# Casimir-Polder interaction between an accelerated two-level system and an infinite plate

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We investigate the Casimir-Polder interaction energy between a uniformly accelerated two-level system and an infinite plate with Dirichlet boundary conditions. Our model is a two-level atom interacting with a massless scalar field, with a uniform acceleration in a direction parallel to the plate. We consider the contributions of vacuum fluctuations and of the radiation reaction field to the atom-wall Casimir-Polder interaction, and we discuss their dependence on the acceleration of the atom. We show that, as a consequence of the noninertial motion of the two-level atom, a thermal term is present in the vacuum fluctuation contribution to the Casimir-Polder interaction. Finally we discuss the relevance of this result for the Unruh effect.

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## I. INTRODUCTION

A striking feature of the quantum description of the electromagnetic field is the existence of zero-point fluctuations. These fluctuations are at the origin of many observable phenomena, such as the Casimir effect [1], the attractive force between two conducting plates at rest in the vacuum, and the Casimir-Polder force between neutral polarizable objects [2]. A crucial feature of these forces is that retardation effects associated with the finite velocity of light modify the dependence of the interaction energy on the distance between the objects. These effects were first investigated theoretically by Casimir and Polder [2], and later many different physical models were proposed, ascribing the origin of the Casimir and Casimir-Polder forces to a change of the energy of vacuum fluctuations, or to the radiation reaction field [3], or both [4] (see also [5] for a review). These investigations are concerned with Casimir-Polder forces between objects (such as atoms or mirrors) at a fixed distance.

A very interesting aspect is to investigate what happens when the motion of the objects is taken into account. In this paper, we investigate the effect of the noninertial motion of an atom (modeled as a two-level system) on the Casimir-Polder interaction between an atom and a plate with Dirichlet boundary conditions. The interest in this issue is motivated by the increasing attention to the physical properties of the quantum vacuum, and, in particular, to the possibility of photon creation from the vacuum. These studies are related to the Unruh effect [6,7], according to which a uniformly accelerated observer perceives the vacuum as a thermal bath at temperature $T=\hbar a/2\pi k_B c$, $a$ being the observer's acceleration. Actually, the question of the appearance of the vacuum in an accelerated frame is a widely controversial problem [8,9]. A closely related phenomenon is the dynamical Casimir effect, which is concerned with the emission of electromagnetic radiation from a single accelerated mirror in the vacuum. Both these phenomena demonstrate the highly nontrivial nature of the quantum vacuum, although the relation between these effects is not yet completely understood. The radiative properties of accelerated atoms in vacuum have been extensively investigated from a theoretical point of view until very recently [10–17]. Unfortunately, both the Unruh effect and the dynamical Casimir effect are extremely weak effects and their detection is therefore very difficult. For example, in order to obtain Unruh radiation corresponding to a temperature of 1 K, an acceleration of the order of $10^{22}$ cm/s$^2$ would be necessary. Experimental schemes have been recently proposed for detecting phenomena related to the Unruh effect, aiming at enhancing the Unruh radiation under specific circumstances [18].

In this paper, we adopt a different point of view. We investigate whether thermal effects due to the acceleration of the two-level atom may modify the Casimir-Polder interaction between the atom and an infinite plate. This is indeed expected because, as is well known, Casimir-Polder interactions are directly related to vacuum field fluctuations [19]. On the other hand, the Unruh effect shows that the accelerated atom perceives vacuum fluctuations as a thermal bath with a temperature proportional to its acceleration. The interest in this subject is also related to the fact that the static Casimir-Polder interaction between an atom at rest and a wall has been recently measured with good precision [20–23]. This suggests the possibility of detecting the Unruh effect through a measurement of the Casimir-Polder interaction between an accelerated atom and a reflecting plate. Also, Casimir-Polder forces in dynamical situations have recently attracted much interest in the literature [24–26]. Recently it has also been suggested that the interatomic Casimir-Polder force may be used as a probe to investigate nonlocal properties of the quantum vacuum [27,28] as well as quantum entanglement of vacuum fluctuations [29].

We stress that, although the physics of a moving detector in a cavity has been extensively investigated, the Casimir-Polder interaction between a uniformly accelerated system and an infinite plate has not been studied, to the best of our knowledge.

We consider a neutral two-level atom uniformly accelerated in a direction parallel to an infinite mirror and calculate the atom-wall Casimir-Polder interaction between the accelerated atom and the mirror. In order to simplify the mathematics involved, we adopt a model consisting of a two-level atom interacting with a massless scalar field, rather than with the electromagnetic field. We first calculate the radiative level shift of the accelerated atom in the presence of the

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mirror. As is well known, the presence of the reflecting plate changes the vacuum field fluctuations. The modification of the Lamb shift of the atom contains terms depending on the atom-mirror distance, yielding the atom-wall Casimir-Polder potential. We identify the contributions of vacuum fluctuations and of radiation reaction to the Casimir-Polder interaction and discuss their dependence on the acceleration of the atom in the limits of small and large accelerations. The relation to the Unruh effect is then considered, as well as the observability of the results obtained.

## II. VACUUM FLUCTUATIONS AND RADIATION REACTION CONTRIBUTIONS TO THE RADIATIVE ENERGY SHIFT

Let us now consider a two-level atom interacting with a real massless scalar field, in the presence of a perfectly reflecting plate located at $z=0$. The atom is modeled as a pointlike system with two internal energy levels $\pm\frac{1}{2}\hbar\omega_0$ relative to the eigenstates $|g\rangle$ and $|e\rangle$. We assume that $\omega_0$ includes any direct modification of the transition frequency due to the acceleration of the atom. The Hamiltonian that describes the atom-field interacting system in the instantaneous inertial frame of the atom is [11,12]

$$H(\tau) = H_A(\tau) + H_F(\tau) + H_{AF}(\tau), \tag{1}$$

where $\tau$ is the proper time and

$$H_A(\tau) = \hbar\omega_0 S_z(\tau), \tag{2}$$

$$H_F(\tau) = \sum_{\mathbf{k}} \hbar\omega_k a_{\mathbf{k}}^\dagger a_{\mathbf{k}} \frac{dt}{d\tau}, \tag{3}$$

$$H_{AF}(\tau) = \mu S_2(\tau)\phi(x(\tau)), \tag{4}$$

$a_{\mathbf{k}}$ and $a_{\mathbf{k}}^\dagger$ are the bosonic operators of the scalar field, and $\mu$ is the atom-field coupling constant. Moreover, we have introduced the pseudospin operators $S_z=(1/2)(|e\rangle\langle e|-|g\rangle\langle g|)$ and $S_2=(i/2)(S_- - S_+)$, where $S_-=|g\rangle\langle e|$ and $S_+=|e\rangle\langle g|$ are the atomic lowering and raising operators. Finally, $\phi(\mathbf{x},t)$ is the scalar field operator,

$$\phi(\mathbf{x},t) = \sum_{\mathbf{k}} \sqrt{\frac{\hbar}{2V\omega_k}} f(\mathbf{k},\mathbf{x})[a_{\mathbf{k}}(t) + a_{\mathbf{k}}^\dagger(t)], \tag{5}$$

where $f(\mathbf{k},\mathbf{x})$ are the appropriate mode functions taking into account Dirichlet boundary conditions for the field operator. The mode functions satisfy the normalization condition

$$\frac{1}{V}\int d^3\mathbf{x}\, f(\mathbf{k},\mathbf{x})\mathbf{f}(\mathbf{k}',\mathbf{x}) = \delta_{\mathbf{k},\mathbf{k}'}. \tag{6}$$

The Hamiltonian $H_F(\tau)$ in (3) governs the evolution of the field in terms of the proper time $\tau$ in the instantaneous inertial frame of the atom. It reduces to the usual free-field Hamiltonian in the simple case of inertial motion, where $dt/d\tau=1$.

We want to evaluate the vacuum fluctuation and radiation reaction contributions to the atom-wall Casimir-Polder interaction energy. This quantity is obtained from the energy level shift of the two-level atom, due to the coupling with the field in the presence of the mirror. Our approach follows that already used in [30,31] for stationary atoms. In order to obtain the equivalent of the Lamb shift of the two-level atom, we consider the time evolution of a generic atomic observable and take the trace over the field degrees of freedom in the equations of motion. The resulting equations can then be partitioned into a vacuum fluctuation part and a radiation reaction part, requiring each part to be an Hermitian operator. This leads to an effective Hamiltonian that governs the time evolution of the atomic observable, consisting of a sum of two terms

$$H_{vf}^{eff} = \frac{i\mu^2}{2\hbar}\int_{\tau_0}^{\tau} d\tau'\, C^F(x(\tau),x(\tau'))[S_2^f(\tau'),S_2^f(\tau)], \tag{7}$$

$$H_{rr}^{eff} = -\frac{i\mu^2}{2\hbar}\int_{\tau_0}^{\tau} d\tau'\, \chi^F(x(\tau),x(\tau'))\{S_2^f(\tau'),S_2^f(\tau)\}, \tag{8}$$

where [ , ] and { , } respectively denote the commutator and anticommutator. The statistical functions $C^F(x(\tau),x(\tau'))$ and $\chi^F(x(\tau),x(\tau'))$ (the correlation function and linear susceptibility, respectively) of the field are expressed as

$$C^F(x(\tau),x(\tau')) = \frac{1}{2}\langle 0|\{\phi^f(x(\tau)),\phi^f(x(\tau'))\}|0\rangle, \tag{9}$$

$$\chi^F(x(\tau),x(\tau')) = \frac{1}{2}\langle 0|[\phi^f(x(\tau)),\phi^f(x(\tau'))]|0\rangle. \tag{10}$$

The expectation values of $H_{vf}^{eff}$ and $H_{rr}^{eff}$ on a generic atomic state $|a\rangle$ give the vacuum fluctuation and radiation reaction contributions to the radiative shift of the atomic level $a$,

$$(\delta E_a)_{vf} = -\frac{i\mu^2}{\hbar}\int_{\tau_0}^{\tau} d\tau'\, C^F(x(\tau),x(\tau'))(\chi^A)_a(\tau,\tau'), \tag{11}$$

$$(\delta E_a)_{rr} = -\frac{i\mu^2}{\hbar}\int_{\tau_0}^{\tau} d\tau'\, \chi^F(x(\tau),x(\tau'))(C^A)_a(\tau,\tau'), \tag{12}$$

where $(C^A)_a(\tau,\tau')$ and $(\chi^A)_a(\tau,\tau')$ are, respectively, the symmetric correlation function and the linear susceptibility of the atom in the state $|a\rangle$,

$$(C^A)_a(\tau,\tau') = \frac{1}{2}\langle a|\{S_2^f(\tau),S_2^f(\tau')\}|a\rangle = \frac{1}{2}\sum_b |\langle a|S_2(0)|b\rangle|^2 (e^{i\omega_{ab}(\tau-\tau')} + e^{-i\omega_{ab}(\tau-\tau')}) \tag{13}$$

$$(\chi^A)_a(\tau,\tau') = \frac{1}{2}\langle a|[S_2^f(\tau),S_2^f(\tau')]|a\rangle = \frac{1}{2}\sum_b |\langle a|S_2(0)|b\rangle|^2 (e^{i\omega_{ab}(\tau-\tau')} - e^{-i\omega_{ab}(\tau-\tau')}). \tag{14}$$

Thus the calculation of the radiative level shifts reduces to the calculation of the statistical functions for the atom and the field. In the next sections we shall use this formalism for evaluating the atom-wall Casimir-Polder interaction between the atom and the plate.

## III. RADIATIVE ENERGY SHIFT FOR AN ATOM AT REST NEAR A REFLECTING PLATE

Let us first consider the atom at rest, located at distance $z_0$ from an infinite perfectly reflecting plate. Its world line $x(\tau)$ is

$$t(\tau)=\tau, \quad x(\tau)=y(\tau)=0, \quad z(\tau)=z_0. \tag{15}$$

We now evaluate the quantities (11) and (12). The statistical functions $C^F(x(\tau),x(\tau'))$ and $\chi^F(x(\tau),x(\tau'))$, in the presence of the reflecting plane boundary, can be calculated from the Wightman function $G(x(\tau),x(\tau'))$ satisfying the Dirichlet boundary conditions on the mirror, $\phi(x)|_{z=0}=0$,

$$G(x(\tau),x(\tau'))=\langle 0|\phi(x(\tau))\phi(x(\tau'))|0\rangle. \tag{16}$$

This function describes the field correlations at two different points $x(\tau)$ and $x(\tau')$. In the presence of a boundary, the Wightman function is the sum of the empty-space contribution $[G_{empty}(x(\tau),x(\tau'))]$ and a part that depends on the presence of the boundary $[G_{bound}(x(\tau),x(\tau'))]$ [34]

$$\begin{aligned}G(x(\tau),x(\tau')) &= G_{empty}(x(\tau),x(\tau'))+G_{bound}(x(\tau),x(\tau'))\\ &= \frac{\hbar}{2}\frac{1}{2\pi^2 c}\left(\frac{1}{[\Delta(\mathbf{x})]^2-(c\Delta t-i\eta)^2}\right.\\ &\quad\left.-\frac{1}{[\overline{\Delta(\mathbf{x})}]^2-(c\Delta t-i\eta)^2}\right),\end{aligned} \tag{17}$$

where we have introduced the variables $\Delta(\mathbf{x})=|\mathbf{x}(\tau)-\mathbf{x}(\tau')|$ [the difference between atomic coordinates $\mathbf{x}(\tau)$ taken at two different proper times] and $\overline{\Delta(\mathbf{x})}=|\mathbf{x}(\tau)-\sigma\mathbf{x}(\tau')|$, where $\sigma\mathbf{x}(\tau')$ is the point corresponding to the reflection of point $\mathbf{x}(\tau')$ on the mirror, and

$$\sigma=\begin{pmatrix}1 & 0 & 0\\ 0 & 1 & 0\\ 0 & 0 & -1\end{pmatrix}. \tag{18}$$

Finally, $\Delta t=t(\tau)-t(\tau')$. Inserting Eqs. (15) in (17) we obtain the symmetric correlation function and the linear susceptibility on the vacuum state for the atom at rest,

$$\begin{aligned}C^F(x(\tau),x(\tau')) &= -\frac{\hbar}{8\pi^2 c}\left(\frac{1}{[c(\tau-\tau')-i\eta]^2}\right.\\ &\quad+\frac{1}{[c(\tau-\tau')+i\eta]^2}-\frac{1}{[c(\tau-\tau')-i\eta]^2-4z_0^2}\\ &\quad\left.-\frac{1}{[c(\tau-\tau')+i\eta]^2-4z_0^2}\right)\end{aligned} \tag{19}$$

and

$$\begin{aligned}\chi^F(x(\tau),x(\tau')) &= -\frac{\hbar}{8\pi^2 c}\left(\frac{1}{[c(\tau-\tau')-i\eta]^2}\right.\\ &\quad-\frac{1}{[c(\tau-\tau')+i\eta]^2}-\frac{1}{[c(\tau-\tau')-i\eta]^2-4z_0^2}\\ &\quad\left.+\frac{1}{[c(\tau-\tau')+i\eta]^2-4z_0^2}\right).\end{aligned} \tag{20}$$

It is convenient to express (19) and (20) as integrals over frequencies. We get

$$\begin{aligned}C^F(x(\tau),x(\tau')) &= \frac{\hbar}{8\pi^2 c^3}\int_0^\infty d\omega\,\omega(e^{-i\omega(\tau-\tau')}+e^{i\omega(\tau-\tau')})\\ &\quad+\frac{\hbar}{8\pi^2 c^3}\,\mathrm{Im}\int_0^\infty \frac{c\,d\omega}{2z_0}(e^{i\omega(\tau-\tau'-2z_0/c)}\\ &\quad-e^{i\omega(\tau-\tau'+2z_0/c)})\end{aligned} \tag{21}$$

and

$$\begin{aligned}\chi^F(x(\tau),x(\tau')) &= \frac{\hbar}{8\pi^2 c^3}\int_0^\infty d\omega\,\omega(e^{-i\omega(\tau-\tau')}-e^{i\omega(\tau-\tau')})\\ &\quad-\frac{\hbar}{8\pi^2 c^3}\,\mathrm{Re}\int_0^\infty \frac{c\,d\omega}{2z_0}(e^{i\omega(\tau-\tau'-2z_0/c)}\\ &\quad-e^{i\omega(\tau-\tau'+2z_0/c)}).\end{aligned} \tag{22}$$

Substituting expressions (13), (14), (21), and (22) in (11) and (12), and taking the limit $\tau\to\infty$ and $\tau_0\to-\infty$, after some algebra we obtain

$$\begin{aligned}(\delta E_a)_{vf} &= \frac{\mu^2}{8\pi^2 c^3}\sum_b|\langle a|S_2(0)|b\rangle|^2\int_0^\infty d\omega\,\omega\left[1-\frac{c}{2z_0\omega}\right.\\ &\quad\left.\times\sin\left(\frac{2z_0\omega}{c}\right)\right]\mathrm{P}\left(\frac{1}{\omega+\omega_{ab}}-\frac{1}{\omega-\omega_{ab}}\right)\end{aligned} \tag{23}$$

and

$$\begin{aligned}(\delta E_a)_{rr} &= -\frac{\mu^2}{8\pi^2 c^3}\sum_b|\langle a|S_2(0)|b\rangle|^2\int_0^\infty d\omega\,\omega\left[1-\frac{c}{2z_0\omega}\right.\\ &\quad\left.\times\sin\left(\frac{2z_0\omega}{c}\right)\right]\mathrm{P}\left(\frac{1}{\omega+\omega_{ab}}+\frac{1}{\omega-\omega_{ab}}\right).\end{aligned} \tag{24}$$

These expressions are, respectively, the contributions of vacuum fluctuations and of radiation reaction to the energy level shift of the atom at rest near the reflecting plate. The presence of the mirror is formally expressed by the $z$-dependent terms, which give an oscillating behavior of the atomic level shift with the atom-plate distance. As expected, when the distance $z_0$ of the atom from the mirror approaches infinity, the function $f(z_0)=1-(c/2z_0\omega)\sin(2z_0\omega/c)$ goes to 1, and we recover the equivalent of the Lamb shift for an atom in the unbounded space. On the contrary, in the limit $z_0\to 0$, $f(z_0)\to 0$, and the two contributions to the atomic level shift vanish. This is a consequence of the Dirichlet boundary conditions on the Wightman function.

The Casimir-Polder interaction energy between the atom at rest and the wall is obtained by considering only the $z$-dependent terms in the vacuum fluctuation and in the radiation reaction contributions, Eqs. (23) and (24), respectively, to the total level shift. For a ground-state atom we obtain

$$E_{CP}=E_{CP}^{(vf)}+E_{CP}^{(rr)} \tag{25}$$

where

$$E_{CP}^{(vf)}=\frac{\mu^2}{8\pi^2c^2}\frac{1}{8z_0}\int_0^\infty d\omega\,\sin\left(\frac{2z_0\omega}{c}\right)\mathrm{P}\left(\frac{1}{\omega+\omega_0}-\frac{1}{\omega-\omega_0}\right) \tag{26}$$

and

$$E_{CP}^{(rr)}=\frac{\mu^2}{8\pi^2c^2}\frac{1}{8z_0}\int_0^\infty d\omega\,\sin\left(\frac{2z_0\omega}{c}\right)\mathrm{P}\left(\frac{1}{\omega+\omega_0}+\frac{1}{\omega-\omega_0}\right) \tag{27}$$

are, respectively, the vacuum fluctuation and the radiation reaction contributions to the atom-wall Casimir-Polder interaction. The integrations in (26) and (27) are easily performed, yielding

$$E_{CP}^{(vf)}=\frac{\mu^2}{8\pi^2c^2}\frac{1}{8z_0}[2f(2\omega_0z_0/c)-\pi\cos(2\omega_0z_0/c)] \tag{28}$$

and

$$E_{CP}^{(rr)}=\frac{\mu^2}{8\pi^2c^2}\frac{1}{8z_0}\pi\cos(2\omega_0z_0/c), \tag{29}$$

where

$$\int_0^\infty d\omega\frac{\sin(2\omega z_0/c)}{\omega+\omega_0}=f(2\omega_0z_0/c) \tag{30}$$

and

$$\int_0^\infty d\omega\frac{\sin(2\omega z_0/c)}{\omega-\omega_0}=-f(2\omega_0z_0/c)+\pi\cos(2\omega_0z_0/c). \tag{31}$$

In the near-zone limit, $\omega_0z_0/c\ll 1$, we obtain

$$E_{CP}^{(vf)}=0\quad\text{and}\quad E_{CP}^{(rr)}\propto 1/z_0, \tag{32}$$

where we have approximated $f(2\omega_0z_0/c)\sim\pi/2$. Therefore the Casimir-Polder interaction near the mirror is due exclusively to the self-reaction contribution and behaves as $1/z_0$. On the other hand, in the far-zone limit, $\omega_0z_0/c\gg 1$, the radiation reaction contribution vanishes and we obtain

$$E_{CP}\sim E_{CP}^{(vf)}=\frac{\mu^2}{8\pi^2c}\frac{1}{8\omega_0z_0^2}. \tag{33}$$

This quantity can be put in the more familiar form

$$E_{CP}\simeq-\frac{\hbar c}{32\pi}\alpha(0)\frac{1}{z_0^2}, \tag{34}$$

where we have defined the "static polarizability"

$$\alpha(0)=-\frac{\mu^2}{2\hbar c^2}\frac{2}{\omega_0}. \tag{35}$$

Therefore, in the far zone, only vacuum fluctuations are responsible for the atom-wall Casimir-Polder potential, as remarked in [4]. We stress that, in contrast with the case of the electromagnetic field where the atom-wall Casimir-Polder force is attractive for ground-state atoms and behaves as $z_0^{-4}$, in the case of the scalar field the Casimir-Polder force is repulsive and behaves as $z_0^{-2}$. This repulsive potential arises as a consequence of the spatial distribution of the vacuum field modes corresponding to Dirichlet boundary conditions chosen for the scalar field. In the case of the electromagnetic field, however, repulsive Casimir-Polder forces are obtained for atoms in excited states and for the three-body component of the Casimir-Polder force between three atoms, depending on their geometrical configuration [32,33].

## IV. RADIATIVE ENERGY SHIFT OF A UNIFORMLY ACCELERATED ATOM NEAR A REFLECTING PLATE

Let us now consider the case of a uniformly accelerated two-level atom, with the acceleration in a direction parallel to the reflecting plate. Let us suppose that the atom is at a distance $z_0$ from the mirror and that it accelerates along the $x$ direction. In the laboratory frame, its trajectory is described, as a function of the proper time, by the equations

$$t(\tau)=\frac{c}{a}\sinh\frac{a\tau}{c},\quad x(\tau)=\frac{c^2}{a}\cosh\frac{a\tau}{c},$$
$$y(\tau)=0\quad z(\tau)=z_0, \tag{36}$$

where $a$ is the proper acceleration. Using the same procedure as in the previous section, we first calculate the Wightman function for the accelerated atom. Substituting (36) into (17), we obtain

$$G(x(\tau),x(\tau'))=-\frac{\hbar}{16\pi^2c^5}P\frac{a^2}{\sinh^2[a(\tau-\tau')/2c]}+\frac{\hbar}{16\pi^2c^5}P\frac{a^2}{\sinh^2[a(\tau-\tau')/2c]-z_0^2a^2/c^4}. \tag{37}$$

From the Wightman function we can obtain the symmetrical correlation function and the linear susceptibility,

$$C^F(x(\tau),x(\tau'))=-\frac{\hbar}{32\pi^2c^5}\left(\frac{a^2}{\sinh^2\{[a(\tau-\tau')/2c]-i\eta\}}+\frac{a^2}{\sinh^2[a(\tau-\tau')/2c+i\eta]}-\frac{a^2}{\sinh^2[a(\tau-\tau')/2c-i\eta]-z_0^2a^2/c^4}-\frac{a^2}{\sinh^2[a(\tau-\tau')/2c+i\eta]-z_0^2a^2/c^4}\right) \tag{38}$$

and

$$\chi^F(x(\tau),x(\tau')) = -i\frac{\hbar a}{8\pi c^4}\frac{1}{\sinh[a(\tau-\tau')/2c]} \times\left[\delta(\tau-\tau') - \frac{1}{2N}\delta\left(\tau-\tau'-\frac{2c}{a}\sinh^{-1}\left(\frac{z_0 a}{c^2}\right)\right) - \frac{1}{2N}\delta\left(\tau-\tau' + \frac{2c}{a}\sinh^{-1}\left(\frac{z_0 a}{c^2}\right)\right)\right], \tag{39}$$

where $N=\sqrt{1+(z_0a/c^2)^2}$. As in the previous section, we can write these statistical functions of the field as integrals over frequencies. We first observe that

$$\frac{1}{\sinh^2(a\tau/2c)} = -\frac{2c^2}{a^2}\int_0^\infty d\omega\, \omega \coth\left(\frac{\pi\omega c}{a}\right)(e^{i\omega\tau}+e^{-i\omega\tau}) \tag{40}$$

and

$$\frac{1}{\sinh^2(a\tau/2c)-(za/c^2)^2} = -\frac{2c^3}{a^2Nz}\int_0^\infty d\omega \coth\left(\frac{\pi\omega c}{a}\right) \times\sin\left[\frac{2\omega c}{a}\sinh^{-1}\left(\frac{az}{c^2}\right)\right](e^{i\omega\tau}+e^{-i\omega\tau}). \tag{41}$$

Using these identities in (38), we obtain the symmetrical correlation function of the field in terms of frequency integrals,

$$C^F(x(\tau),x(\tau')) = \frac{\hbar}{8\pi^2c^3}\left(\int_0^\infty d\omega\,\omega\coth\left(\frac{\pi\omega c}{a}\right)(e^{i\omega(\tau-\tau')} + e^{-i\omega(\tau-\tau')}) - \frac{c}{2z_0N}\int_0^\infty d\omega\left\{\coth\left(\frac{\pi\omega c}{a}\right) \times \sin\left[\frac{2\omega c}{a}\sinh^{-1}\left(\frac{az_0}{c^2}\right)\right](e^{i\omega(\tau-\tau')} + e^{-i\omega(\tau-\tau')})\right\}\right). \tag{42}$$

Similarly, for the antisymmetric correlation function (40),

$$\chi^F(x(\tau),x(\tau')) = -\frac{\hbar}{8\pi^2c^3}\left\{\int_0^\infty d\omega\,\omega(e^{i\omega(\tau-\tau')}+e^{-i\omega(\tau-\tau')}) - \frac{c}{2z_0N}\int_0^\infty d\omega \sin\left[\frac{2\omega c}{a}\sinh^{-1}\left(\frac{az_0}{c^2}\right)\right] \times(e^{i\omega(\tau-\tau')}+e^{-i\omega(\tau-\tau')})\right\}. \tag{43}$$

Substituting Eqs. (13), (14), (42), and (43) into (1) and (12) and taking the limits $\tau_0\to-\infty$, $\tau\to\infty$, after some algebra we obtain

$$(\delta E_a)_{vf} = \frac{\mu^2}{8\pi^2c^3}\sum_b |\langle a|S_2(0)|b\rangle|^2 \times\int_0^\infty d\omega\,\omega\left[1-\frac{c}{2z_0\omega N}\sin\left(\frac{2c\omega\sinh^{-1}(z_0a/c^2)}{a}\right)\right] \times \mathrm{P}\left(\frac{1}{\omega+\omega_{ab}} - \frac{1}{\omega-\omega_{ab}}\right)\left(1+\frac{2}{e^{2\pi\omega c/a}-1}\right) \tag{44}$$

and

$$(\delta E_a)_{rr} = -\frac{\mu^2}{8\pi^2c^3}\sum_b |\langle a|S_2(0)|b\rangle|^2 \times\int_0^\infty d\omega \times\omega\left[1-\frac{c}{2z_0\omega N}\sin\left(\frac{2c\omega\sinh^{-1}(z_0a/c^2)}{a}\right)\right] \times\mathrm{P}\left(\frac{1}{\omega-\omega_{ab}}+\frac{1}{\omega+\omega_{ab}}\right). \tag{45}$$

These expressions give the contributions of vacuum fluctuations and of the radiation reaction to the energy level shift of the accelerated atom near the mirror. In contrast to the case of unbounded space (where the radiation reaction term is not affected by the acceleration of the atom), in the present case both contributions explicitly depend on the acceleration of the atom. In the limit $z_0\to\infty$, the function $f(z_0)=1-(c/2z_0\omega N)\sin[2c\omega\sinh^{-1}(z_0a/c^2)/a]$ tends to 1 and the results obtained for an accelerated atom in the unbounded space are recovered [12]. Moreover, a comparison of (44) with (23) shows that the effect of the uniform acceleration is a thermal-like correction to the Unruh temperature $T=\hbar a/2\pi ck_B$, due to the presence of the $\coth(\pi\omega c/a)$ function in the symmetric correlation function.

As in Sec. III, we now calculate the atom-wall Casimir-Polder interaction by considering only terms containing the distance $z_0$ between the atom and the wall. For a ground-state atom we get

$$E_{CP} = E_{CP}^{(vf)} + E_{CP}^{(rr)}, \tag{46}$$

where

$$E_{CP}^{(vf)} = -\frac{\mu^2}{8\pi^2c^2}\frac{1}{8z_0N}\int_0^\infty d\omega\sin\left(\frac{2c\omega\sinh^{-1}(z_0a/c^2)}{a}\right) \times\mathrm{P}\left(\frac{1}{\omega-\omega_0}-\frac{1}{\omega+\omega_0}\right)\left(1+\frac{2}{e^{2\pi\omega c/a}-1}\right) \tag{47}$$

and

$$E_{CP}^{(rr)} = \frac{\mu^2}{8\pi^2c^2}\frac{1}{8z_0N}\int_0^\infty d\omega\sin\left(\frac{2c\omega\sinh^{-1}(z_0a/c^2)}{a}\right) \times\mathrm{P}\left(\frac{1}{\omega+\omega_0}+\frac{1}{\omega-\omega_0}\right). \tag{48}$$

Using the relation

$$f\left(\frac{2\omega_0 c}{a}\sinh^{-1}\left(\frac{z_0 a}{c^2}\right)\right)=\int_0^\infty d\omega\frac{\sin[2c\omega\sinh^{-1}(z_0a/c^2)/a]}{\omega+\omega_0},\tag{49}$$

the expressions above can be written in the following form:

$$E_{CP}^{(vf)}=-\frac{\mu^2}{8\pi^2c^2}\frac{1}{8z_0N}\times\left\{-2f\left(\frac{2\omega_0c}{a}\sinh^{-1}\left(\frac{z_0a}{c^2}\right)\right)+\pi\cos\left[\frac{2\omega_0c}{a}\sinh^{-1}\left(\frac{z_0a}{c^2}\right)\right]-\frac{a}{\omega_0c}\left[\cos\left(\frac{2c\omega_0\sinh^{-1}(z_0a/c^2)}{a}\right)-1\right]\right\}\tag{50}$$

and

$$E_{CP}^{(rr)}=\frac{\mu^2}{8\pi^2c^2}\frac{1}{8z_0N}\cos\left[\frac{2\omega_0c}{a}\sinh^{-1}\left(\frac{z_0a}{c^2}\right)\right].\tag{51}$$

As expected, the Casimir-Polder interaction depends explicitly on the acceleration of the atom. It is now interesting to investigate the behavior of the Casimir-Polder interaction as a function of $a$ and $z_0$, in the limits $2\omega_0c/a\sinh^{-1}(z_0a/c^2)\ll 1$ and $2\omega_0c/a\sinh^{-1}(z_0a/c^2)\gg 1$. These two limits single out two different *regions* of the space, $z_0\ll c^2/a\sinh(a/2\omega_0c)$ and $z_0\gg c^2/a\sinh(a/2\omega_0c)$, respectively, in analogy with the near and far-zone limits of the inertial atom-wall Casimir-Polder interaction. In other words, in the case of accelerated atoms, we can define a new near zone and a new far zone limit for the Casimir-Polder interaction, which depend also on the acceleration of the atom. We now investigate the behavior of the Casimir-Polder interaction in these two regions and in the limits $a<\omega_0c$ and $a>\omega_0c$. This is equivalent to considering the two cases $k_BT<\hbar\omega_0$ and $k_BT>\hbar\omega_0$.

In the limit of acceleration small compared with $\omega_0c$, these two regions coincide with the usual near zone and far zone of the stationary Casimir-Polder interaction for inertial atoms. From Eqs. (50) and (51), we obtain

$$E_{CP}^{(vf)}=-\frac{\mu^2}{8\pi^2c^2}\frac{1}{8z_0}\times\left[-2f\left(\frac{2\omega_0z_0}{c}\right)+\pi\cos\left(\frac{2\omega_0z_0}{c}\right)\right]\tag{52}$$

and

$$E_{CP}^{(rr)}=\frac{\mu^2}{8\pi^2c^2}\frac{1}{8z_0}\pi\cos\left(\frac{2\omega_0z_0}{c}\right),\tag{53}$$

where we have approximated $2\omega_0c/a\sinh^{-1}(z_0a/c^2)\sim 2\omega_0z_0/c$. Therefore, as expected, in the limit of small acceleration we recover the usual stationary atom-wall Casimir-Polder potential,

$$E_{CP}=\begin{cases}\dfrac{\mu^2}{8\pi^2c^2}\dfrac{\pi}{8z_0}, & \text{near zone}, \quad (54)\\[2ex] \dfrac{\mu^2}{8\pi^2c}\dfrac{1}{8\omega_0z_0^2}, & \text{far zone}. \quad (55)\end{cases}$$

We now consider the case $a\geq\omega_0c$. For typical values of the atomic transition frequency ($\omega_0\sim 10^{15}\ \mathrm{s}^{-1}$), this corresponds to accelerations larger than $a\sim 10^{25}\ \mathrm{cm/s}^2$. In this limit, $2\omega_0c/a\sinh^{-1}(z_0a/c^2)\ll 1$ and from Eqs. (50) and (51), we obtain

$$E_{CP}=E_{CP}^{(vf)}+E_{CP}^{(vf)}\sim\frac{\mu^2}{8\pi^2c^2}\frac{1}{8z_0\sqrt{1+(z_0a/c^2)^2}}\times\left(\pi+\frac{a}{\omega_0c}\left\{\cos\left[2\omega_0c/a\sinh^{-1}\left(\frac{z_0a}{c^2}\right)\right]-1\right\}\right),\tag{56}$$

which depends explicitly on the acceleration $a$. For distances $z_0$ such that $z_0a/c^2\ll 1$, we have

$$E_{CP}=E_{CP}^{(vf)}+E_{CP}^{(vf)}\sim\frac{\mu^2}{8\pi^2c^2}\frac{1}{8z_0}\times\left(\pi+\frac{a}{\omega_0c}\left\{\cos\left[2\omega_0c/a\sinh^{-1}\left(\frac{z_0a}{c^2}\right)\right]-1\right\}\right).\tag{57}$$

On the contrary, in the limit $z_0a/c^2\gg 1$, we obtain

$$E_{CP}\sim\frac{\mu^2}{8\pi^2}\frac{1}{8z_0^2a}\times\left(\pi+\frac{a}{\omega_0c}\left\{\cos\left[2\omega_0c/a\sinh^{-1}\left(\frac{z_0a}{c^2}\right)\right]-1\right\}\right),\tag{58}$$

which gives the Casimir-Polder interaction between the accelerated atom and the wall for high accelerations. The most striking effect of the acceleration of the atom is the presence of an oscillatory term in the interaction energy, which modulates the interaction as a function of $z_0$ and $a$. This oscillatory behavior is reminiscent of the stationary Casimir-Polder interaction between an excited atom and a mirror, where a spatially oscillating term is present. This can be explained by observing that the limit $a\geq\omega_0c$ corresponds to a temperature $T\geq\hbar\omega_0/k_B$. In this limit the excitation probability of the atom is nonvanishing, and this is reflected in the oscillatory behavior of the Casimir-Polder interaction. We emphasize that in the *far-zone* limit the Casimir-Polder interaction is essentially due to the vacuum fluctuation contribution, where a "thermal" term is present, due to the acceleration of the atom. Thus our results show that thermal effects of acceleration may induce observable effects in the far-zone Casimir-Polder interaction between an accelerated atom and a wall, at least in the case of a scalar field considered here.

It is worth comparing our results with the Casimir-Polder interaction between an atom at rest and a plate immersed in a thermal bath. As is well known [35], the atom-wall Casimir-Polder interaction at temperature $T$ is proportional to the temperature of the bath. A comparison of the results in [35] with Eqs. (56)–(58) immediately shows that the Casimir-Polder interaction between the accelerated atom and the plate is qualitatively different from the static counterpart at the Unruh temperature $T$, because of the nontrivial dependence on the acceleration $a$ in (57) and (58). This consideration reveals that, in general, uniformly accelerated atoms behave differently from static ones in a thermal bath at the Unruh temperature, contrary to our expectations. In a different context, this aspect has been discussed in [16,17].

We conclude by observing that we have considered the interaction of a two-level atom with a scalar field. Recently, the radiative level shifts of an accelerated atom in the framework of quantum electrodynamics have been considered. It has been shown that the effects of electromagnetic vacuum fluctuations on the atomic level shifts are not totally equivalent to that of a thermal field, because an extra term is present [13]. This consideration suggests that, for Casimir-Polder interactions between a uniformly accelerated atom and a wall also, nonthermal terms may appear when the electromagnetic field is considered instead of the scalar field. We hope to investigate this point in the future.

## V. CONCLUSION

In this paper, we have investigated the Casimir-Polder interaction between a uniformly accelerated two-level system interacting with a scalar field and a plate with Dirichlet boundary conditions. We have considered the contributions of vacuum fluctuations and of the radiation reaction to the Casimir-Polder interaction energy and discussed their dependence on the acceleration of the two-level atom, in the limits of the near and far zones. We have shown that the atom-wall Casimir-Polder interaction in the limit of small accelerations coincides with the stationary atom-wall Casimir-Polder potential. On the contrary, for high accelerations of the two-level atom, the Casimir-Polder interaction depends on the acceleration of the two-level atom and exhibits an oscillatory behavior in space. This behavior is a consequence of the presence of a thermal term in the vacuum fluctuation contribution to the Casimir-Polder interaction. Therefore it appears that thermal effects due to the acceleration of the atom may become evident in the atom-wall Casimir-Polder interaction and that, in principle, they should be observable.

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