Decoherence of a multiparticle system in vacuum

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In this work, decoherence of a system from an entangled state is studied theoretically by computing the time-dependent probability amplitude with which the system remains in that state. The system is composed of, in addition to vacuum, identical two-level atoms that interact indirectly with each other through photons and have a uniform distribution in the vacuum. Under the assumption that in the initial entangled state only one atom is allowed to be in the excited state, a general expression for the probability amplitude is obtained and applied to two specific entangled states, one symmetric and one antisymmetric, to demonstrate that the system decoheres from these two states at the same rate under certain conditions.

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I. INTRODUCTION

Any open quantum system inevitably interacts with its environment and changes the coherence among its constituents as a result. Such a process is referred to as decoherence. Decoherence of various quantum systems is studied largely because this process not only sheds light on the fundamental problem of where the possible boundary between the quantum and classical worlds lies [1] but also is important for research in the theory of quantum information [2]. To address these two applications properly, it is necessary to note that any macroscopic classical object is an ensemble of many atoms, and that any practical application of quantum-information processing also involves many atoms. Thus knowledge about decoherence obtained from those systems composed of a few particles [3–5] has to be extended by considering different quantum systems containing more atoms [6–11]. In this work, decoherence of an ensemble of N ($N \to \infty$) identical two-level atoms is studied.

Unlike the system studied in Refs. [6,8] (where atoms are placed inside a cavity with a single mode, and have practically the same location), the atoms considered in the present work are assumed to be distributed uniformly throughout vacuum, that is, if the volume of the vacuum is V_{space}, then the density of the atoms $n = NV_{space}^{-1}$ is finite. For an arbitrary atom, the jth atom located at $\vec{R}_j$ for example, its excited state is denoted as $|E^j\rangle$ with energy $\hbar\omega_E$, and its ground state as $|G^j\rangle$ with energy $\hbar\omega_G$; the difference between $\hbar\omega_E$ and $\hbar\omega_G$ is defined as $\hbar\omega_0 \equiv \hbar\omega_E - \hbar\omega_G$. The interaction between the atoms and field modes inside the vacuum is represented by the following Hamiltonian V (and its conjugate $V^\dagger$) in the minimal-coupling form:

$$V = \sum_{j,\alpha} \vec{\mu}_{GE} \cdot \vec{g}_\alpha |G^j\rangle\langle E^j| e^{-i\vec{k}_\alpha \cdot \vec{R}_j} a_\alpha^\dagger \equiv \sum_{j,\alpha} V_j, \tag{1}$$

where $\vec{g}_\alpha = i\sqrt{2\pi\hbar\omega_0^2/(L^3\omega_\alpha)}\vec{\epsilon}_\alpha$ is the amplitude (containing a polarization unit vector $\vec{\epsilon}_\alpha$) of the αth quantized electromagnetic mode ω_α ($|\vec{k}_\alpha| = \omega_\alpha c^{-1}$, where c is the speed of light in the vacuum), and $\vec{\mu}$ is the electric dipole moment operator of the atoms. As in Ref. [12], all the dipole moment operators are assumed to be the same. The symbol $\vec{\mu}_{GE}$ is used to represent the matrix element of $\vec{\mu}$ between the ground and excited states of the atoms. The field creation (annihilation) operator for the αth mode is denoted by the notation $a_\alpha^\dagger$ (a_α). It is through the vacuum fields that the atoms achieve mutual indirect interaction. Throughout the paper, greek letters are reserved for the field modes. For simplicity, the counter-rotating terms are excluded from V, because these terms merely play an insignificant role in the atomic transitions studied in the present discussion [13,14]. Also for simplicity, the atoms are assumed to be stationary; see Ref. [5] for a discussion of quantum effects due to the center-of-mass motion.

The Hamiltonian H of the system, comprising the atomic ensemble and the vacuum, is constructed as usual to include the interaction Hamiltonians V and $V^\dagger$ and the unperturbed Hamiltonian H_0:

$$H = H_0 + V + V^\dagger, \tag{2}$$

where $H_0 = \Sigma_j(\hbar\omega_E|E^j\rangle\langle E^j| + \hbar\omega_G|G^j\rangle\langle G^j|) + \Sigma_\alpha \hbar\omega_\alpha a_\alpha^\dagger a_\alpha$. Since it is constant and contributes little to the time evolution of the system, the zero-point energy of the vacuum is excluded from H. As in Ref. [6], a particular family of entangled states of the atomic ensemble is considered, in which only one atom is in the excited state, $\Sigma_j c_j |G^1\rangle \cdots |E^j\rangle \cdots |G^N\rangle \equiv \Sigma_j c_j ||E^j\rangle$, where the parameters c_i are subject to the normalization requirement $\Sigma_j |c_j|^2 = 1$. In principle, such entangled states can be prepared by allowing ground-state atoms to interact with an incident photon. Note that it is the equal chance for each atom to absorb the photon and to enter the excited state that causes the desired entanglement to be established. See, for example, Ref. [14]. If initially no photon is present, the initial state $|\psi(0)\rangle$ of the system reads

$$|\psi(0)\rangle = \sum_j c_j ||E^j\rangle \otimes |0\rangle. \tag{3}$$

The present initial condition is different from that considered in Ref. [7], where entanglement is assumed to exist only among some of the atoms. In the following, the time evolution of the system is analyzed in the Schrödinger picture, so

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that the state $|\psi(t)\rangle$ of the system at time t is related to $|\psi(0)\rangle$ through an integral relationship:

$$|\psi(t)\rangle = -\frac{1}{2\pi i}\int_{-\infty}^{\infty} dz \frac{e^{-izt/\hbar}}{z-H}|\psi(0)\rangle, \tag{4}$$

where it is understood that the denominator of the integrand contains an imaginary component $i\eta$ ($\eta\to 0^+$). There are two advantages of working in the Schrödinger picture rather than in the Heisenberg picture. First, in the Schrödinger picture the system's evolution can be discussed straightforwardly as a result of atomic transitions and photon emission and absorption [3]. Second, in the Schrödinger picture, atomic operators and field operators commute, so that the common problem of how the atomic and field operators should be ordered does not exist [13]. For the present system, decoherence is caused by dissipation; see Refs. [9,11] for a discussion of decoherence due to depolarization.

Decoherence of the system from its initial state is studied in this paper via calculating the time-dependent probability amplitude $A(t)$ that the system remains in its initial state [3]. This approach little resembles either the concurrence approach [10] or other approaches [7,9]. In Sec. II, all those atomic transitions responsible for the evolution of the system are examined. From these transitions, the probability amplitude is obtained and expressed as a series in Sec. III. The series is then converted into closed-form expressions for two special initial states, one symmetric and one antisymmetric. It is shown that the decoherence rates of the system from these states can be equal when certain conditions are met. The paper is summarized in Sec. IV.

II. TIME EVOLUTION OF THE SYSTEM

In the Schrödinger picture, time evolution of the system results from atomic transitions driven by the function $1/(z-H)$, known as the Green function in Eq. (4). In order to examine these transitions, it is convenient to expand the Green function in ascending powers of V and $V^\dagger$ [3,13]:

$$\frac{1}{z-H} = \frac{1}{z-H_0} + \frac{1}{z-H_0}(V+V^\dagger)\frac{1}{z-H_0} + \frac{1}{z-H_0}(V+V^\dagger)\frac{1}{z-H_0}(V+V^\dagger)\frac{1}{z-H_0} + \cdots . \tag{5}$$

Since H is under the rotating-wave approximation, and since initially no photon is present, a ground-state atom is never able to change its state by itself. On the other hand, as a result of interaction with the vacuum fields, an excited atom has a chance to go to the ground state and to emit one photon spontaneously. Thus the evolution of the system in fact starts from the atom already in the excited state. Besides, before approaching a different atom, the photon emitted by the excited atom can be absorbed and released by the atom many times. Such a process of repeated emission and absorption of a photon by the same atom is known as radiation reaction and has been demonstrated to be the origin of atomic spontaneous emission [13].

As the first step to discuss the atoms' transitions, the radiation reactions of an excited atom have to be treated carefully. Consider, for example, the (excited) jth atom in a particular component $\|E^j\rangle\otimes|0\rangle$ of the initial state. The radiation reactions of this atom certainly can only be produced by that part of the Green function in Eq. (4) G_j that contains operators V_j and $V_j^\dagger$ alone,

$$G_j = \frac{1}{z-H_0} + \frac{1}{z-H_0}(V_j+V_j^\dagger)\frac{1}{z-H_0} + \frac{1}{z-H_0}(V_j+V_j^\dagger)\frac{1}{z-H_0}(V_j+V_j^\dagger)\frac{1}{z-H_0} + \cdots . \tag{6}$$

When the first term $1/(z-H_0)$ on the right-hand side (RHS) of the preceding equation is operated on by $\|E^j\rangle\otimes|0\rangle$, one obtains

$$\frac{1}{z-H_0}\|E^j\rangle\otimes|0\rangle = \frac{1}{z-E_0}\|E^j\rangle\otimes|0\rangle, \tag{7}$$

where $E_0=(N-1)\hbar\omega_G+\hbar\omega_E$. Physically, Eq. (7) represents one possibility that even under the action of G_j the jth atom can stay in its initial state. The excited jth atom's transition into the ground state and the associated emission of one photon are all initiated by V_j ($V_j^\dagger$ has no effect at this stage) in the second term:

$$\frac{1}{z-H_0}(V_j+V_j^\dagger)\frac{1}{z-H_0}\|E^j\rangle\otimes|0\rangle = \frac{1}{z-E_0}\sum_\alpha \frac{\vec{\mu}_{GE}\cdot\vec{g}_\alpha\|G\rangle\otimes|1_\alpha\rangle}{z-E_1-\hbar\omega_\alpha}e^{-i\vec{k}_\alpha\cdot\vec{R}_j}, \tag{8}$$

where $E_1\equiv N\hbar\omega_G$ is the energy of the atomic ensemble when all the atoms are in the ground state $\|G\rangle\equiv|G^1\rangle|G^2\rangle\cdots|G^N\rangle$, and the emitted photon can be in any state $|1_\alpha\rangle$. When the emitted photon is absorbed by the jth atom (now in the ground state) through the left $V_j^\dagger$ in the third term on the RHS of Eq. (6), the system returns to its initial state $\|E^j\rangle\otimes|0\rangle$:

$$\frac{1}{z-H_0}(V_j+V_j^\dagger)\frac{1}{z-H_0}(V_j+V_j^\dagger)\frac{1}{z-H_0}\|E^j\rangle\otimes|0\rangle = \frac{1}{(z-E_0)^2}\sum_\alpha \frac{|\vec{\mu}_{GE}\cdot\vec{g}_\alpha|^2}{z-E_1-\hbar\omega_\alpha}\|E^j\rangle\otimes|0\rangle. \tag{9}$$

In general, it is found that, while the even orders of V_j and $V_j^\dagger$ in the serial expansion of G_j leave the system in $\|E^j\rangle\otimes|0\rangle$, the odd orders allow the system to reside in $\|G\rangle\otimes|1_\alpha\rangle$. After every term of G_j is analyzed, the radiation reactions of the jth atom are noted to turn the system into the following state:

$$G_j\|E^j\rangle\otimes|0\rangle = \frac{1}{z-E_0-E_A}\|E^j\rangle\otimes|0\rangle + \frac{1}{z-E_0-E_A}\times\sum_\alpha \frac{\vec{\mu}_{GE}\cdot\vec{g}_\alpha}{z-E_1-\hbar\omega_\alpha}e^{-i\vec{k}_\alpha\cdot\vec{R}_j}\|G\rangle\otimes|1_\alpha\rangle, \tag{10}$$

where $E_A \equiv \Sigma_\beta |\vec{\mu}_{GE}\cdot\vec{g}_\beta|^2/(z-E_1-\hbar\omega_\beta)$ represents the correction to the jth atom's excited-state level as a result of the radiation reactions. By following the method used in Ref. [13], E_A is found to be given by

$$E_A = -\frac{\Gamma_0\hbar}{2\omega_0\pi}\left[\Omega + \omega_0 \ln\left(\frac{\Omega-\omega_0}{\omega_0}\right)\right] - i\frac{\Gamma_0\hbar}{2} \equiv -\hbar(a+ib), \tag{11}$$

where Ω denotes the cutoff frequency of the vacuum modes, a parameter needed to make the nonrelativistic Hamiltonian H valid in the present discussion [15], and $\Gamma_0 = 4|\vec{\mu}_{GE}|^2\omega_0^3/(3\hbar c^3)$ is the familiar spontaneous emission rate of an isolated excited atom in vacuum. From Eq. (10), it is evident that after spontaneous emission the jth atom becomes entangled with the photon it creates. Entanglement among different atoms can be established if the atoms all interact with a common system (the vacuum in the present paper) with many degrees of freedom [16,17].

The energy-level correction E_A of the jth atom is due to the radiation reactions alone and is immune to the presence of other atoms around this atom, because in the radiation reactions the photon released by the jth atom interacts only with the jth atom. For the same reason, in Eq. (14), when the lth atom completes its own radiation reactions, it also creates the same energy-level correction E_A. Nevertheless, in the following, it will be demonstrated that E_A is not the only energy-level correction the jth atom receives. The reason is that, when the photon $|1_\alpha\rangle$ emitted by the jth atom is scattered back to it from other atoms, the energy level of the jth atom can be additionally changed.

As the photon emitted from the jth atom approaches a different ground-state atom, atom-atom (indirect) interaction through photons is established. (See a discussion of such interaction between two atoms in Ref. [3].) Aided by this photon, the latter ground-state atom (denoted as the lth atom) is able to transit to the excited state and conduct the same radiation reactions thereafter before emitting its own photon. The Green function $G_{j\to l}$ that describes such events as the radiation reactions of the jth atom, photon propagation from the jth atom to the lth atom, and the subsequent radiation reactions of the lth atom still comes from the Green function in Eq. (5) after the atomic operators involving atoms j and l are properly arranged:

$$G_{j\to l} = \left(\frac{1}{z-H_0}(V_l+V_l^\dagger) + \frac{1}{z-H_0}(V_l+V_l^\dagger)\frac{1}{z-H_0}\times(V_l+V_l^\dagger)+\cdots\right)G_j. \tag{12}$$

As an illustration, the first operator $(z-H_0)^{-1}(V_l+V_l^\dagger)G_j$ on the RHS of Eq. (12) is found to transit the lth atom (initially in the ground state) to the excited state for the first time after the lth atom absorbs the photon from the jth atom:

$$\frac{1}{z-H_0}(V_l+V_l^\dagger)G_j\|E^j\rangle\otimes|0\rangle = (z-E_0-E_A)^{-1}\frac{1}{z-E_0}\|E^l\rangle\otimes|0\rangle \times\sum_\alpha e^{i\vec{k}_\alpha\cdot\vec{R}_l}\frac{|\vec{\mu}_{GE}\cdot\vec{g}_\alpha|^2}{z-E_1-\hbar\omega_\alpha}e^{-i\vec{k}_\alpha\cdot\vec{R}_j}. \tag{13}$$

Note that the photon $|1_\alpha\rangle$ emitted by the jth atom is responsible only for the excitation of the lth atom and not for the latter atom's subsequent radiation reactions. After completing its own radiation reactions, the lth atom can also reside in either the excited state or the ground state, and the state of the system becomes

$$G_{j\to l}\|E^j\rangle\otimes|0\rangle = (z-E_0-E_A)^{-2}\|E^l\rangle\otimes|0\rangle\sum_\alpha e^{i\vec{k}_\alpha\cdot\vec{R}_l}\frac{|\vec{\mu}_{GE}\cdot\vec{g}_\alpha|^2}{z-E_1-\hbar\omega_\alpha}e^{-i\vec{k}_\alpha\cdot\vec{R}_j} + (z-E_0-E_A)^{-2}\sum_\gamma\frac{\vec{\mu}_{GE}\cdot\vec{g}_\gamma}{z-E_1-\hbar\omega_\gamma}\|G\rangle \otimes|1_\gamma\rangle e^{-i\vec{k}_\gamma\cdot\vec{R}_l}\sum_\alpha e^{i\vec{k}_\alpha\cdot\vec{R}_l}\frac{|\vec{\mu}_{GE}\cdot\vec{g}_\alpha|^2}{z-E_1-\hbar\omega_\alpha}e^{-i\vec{k}_\alpha\cdot\vec{R}_j}. \tag{14}$$

Similarly, the photon ($|1_\gamma\rangle$) in Eq. (14), which is released by the lth atom, can also afford another ground-state atom (the mth atom) the possibility to enter its excited state. Finally, the state of the system $|T^j\rangle$ evolving from $\|E^j\rangle\otimes|0\rangle$ as a result of the jth atom's spontaneous emission is obtained after the transitions of each atom are all taken into consideration:

$$|T^j\rangle = (z-E_0-E_A)^{-1}\|E^j\rangle\otimes|0\rangle + (z-E_0-E_A)^{-2}\sum_{l\neq j}\|E^l\rangle\otimes|0\rangle\sum_\alpha e^{i\vec{k}_\alpha\cdot\vec{R}_l}\frac{|\vec{\mu}_{GE}\cdot\vec{g}_\alpha|^2}{z-E_1-\hbar\omega_\alpha}e^{-i\vec{k}_\alpha\cdot\vec{R}_j} + (z-E_0-E_A)^{-3}\sum_{m\neq l}\|E^m\rangle \otimes|0\rangle\sum_{l\neq j}\sum_\gamma e^{i\vec{k}_\gamma\cdot\vec{R}_m}\frac{|\vec{\mu}_{GE}\cdot\vec{g}_\gamma|^2}{z-E_1-\hbar\omega_\gamma}e^{-i\vec{k}_\gamma\cdot\vec{R}_l}\sum_\alpha e^{i\vec{k}_\alpha\cdot\vec{R}_l}\frac{|\vec{\mu}_{GE}\cdot\vec{g}_\alpha|^2}{z-E_1-\hbar\omega_\alpha}e^{-i\vec{k}_\alpha\cdot\vec{R}_j}+\cdots, \tag{15}$$

where $\Sigma_{l\neq j}$ means that $\vec{R}_l$ is summed over as long as it is different from $\vec{R}_j$. Physically, the first term on the RHS of Eq. (15) represents the spontaneous emission of the jth atom, and the second term illustrates that the spontaneously emitted photon from the jth atom causes the lth atom to enter the excited state, from where the lth atom conducts its own radiation reactions. The photon emitted from the lth atom can also excite another ground-state atom, the mth atom, as the third term shows. Note, in the third term, when $m=j$, the photon emitted by the lth atom (or the photon first emitted by the jth atom and then scattered from the lth atom) in fact returns to and reexcites the jth atom (now in the ground state), so that the jth atom, once excited, can conduct the radiation reactions again. The (excited) energy level of the jth atom is thus, as already mentioned before, further corrected by its neighboring atoms, and has a correction not limited to E_A. Higher orders of scattering of the photon emitted by the jth atom and higher orders of correction to the energy level of the same atom are all given in the remaining terms on the RHS of Eq. (15).

During its evolution, the system can certainly be in those states where the atoms are all in the ground state with one photon emitted into any mode; see the second term on the RHS of Eq. (14) as an example, when only the jth and lth atoms are concerned. Such states are ignored in Eq. (15), because they are irrelevant to the planned calculation of the probability amplitude $A(t)$. After $|T^j\rangle$ is summed over j, a result characterizing the evolution of the system from its initial state $|\psi(0)\rangle$ is obtained. Note that $||E^j\rangle\otimes|0\rangle$ is merely one component of $|\psi(0)\rangle$ and that the jth atom is not the only atom that can be in the excited state. In the following section, $A(t)$ is calculated based on $\Sigma_j c_j|T^j\rangle$.

III. DECOHERENCE OF THE SYSTEM

It is convenient to obtain the scalar product $A \equiv \langle\psi(0)|\Sigma_j c_j|T^j\rangle$ first:

$$A = (z-E_0-E_A)^{-1} + (z-E_0-E_A)^{-2} \times \sum_{l\neq j} c_l^* \sum_j \sum_\alpha e^{i\vec{k}_\alpha\cdot\vec{R}_l} \frac{|\vec{\mu}_{GE}\cdot\vec{g}_\alpha|^2}{z-E_1-\hbar\omega_\alpha} e^{-i\vec{k}_\alpha\cdot\vec{R}_j} c_j + (z-E_0-E_A)^{-3} \sum_{m\neq l} c_m^* \sum_{l\neq j} \sum_\gamma e^{i\vec{k}_\gamma\cdot\vec{R}_m} \frac{|\vec{\mu}_{GE}\cdot\vec{g}_\gamma|^2}{z-E_1-\hbar\omega_\gamma} e^{-i\vec{k}_\gamma\cdot\vec{R}_l} \times \sum_j \sum_\alpha e^{i\vec{k}_\alpha\cdot\vec{R}_l} \frac{|\vec{\mu}_{GE}\cdot\vec{g}_\alpha|^2}{z-E_1-\hbar\omega_\alpha} e^{-i\vec{k}_\alpha\cdot\vec{R}_j} c_j + \cdots. \quad (16)$$

Sums over the vacuum modes in the preceding equation can be determined by using the mode-continuum-limit approximation [3] and by averaging over the orientation of $\vec{\mu}$, which is never observed [18]. For example, it is found that the second term A_2 on the RHS of Eq. (16) is approximately equal to

$$A_2 = -B(z-E_0-E_A)^{-2} \sum_{l\neq j} \sum_j c_l^* c_j \frac{e^{i\tilde{k}_0|\vec{R}_l-\vec{R}_j|}}{|\vec{R}_l-\vec{R}_j|}, \quad (17)$$

where $\tilde{k}_0=(\omega_0-a)c^{-1}$ and $B=k_0^2|\vec{\mu}_{GE}|^2/2$, and that the third term A_3 is equal to

$$A_3 = B^2(z-E_0-E_A)^{-3} \sum_{m\neq l} \sum_{l\neq j} \sum_j \frac{e^{i\tilde{k}_0|\vec{R}_m-\vec{R}_l|}}{|\vec{R}_m-\vec{R}_l|} \frac{e^{i\tilde{k}_0|\vec{R}_l-\vec{R}_j|}}{|\vec{R}_l-\vec{R}_j|} c_m^* c_j. \quad (18)$$

Thus, the indirect atom-atom interaction through photons [see the sums over α and γ in Eq. (16)] is represented by $g_c(\vec{r})=e^{i\tilde{k}_0 r}/r$, which is nothing other than the retarded classical Green function. This result is expected, because photon propagation through an atomic ensemble follows classical laws [14]. In terms of the $g_c(\vec{r})$ function, A becomes

$$A = (z-E_0-E_A)^{-1} + (-B)(z-E_0-E_A)^{-2} \times \sum_{l\neq j} \sum_j c_l^* c_j g_c(\vec{R}_l-\vec{R}_j) + (-B)^2(z-E_0-E_A)^{-3} \times \sum_{m\neq l} \sum_{l\neq j} \sum_j c_m^* c_j g_c(\vec{R}_m-\vec{R}_l) g_c(\vec{R}_l-\vec{R}_j) + \cdots. \quad (19)$$

The serial expression of A in Eq. (19) can be further simplified by noting that the atoms are already assumed to have a uniform distribution n. Replace all the sums over atomic locations in the same equation by integrals to rewrite A approximately as

$$A \simeq (z-E_0-E_A)^{-1} + (-B)n^2(z-E_0-E_A)^{-2} \times \int d\vec{R}_l d\vec{R}_j c^*(\vec{R}_l) c(\vec{R}_j) g_c(\vec{R}_l-\vec{R}_j) + (-B)^2 n^3 (z-E_0-E_A)^{-3} \times \int d\vec{R}_m d\vec{R}_l d\vec{R}_j c^*(\vec{R}_m) c(\vec{R}_j) g_c(\vec{R}_m-\vec{R}_l) g_c(\vec{R}_l-\vec{R}_j) + \cdots. \quad (20)$$

Still, the series in Eq. (20) cannot be summed into a closed-form expression without first specifying the coefficients c_j. Two special cases are next considered.

A. Antisymmetric state

The initial state $|\psi(0)\rangle$ is first assumed to be an antisymmetric state, in which c_j are equal to $\sqrt{N^{-1}}$ but with alternative positive and negative signs:

$$|\psi(0)\rangle = \sqrt{N^{-1}}(||E^1\rangle - ||E^2\rangle + ||E^3\rangle - \cdots) \otimes |0\rangle, \quad (21)$$

which reduces to one of the well-known Bell states $\sqrt{2^{-1}}(|G^1\rangle|E^2\rangle - |E^1\rangle|G^2\rangle)\otimes|0\rangle$, when $N=2$. If it is additionally assumed that, in any macroscopically small volume element, there are roughly equal numbers of states $||E^i\rangle$ that take either $-\sqrt{N^{-1}}$ or $\sqrt{N^{-1}}$ (this assumption, along with the uniform atomic distribution, is addressed below as the uniform configuration condition), then one is capable of ignoring practically all the integrals in the serial expression of A, because $\int d\vec{R}_j c(\vec{R}_j) g_c(\vec{R}_l - \vec{R}_j) \simeq 0$. So, in Eq. (20), only the first term is left:

$$A \simeq (z - E_0 - E_A)^{-1}. \tag{22}$$

The time-dependent probability amplitude $A(t)$ for the system to stay in its initial state is related to A through the following relation [see Eq. (4)]:

$$A(t) = -\frac{1}{2\pi i}\int_{-\infty}^{\infty} dz\, e^{-itz\hbar^{-1}} A, \tag{23}$$

which, in the present case, leads to

$$A(t) = -\frac{1}{2\pi i}\int_{-\infty}^{\infty} dz \frac{e^{-itz\hbar^{-1}}}{z - E_0 - E_A} = e^{-it[(N-1)\omega_G + \omega_E - a] - tb}. \tag{24}$$

The preceding equation evidently shows that the entangled state of a multiparticle system, if restricted to the uniform configuration condition, decoheres in vacuum at a rate as if the system were a single excited atom. Physically this result can be understood by noting that, in the present situation, influences on one atom from other atoms practically cancel out, so that each atom evolves independently and has its excited-state level changed by the radiation reactions alone. In Ref. [6], when atoms are instead placed in a single-mode cavity, and when both the atoms and the cavity mode are allowed to interact with a broadband field outside, it is predicted that an antisymmetric state is a decoherence-free state.

B. Symmetric state

Alternatively, the expression in Eq. (20) also permits an analytical study when each c_j is chosen to be a positive constant $\sqrt{N^{-1}}$. In this case, the initial state $|\psi(0)\rangle \equiv \sqrt{N^{-1}}\Sigma_j \| E^j\rangle \otimes |0\rangle$ becomes fully symmetric and is often referred to as a W state [19]. It is then a straightforward matter to find that

$$\begin{aligned} A &= (z - E_0 - E_A)^{-1} + (4\pi n B\tilde{k}_0^{-2})(z - E_0 - E_A)^{-2} \\ &\quad + (4\pi n B\tilde{k}_0^{-2})^2 (z - E_0 - E_A)^{-3} + \cdots \\ &= (z - E_0 - E_A - 4\pi n B\tilde{k}_0^{-2})^{-1}. \end{aligned} \tag{25}$$

From Eq. (23), the time-dependent probability amplitude for the system to stay in the symmetric state reads

$$\begin{aligned} A(t) &= -\frac{1}{2\pi i}\int_{-\infty}^{\infty} dz \frac{e^{-izt\hbar^{-1}}}{z - E_0 - E_A - 4\pi n B\tilde{k}_0^{-2}} \\ &= e^{-it[(N-1)\omega_G + \omega_E - a + 4\pi n B\tilde{k}_0^{-2})-tb}. \end{aligned} \tag{26}$$

Unlike the antisymmetric case, if the system is in a symmetric state, the influences on one atom from the other atoms do not cancel out and, instead, cause the energy level of the atom to be further changed: the real part of E_A is additionally shifted by $4\pi n B\tilde{k}_0^{-2}$, while the imaginary part remains unchanged. This process is what Eqs. (25) and (26) imply. Thus, no matter whether the initial state of the system is symmetric or antisymmetric, the system can decohere at the same rate $2b$ under certain conditions; see Eqs. (24) and (26). Still, inside the vacuum, when $N=2$, the symmetric and antisymmetric states are also found to decohere following the same (nonexponential) law [3].

In other situations, a W state is predicted to be either a decoherence-free state [4] or a state whose decoherence state increases with N [10]. Thus, the decoherence rate of a system does not always grow linearly with the size of the system. Moreover, it is even argued that the robustness of Greenberger-Horne-Zeilinger states increases with the number of qubits [9,11].

Two points are worthy of note. First, in the present discussion, direct atom-atom interaction is ignored in H. If the dipole-dipole interaction is assumed to be the only direct interaction among the atoms, then the results obtained are valid only when n is much smaller than $\hbar\omega_0|\vec{\mu}_{GE}|^{-2}$, because under this condition the spatial separation between two neighboring atoms is large enough to prevent the dipole-dipole interaction from changing the atomic energy levels significantly. Second, although Eq. (4) is valid at any time $t>0$, the relations in Eqs. (24) and (26) are only meaningful as $t \to \infty$, because these relations are derived by considering atomic transitions aided by photons coming from atoms located anywhere in the vacuum. See Eqs. (12)–(15) for example.

IV. CONCLUSION

Decoherence of a multiparticle system in vacuum is discussed in the Schrödinger picture through calculating the time-dependent probability amplitude for the system to remain in its initial entangled state. Two initial states, one antisymmetric and one symmetric, are studied in particular, and found to decohere at the same rate when the antisymmetric state is subject to the uniform configuration condition.

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