in-phase) mode in the launched signal shown in Fig. 7 as a function of the total power for confinement parameter values (Γ_A=1.112Γ_S). At such values of Γ_A and Γ_S the small signal gain of the antisymmetric mode is

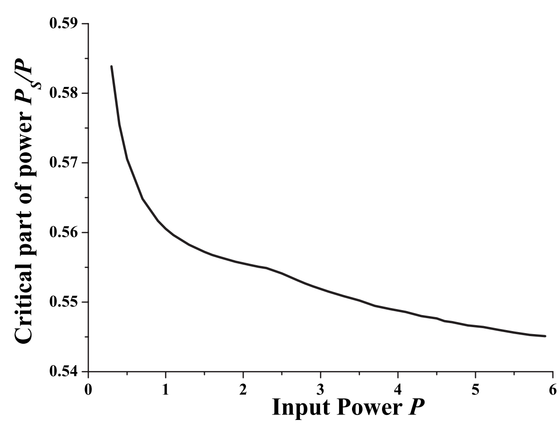

FIG. 7. Critical fraction of the in-phase mode in the input total power P=$P_{A0}+P_{S0}$, above which this mode dominates. The quantities are dimensionless, Γ_A=1.112Γ_S.

greater than that of the symmetric one. Despite this fact, the in-phase mode dominates at the output of the amplifier when its fraction in the total launched power is above the curve shown in Fig. 7. The higher the total launched power, the lower the critical fraction of the in-phase mode. It follows from this figure that 40% excess of in-phase mode power in the input signal is sufficient to suppress the out-of-phase mode power for $P > 0.3$. The analysis performed is strictly valid for the model system under consideration. However, the mechanism of weak-mode suppression by a strong mode is of quite general nature. Thus, it is expected that this mechanism will work in any fiber amplifier provided the input signal has a sufficiently narrow spectral width in order that results found for a monochromatic wave field are applicable for a real signal.

VI. CONCLUSIONS

A theory of monochromatic wave field amplification in a waveguide array is developed. An approach based on expansion of the wave field in terms of guided array modes leads to the appearance of additional terms in the system of ordinary evolution equations for the mode amplitudes. These terms have the meaning of cross-modal gain and, as shown, completely change the behavior of the amplified wave field. Analysis of two-mode amplification reveals unusual features in its characteristics. Instead of unlimited growth of both modes for incoherent fields, the effect of weak-mode suppression by a strong one takes place. A detailed analysis is made for an amplifier composed of a pair of ultrathin waveguides. The critical values of waveguides and input signal parameters are found at which the in-phase mode dominates at the amplifier output. The conditions for asymptotically stable single-mode amplification are found. The model developed is applicable for studies on multimode competition in a multicore fiber amplifier.

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