Decoherence dynamics of two charge qubits in vertically coupled quantum dots

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The decoherence dynamics of two charge qubits in a double quantum dot is investigated theoretically. We consider the quantum dynamics of two interacting electrons in a vertically coupled quantum dot driven by an external electric field. We derive the equations of motion for the density matrix, in which the presence of an electron confined in the double dot represents one qubit. A Markovian approach to the dynamical evolution of the reduced density matrix is adopted. We evaluate the concurrence of two qubits in order to study the effect of acoustic phonons on the entanglement. We also show that the disentanglement effect depends on the double dot parameters and increases with the temperature.

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I. INTRODUCTION

Various candidates for realizing building blocks of quantum information processors with nanoscale solid-state structures have been proposed and also partially realized in ground-breaking experiments. An important class of proposals consists of using the charge degree of freedom in semiconducting double dots [1,2] (DQDs) to realize a quantum mechanical two-state system or qubit. The position of a single electron in a double dot defines the logical states $|0\rangle$ and $|1\rangle$ [3–7].

Charge qubits in semiconductors have the substantial advantage of being easy to manipulate and to measure since the experimental techniques for measuring single electron charges in semiconductors are extremely well developed (note, however, that spin qubits [8–13] have been shown to read-out very efficiently in the presence of an efficient spin-charge conversion mechanism [14]). The electrically controlled charge qubit in semiconductor DQDs has a potential advantage for large systems and is compatible with the current microelectronics technology. The price one pays for the relative ease in the manipulation and read-out of single-charge states is, of course, the strong decoherence and the rather short decoherence time of the orbital charge states because they strongly couple to the environment through the long-range Coulomb interaction.

This fast decoherence of orbital states makes semiconductor charge qubits rather unlikely candidates for a stable quantum computer architecture. However, the strong interactions make the orbital states an excellent choice for studying qubit dynamics and qubit coupling in a solid-state nanostructure environment. The double dot system [15–21] is also extremely useful in basic physics as it enables us to investigate the decoherence and dissipation of a small quantum system interacting with the environment. The system of two vertically QDs has been experimentally [22–27] and theoretically [28–30] studied.

Entanglement, nonlocal quantum correlations between subsystems, is not only one of the basic concepts in quantum mechanics [31] but also central to quantum computation and quantum information [32]. Decoherence, loss of phase relations between the states, is essential in understanding how a quantum system becomes effectively classical [33]. Therefore how an entangled system undergoes decoherence or how the entanglement changes as a result of interaction with the environment is an important issue. The environment decoherence leads to deterioration of the performance of quantum logic operations and also strongly influences entanglement between qubits [34] necessary for quantum gate operation.

Recent theoretical studies have been carried out to deal with the charge qubit(s) decoherence [4,35–39] in double dot due to electron-phonon interaction. These studies were focused on the decoherence properties of charge qubits in semiconductors [4,40,41].

In this paper, we investigate theoretically the decoherence dynamics of two charge qubits driven by an oscillatory electric field as a result of weak interaction with a bath of acoustic phonons in order to study the effect of decoherence on entangled states. The qubit is encoded in the presence of a single electron confined in vertically coupled quantum dots. We assume that the parameters of the GaAs DQDs are selected appropriately and that the temperature is low enough to neglect the effects of the electron transitions to the higher energy. These parameters apply to self-assembled QDs. We use a Marcovian approach to the dynamical evolution of the reduced density matrix.

In order to study the measure of the decoherence we adopt three possible measurements here. For a quantitative description of the entanglement decay of the system, a measure of entanglement that may be calculated from the system is needed. We adopt the concurrence $C(\rho)$ in order to quantify the evolution of the two-qubit entanglement degree in the presence of a bath of acoustic phonons. For a second measure, we consider the fidelity $\mathcal{F}(t)$ in order to quantify the stability of the quantum system under the action of the phonon-electron interaction. Finally, we explore the linear entropy $S(\rho)$ in order to study the mixed character of a system described by a density matrix ρ.

This paper is organized as follows. In Sec. II we introduce the model Hamiltonian for the charge qubits. The DQD electrons states are described within the effective mass scheme. In Sec. III, we numerically analyze the two-qubit decoherence dynamics due to a weak interaction with a bath of acoustic phonons. A Markovian approach to the dynamical

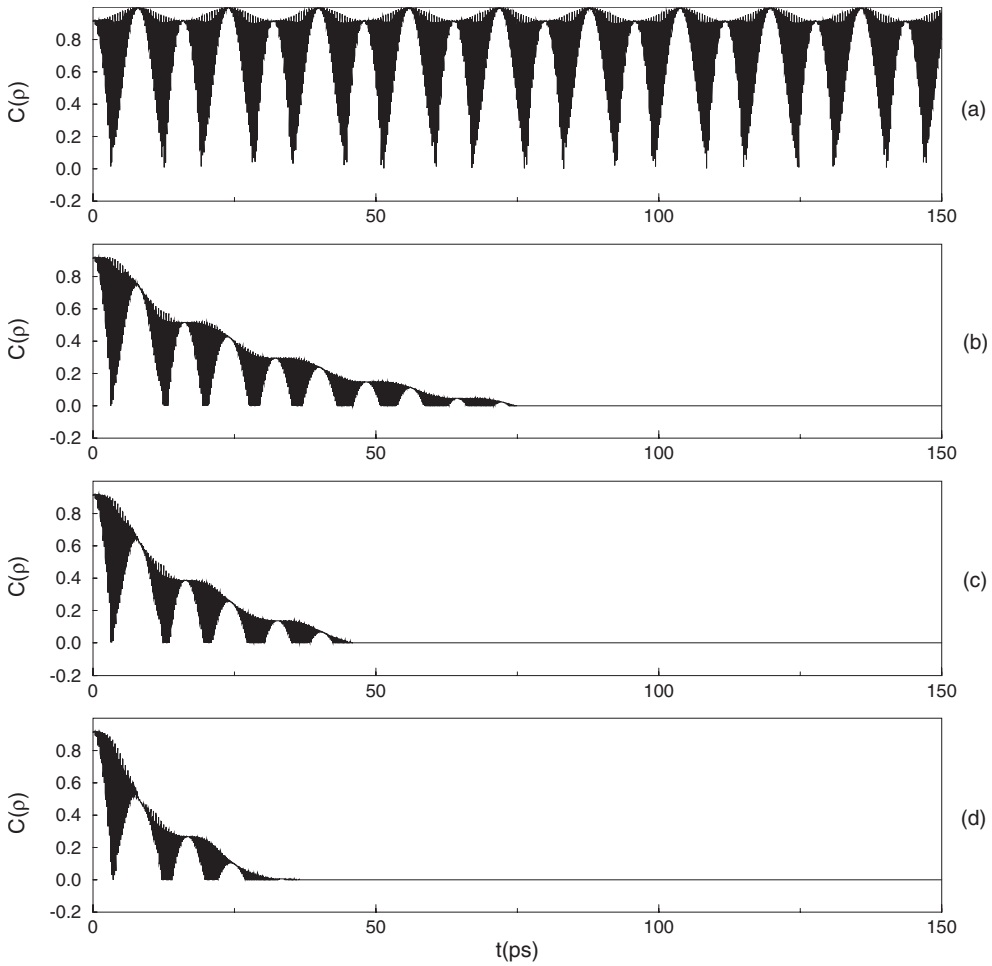

FIG. 1. Time evolution of the concurrence. (a) In the absence of phonons for the set of parameters: $d=14.5$ nm, $\hbar\omega_0=E_{S2}-E_{S1}=9.18$ meV, $F_0=0.5$ kV/cm, and $T=5$ K. We consider the presence of phonons due to deformation potential interaction (b), piezoelectric interaction (c), and both mechanisms (d).

evolution of the reduced density matrix is presented. Finally, the conclusion is given in Sec. IV.

II. MODEL

We assume that the decoherence due to electron-phonon coupling in GaAs is the dominant decoherence mechanism in coupled quantum-dot setting. The total Hamiltonian is given by $H=H_S+H_B+H_{SB}$ where S and B stand for the system and bath, respectively.

Here, $H_S=H_0+V(t)$ is the Hamiltonian of two electrons confined in DQDs under the action of an oscillatory electric field applied along the z direction, of the form $F(t)=F_0\cos(\omega_0 t)$, where $V(t)=e\ F(t)(z_1+z_2)$. The DQDs consist of two vertically coupled QDs at mutual distance $2d$. We assume throughout this work that the two QDs have identical shape and size. Electrons states are described within the effective mass approximation. The Hamiltonian H_0 [42–44] is given by

$$H_0=\sum_{i=1,2}\left(\frac{p_i^2}{2m^*}+\frac{m^*w_z^2\alpha^2}{2}(x_i^2+y_i^2)+\frac{m^*w_z^2}{8d^2}(z_i^2-d^2)^2\right)+\frac{e^2}{\epsilon_r|\vec{r_1}-\vec{r_2}|}, \quad (1)$$

where α is the anisotropy parameter which determines the strength of the vertical confinement relatively to the lateral one and $\hbar\omega_z$ is the quantization energy. The last term in Eq. (1) represents the Coulomb interaction. We take as material parameters for GaAs $m^*=0.067m_0$ for the electron effective mass where m_0 is the bare electron mass and $\epsilon_r=13.1$ for the dielectric constant. We consider the four lowest eigenstates of H_0. In our work, we consider $\alpha=0.5$ and $\hbar\omega_z=20$ meV. We use the linear combination of atomic orbitals (LCAO) [45] in order to construct the one particle molecular orbital states. The single electron wave function $|\phi_\pm\rangle$ is given by a superposition of the single dot wave functions $|\phi_\pm\rangle=a_\pm(|0\rangle\pm|1\rangle)$, where $a_\pm$ are the normalization coefficient which depends on the parameters of the system. The logical states $|0\rangle=|\varphi_{-d}\rangle$ and $|1\rangle=|\varphi_{+d}\rangle$ represent the ground-state so-

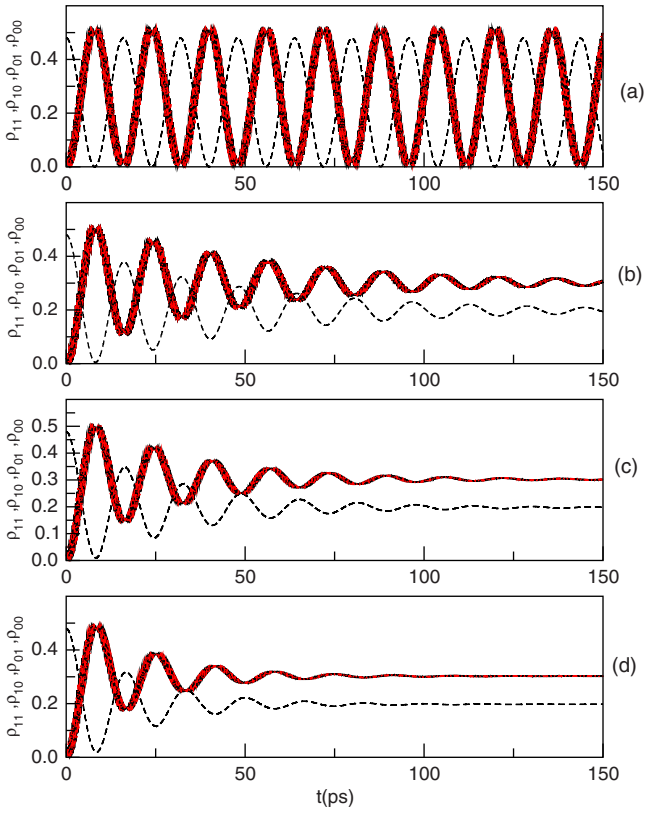

FIG. 2. (Color online) Time evolution of diagonal elements of the density matrix under the action of oscillatory electric field ρ_{00} (solid line), ρ_{11} (dotted red line), $\rho_{10}(\rho_{01})$ (dashed line), for the same parameters as in Fig. 1. (a) In the absence of phonons, (b) in the presence of phonons due to deformation potential interaction, (c) piezoelectric interaction, and (d) both mechanisms.

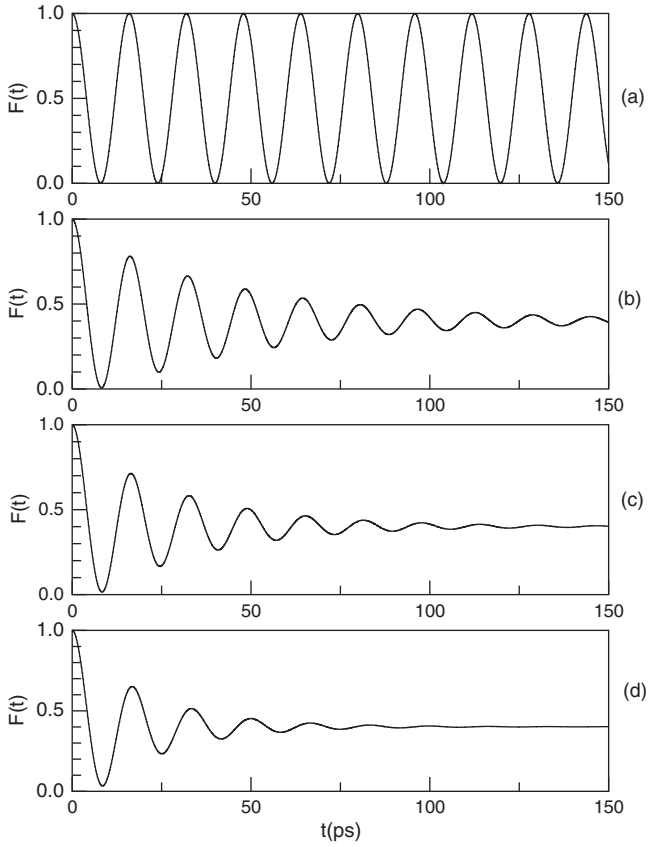

FIG. 3. Time evolution of the fidelity for the same parameters as used in Fig. 1. (a) In the absence of phonons, (b) in the presence of phonons due to deformation potential interaction, (c) piezoelectric interaction, and (d) both mechanisms.

lution of the one particle Hamiltonian of the two isolated dots, centered, respectively, on $z=-d$ and $z=+d$, $\varphi_{\pm d}(x,y,z)=\left(\frac{m^*\omega_z}{\pi\hbar}\right)^{3/4}\sqrt{\alpha}\exp\left\{-\frac{m^*\omega_z}{2\hbar}[\alpha(x^2+y^2)+(z\mp d)^2]\right\}$.

Obviously, $V(t)$ does not mix singlet and triplet states, and thus the spin-triplet state of H_0 is insensitive to the applied field within our truncated basis. Hence we will consider the three lowest singlets states (E_{s1}, E_{s2}, E_{s3}). The three levels can be written in the standard basis $\{|11\rangle,|10\rangle,|01\rangle,|00\rangle\}$:

$$|\psi_1\rangle = \gamma_1|\phi_+\phi_+\rangle + \delta_1|\phi_-\phi_-\rangle = \alpha_1(|11\rangle + |00\rangle) + \beta_1(|10\rangle + |01\rangle), \tag{2}$$

$$|\psi_2\rangle = \frac{1}{\sqrt{2}}(|\phi_+\phi_-\rangle - |\phi_-\phi_+\rangle) = \beta_2(|10\rangle - |01\rangle), \tag{3}$$

$$|\psi_3\rangle = \delta_1|\phi_+\phi_+\rangle - \gamma_1|\phi_-\phi_-\rangle = \alpha_3(|11\rangle + |00\rangle) + \beta_3(|10\rangle + |01\rangle). \tag{4}$$

We note that the states $\{|0\rangle,|1\rangle\}$ are not orthogonal $\langle 0|1\rangle=s$, where s denotes the overlap integral. For large interdot separation the overlap decrease to zero and the logical states becomes orthogonal, which will be the case of our study. The coefficients (γ_i,δ_i) depend on the interdot separation and on the confinement.

The Hamiltonian for the phonon bath is, as usual, given by

$$H_B = \sum_k \hbar\omega_k a_k^\dagger a_k, \tag{5}$$

where $a_k^\dagger$ and a_k are the creation and annihilation operators, respectively, of the phonons with the wave vector k satisfying $[a_k^\dagger,a_{k'}]=\delta_{k,k'}$. We consider isotropic acoustic phonons with the linear dispersion law $\omega_k=c_s k$.

The effect of phonons bath is described by the electron-phonon interaction term [46]

$$H_{\text{int}} = \sum_{i=1,2}\sum_k (M_k^* e^{-i\vec{k}\cdot\vec{r}_i}a_k^\dagger + M_k e^{i\vec{k}\cdot\vec{r}_i}a_k) \tag{6}$$

corresponding to the emission or the absorption of a phonon. The bulk matrix element M_k depends on the type of the interaction. The bulk matrix element for the deformation potential coupling is given by

$$M_k^{\mathrm{def}} = \left(\frac{\hbar k}{2V\sigma c_s}\right)^{1/2} D_c, \tag{7}$$

where c_s is the phonon sound velocity, σ the crystal density of GaAs, V the normalization volume, and D_c the deformation potential coupling constant. For our calculations, we consider $D_c=8.6$ eV, $\sigma=5.3$ g cm^{-3}, and $c_s=37\times10^4$ cm s^{-1}.

The bulk matrix element for the piezoelectric potential is given by

$$M_k^{\mathrm{pz}} = \frac{ee_{14}}{\epsilon_0\epsilon_r}\left(\frac{\hbar}{2V\sigma c_s k}\right)^{1/2}, \tag{8}$$

where $e_{14}=0.16$ C/m^2 is the piezoelectric coupling constant [47]. One of the central points in quantum physics is the loss of coherence of quantum systems. In this paper, we will focus on the phonon effects on the dynamics of two qubits. Only low-energy (acoustical) phonons will be considered in the next section. Indeed, the interaction with optical phonon (of about 36 meV in these systems) is strongly inhibited at low temperatures. Moreover, since in self-assembled QDs the two confined electrons usually come from doping and in our work the intensity of the applied field is expected to be much bigger than the (small) one of the fluctuating field stemming from contacts, we can disregard additional decoherence coming from gate-related noise [48].

III. TWO CHARGE QUBITS DYNAMICS

In a double quantum dot, scattering by phonons can cause considerable loss of coherence. The dynamics of the qubits is determined by the reduced density matrix $\rho=\mathrm{Tr}_{\mathrm{bath}}\rho_{\mathrm{tot}}$, where the trace is carried out over the degrees of freedom of the bath. Assuming initial decorrelation of the system and bath, a perturbative treatment of the system bath coupling Hamiltonian results in the master equation. The Markovian master equation [49,50] of the reduced density matrix into the eigenstate basis of H_0 with the secular approximation is given by

$$\dot{\rho_{nm}} = -i\omega_{nm}\rho_{nm} - \frac{i}{\hbar}\langle\phi_n|[V(t),\rho]|\phi_m\rangle + \sum_{j,l} R_{nmjl}\rho_{jl}, \tag{9}$$

where the dummy indices n, m, j, and l run over the three singlet states and $\omega_{nm}=E_n-E_m/\hbar$, $V(t)$ describes the coupling between the electrons and the oscillatory electric field applied along the z direction. The first term on the right-hand side of the Eq. (9) denotes the unitary evolution and the Redfield relaxation tensor R_{nmjl}, which are given through the golden rule rates incorporating the decoherence effects

$$\sum_{j,l} R_{nmjl}\rho_{jl} = \begin{cases} \sum_{j,l\neq m}\delta_{jl}(\rho_{jj}\Gamma_{j\to m} - \rho_{mm}\Gamma_{m\to j}), & (m=n) \\ -\frac{1}{2}\sum_{j,l}\delta_{nj}\delta_{ml}(\sum_{i\neq n}\Gamma_{n\to i} + \sum_{i\neq m}\Gamma_{m\to i})\rho_{nm}, & (m\neq n) \end{cases}. \tag{10}$$

In the Schrödinger representation, the master equation expanded over the basis of eigenstates of H_0, has the structure of a linear differential system. The Redfield tensor and the time evolution of the density matrix are evaluated numerically to determine the decoherence properties of the system due to a weak electron phonon coupling. The electron-phonon interaction effects affecting the two-qubit system lead to a decoherence, which manifests itself in two ways: relaxation and dephasing. The decoherence rates, i.e., the relaxation and dephasing rates are defined according to $\Gamma_R=\Sigma_n\Lambda_n$ where Λ_n are the eigenvalues of the matrix composed of the $R_{n,n,m,m}$ elements, $n,m=1,\ldots,3$ and $\Gamma_{\varphi_{nm}}=-\mathrm{Re}(R_{n,m,n,m})$ for nondegenerate levels $|\omega_{nm}|>|R_{n,m,n,m}|$ and in the absence of Liouvillian degeneracy, $|\omega_{nm}-\omega_{jl}|>|R_{a,b,c,d}|$ $a,b,c,d,\in j,l,m,n$, respectively [51–53]. In this notation, $\Gamma_{\varphi_{nm}}$ is the rate at which a superposition of energy eigenstates n and m decays into a classical mixture.

Now, that we have determined the decoherence rates, we are ready to study the dynamics of two charge qubits driven by external electric field. To study the decoherence dynamics, we fixed the distance between the dots at $d=14.5$ nm where the effect of acoustic phonons is considerable, as we show in the Appendix, and we consider $s=\langle 0|1\rangle\simeq 0$. We assume that the system has the initial state $|\Psi(t=0)\rangle=|\psi_1\rangle$. The amplitude F_0 of the electric field affects the oscillation period of the system evolution. The smaller the amplitude, the larger the period. We applied an oscillatory electric field having an amplitude $F_0=0.5$ kV/cm and the frequency is equal to the one corresponding to the difference between the lowest states $\hbar\omega_0=E_{s2}-E_{s1}=9.18$ meV.

The decoherence rates, describing the electron-phonon interaction destroy the coherent dynamics. To study the effect of these terms, we evaluate the concurrence C [54], as defined by Wootters, which measure the entanglement of two charge qubits. The concurrence provides mathematically complete information about two-qubit entanglement. It varies from $C=0$ for unentangled state to $C=1$ for a maximally entangled state. The concurrence may be calculated explicitly from the density matrix ρ:

$$C(\rho) = \max\{0,\lambda_1-\lambda_2-\lambda_3-\lambda_4\}, \tag{11}$$

where the quantities λ_i are the square roots of the eigenvalues in a decreasing order of the matrix

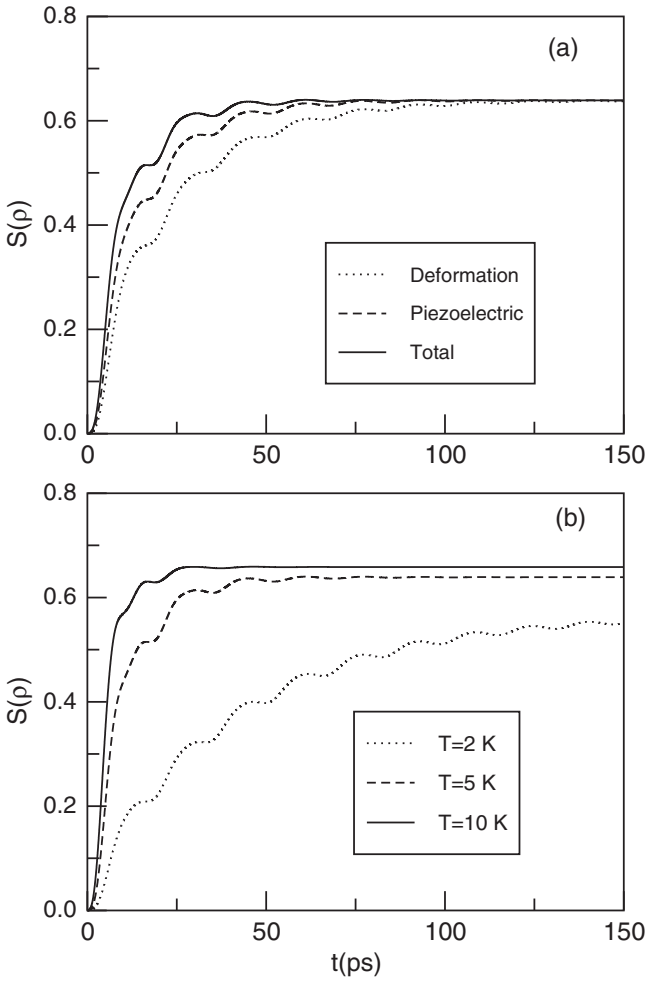

FIG. 4. Time evolution of the linear entropy $S(\rho)$ (a) for $T=5$ K, (b) for different temperatures.

$$\rho' = \rho(\sigma_y \otimes \sigma_y)\rho^*(\sigma_y \otimes \sigma_y), \tag{12}$$

where ρ^* denotes the complex conjugate of ρ in the standard basis. To calculate the concurrence, we need to compute the values of the density matrix. The solution of master equation for the time evolution of the reduced density matrix is carried out numerically using a Runge-Kutta fourth order algorithm.

In Fig. 1, we plot the concurrence as function of the time at $T=5$ K. When we neglected the presence of phonons, the concurrence oscillates between 0 and 1 indicating that the two qubits evolve between maximally entangled and unentangled states. According to Eq. (9), the evolution of the density matrix elements depends on two frequencies: one is related to the electric field and the second to the frequency ω_{nm} of coherencies.

In Figs. 1(b)–1(d), we show the decay of the entanglement between qubits. The electron-phonon interaction affects the oscillations and destroys the entanglement. The time at which the concurrence vanishes is shorter for the piezoelectric interaction than that for the interaction via the deformation potential. When we consider both mechanisms the time at which the concurrence vanishes completely is about t_c $=35$ ps. This time depends on the temperature, interdot distance, and confinement. As temperature increases, the concurrence decays faster. Since that the decoherence rates increase with the rise of temperature, as we show in the Appendix. The time t_c depends on the interdot distance.

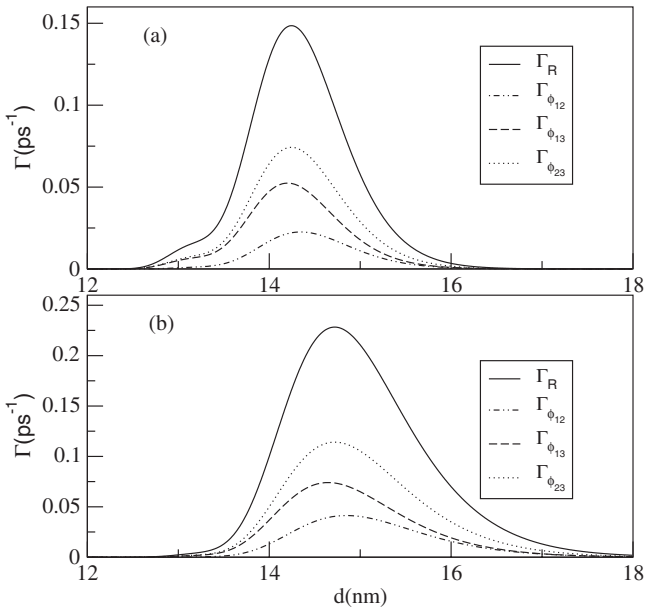

FIG. 5. Relaxation and dephasing rates as a function of the half interdot distance d at $T=5$ K due to (a) the interaction via the deformation potential and (b) the piezoelectric interaction.

The corresponding dynamics of the density matrix elements in the standard basis (ρ_{00}, ρ_{01}, ρ_{10}, ρ_{11}) is shown in Fig. 2. The density matrix element $\rho_{00}=\langle 00|\rho|00\rangle$ represents the probability to have the two electrons on the lower dot and $\rho_{01}=\langle 01|\rho|01\rangle$describes the probability to find one electron on the lower dot and the other one on the upper. For equal quantum dots, we find that $\rho_{10}=\rho_{01}$ and $\rho_{11}\sim\rho_{00}$.

It is worth stressing that the diagonal elements oscillate under the effect of oscillatory electric field. We note that for selected times, we have $\rho_{10}=\rho_{01}=0$ and $\rho_{00}=\rho_{11}=0.5$ corresponding to $C=1$ [Fig. 1(a)], which means that we obtain the Einstein-Podolsky-Rosen (EPR) state $|11\rangle+|00\rangle$. However, when we consider the presence of phonons, deformation potential; piezoelectric; or both mechanisms [Figs. 2(b)–2(d)], we do not have anymore EPR states and the amplitude of oscillations decrease to a stable value. The reduce of amplitude is more shorter in the presence of the piezoelectric mechanism than for the deformation potential and it becomes faster when we consider both mechanisms. We show that when the stable value is obtained, the probability of finding the two electrons in the same dot is larger than that corresponding to the probability for finding one electron in each dot. According to Figs. 1(b)–1(d) and Fig. 2(b)–2(d), we note that we obtain $\rho_{10}=\rho_{01}\simeq\rho_{00}=\rho_{11}\simeq 0.25$ at a time almost equal to the time t_c corresponding to $C=0$. After this time we have $\rho_{00}=\rho_{11}>\rho_{10}=\rho_{01}$.

To study the coherent evolution of the system, we explore the fidelity $\mathcal{F}(t)$. $\mathcal{F}(t)$ was introduced as a measure of the stability of quantum motion with respect to changing some

control parameter [55]. The fidelity recently evoked considerable interest as an alternative route for the study of the effects of perturbations on the coherent evolution of quantum systems, particularly in the context of quantum information [56]. Starting from an eigenstate $|\Psi(t=0)\rangle=|\psi_1\rangle$ of H_0, the fidelity

$$\mathcal{F}(t)=\langle\Psi(t=0)|\Psi(t)\rangle^2=\langle\psi_1|\rho|\psi_1\rangle. \tag{13}$$

This represents the probability of the system to be in the ground state. For initial eigenstates, the fidelity is equivalent to the survival probability [57]. Figure 3 shows the evolution of the fidelity at $T=5$ K. We observe that in the case where we neglected the electron-phonon interaction [Fig. 3(a)], the fidelity oscillates between 0 and 1. When we consider the electron-phonon interaction [Figs. 3(b)–3(d)], the fidelity decays to a stable value and saturates at a constant: the decoherence dampens the oscillation down. This is explained by the effect of phonons which cause transitions between energy levels. The decay of fidelity depends on the strength of the perturbation. We find that the fidelity decay due to piezoelectric interaction is more important. The saturating value of the fidelity depends essentially on the temperature and on the interdot distance.

This stable value decreases with the rise of temperature. The time at which the fidelity takes this value is about 80 ps when we consider both mechanisms [Fig. 3(d)]. This time is different from t_c at which the concurrence vanishes. According to Figs. 1 and 3, we can say that when the amplitude of the fidelity is less than 50%, we can predict that the concurrence vanishes.

A measure of the mixed character of the system described by a density matrix ρ is provided by the linear entropy [58]

$$S(\rho)=\mathrm{Tr}(\rho-\rho^2). \tag{14}$$

This quantity is zero for a pure state, since $\mathrm{Tr}(\rho)=\mathrm{Tr}(\rho^2)=1$. Nonzero values of $S(\rho)$ then provide a quantitative measure of the nonpurity of the system state. When monitored in time, the linear entropy provides a convenient measure of how fast the loss of quantum purity occurs in a system in contact with a bath. We explore the influence of the temperature and electron-acoustic phonon interaction on $S(\rho)$.

In the present case, in the absence of phonons, we have a pure state. It is a linear combination of states $|11\rangle$, $|00\rangle$, $|10\rangle$, and $|01\rangle$, which means that $S(\rho)=0$. Knowing the time evolution of the density matrix we can also calculate the linear entropy of the system in order to monitor the degree of nonpurity introduced during the switching process by the thermal bath. In Fig. 4 we plot the time evolution of the linear entropy for both mechanisms, deformation potential and piezoelectric, and for different temperatures. In Fig. 4(a), at $T=5$ K, we show that starting from a pure state and in the presence of acoustic phonons the state becomes mixed indicating that the environment quickly destroys the vast majority of the superpositions. We see that when we consider the deformation potential and/or piezoelectric mechanism, the linear entropy increases to a final stable value which depends on the temperature.

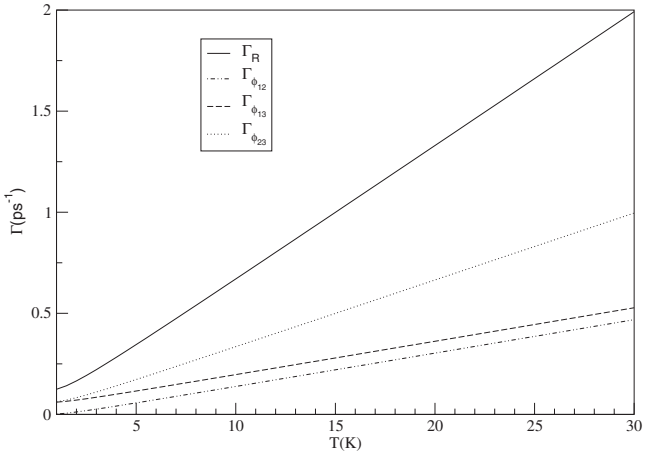

FIG. 6. Relaxation and dephasing rates as a function of the temperature T at $d=14.5$ nm due to both mechanisms.

Considering both mechanisms, we plot the evolution of $S(\rho)$ at different temperatures [Fig. 4(b)]. We see that the linear entropy increases when the temperature increases and for each temperature $S(\rho)$ increases to a final stable value. The time at which the linear entropy reaches the final stable value is longer when the temperature decreases.

IV. CONCLUSION

We have studied the effects of acoustic phonons on entangled states. The entanglement of two electrons in a double quantum dot is dynamically manipulated by an external electric field. We have analyzed decoherence effects through calculation of the concurrence, the fidelity, and the linear entropy. We found that the acoustic phonons completely destroy the coherences between two qubits. This effect is more important with the rise of temperature. It appears that phonon-assisted decoherence can be suppressed by a careful choice of system parameters leading to a maximum entanglement and higher fidelity. We mention that this theoretical model using the master equation for studying the dynamics of two electrons in the presence of the bath of phonons is valid at low temperature (a few K).

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APPENDIX: DECOHERENCE RATE

In this appendix, we estimate the decoherence rate due to the interaction with acoustic phonons. The coupling between electrons and acoustic phonons in a semiconductor have three mechanisms: the deformation potential and piezoelectric and ripple mechanisms [59]. In our work, we consider just the two first mechanisms. The major parameter of dots influencing the interaction with phonons is their size [60]. For DQDs the interdot distance influences this interaction too [36].

The scattering rate involving two electrons levels can be evaluated using Fermi's golden rule

$$\Gamma_{i\to j} = \frac{V}{\hbar(4\pi^2)} \int d^3k |M_k|^2 |\langle \phi_i | e^{\pm i\vec{k}\cdot\vec{r_1}} + e^{\pm i\vec{k}\cdot\vec{r_2}} | \phi_j \rangle|^2 \times \delta(E_m - E_n \mp \hbar\omega_k) \left(n_k(\hbar\omega_k, T) + \frac{1+s}{2} \right), \quad \text{(A1)}$$

where M_k is the bulk matrix element for the deformation potential coupling or the piezoelectric potential given, respectively, by Eqs. (7) and (8), $n_k(\hbar\omega_k, T) = 1/[\exp(\hbar\omega_k/k_BT) - 1]$ is the Bose occupation function for a bath of phonons at temperature$T$, "±" corresponds to the absorption or emission of phonon by the confined system ($s = 1$ for emission and $s = -1$ for absorption).

In our calculations we find that

$$\Gamma_{1\to 2} = \sqrt{2}(\gamma_1 + \delta_1)\Gamma_{1\mathrm{ph}}, \quad \text{(A2)}$$

$$\Gamma_{2\to 3} = \sqrt{2}(\delta_1 - \gamma_1)\Gamma_{1\mathrm{ph}}, \quad \text{(A3)}$$

$$\Gamma_{1\to 3} = 4\delta_1\gamma_1 I(\vec{k}), \quad \text{(A4)}$$

where $\Gamma_{1\mathrm{ph}}$ is the scattering rate for a one charge qubit (one electron in the double dot) and $I(\vec{k})$ is the dephasing matrix element for one charge qubit [36,61]. The shape of the scattering rate as function of the interdistance d of two electrons in DQDs is the same as for one electron.

In Fig. 5 we plot the relaxation and dephasing rates as a function of the interdot distance d due to the interaction via the deformation potential and piezoelectric interaction. The relaxation process dominates. For small interdot distance, the relaxation rate is dominating because of the deformation potential mechanism. However, for large interdot separation the relaxation rate is dominating because of the piezoelectric mechanism. Our results show that the acoustic phonons can be considered as a source of decoherence in the system for selected values of interdot distance and confinement.

Figure 6 shows the decoherence rates as a function of the temperature. The variation is linear. The dependence on temperature is more important for the relaxation rate than for the dephasing rate.

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