Comment on "Alternative perspective on photonic tunneling" and "Theoretical evidence for the superluminality of evanescent waves"

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In a pair of related papers, Wang *et al.* [Phys. Rev. A **75**, 013813 (2007); **75**, 042105 (2007)] claim to present evidence, based on quantum field theory, for the superluminality of evanescent modes in undersized waveguides. Here we show that the conclusion of the authors is false and is based on an error: they mistake a nonzero propagator for a nonzero commutator. The commutator of the field operator at two points separated by a spacelike interval is strictly zero, which makes true superluminality impossible.

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In Refs. [1,2], the authors claim to provide evidence that the propagation of photons along an undersized waveguide is superluminal. They base this claim on a calculation that uses the heavy machinery of quantum field theory on a simple problem of classical electrodynamics. In this Comment, I point out that the authors' interpretation of their calculation is wrong since the system under study is completely causal and does not permit true superluminality. Contrary to their statements, the equal-time commutator between the field operators at two different points is strictly zero, which means that a measurement at one point cannot influence a measurement at the other. What they mistake for superluminal propagation is simply the behavior of an exponentially decaying standing wave in which all spatial points oscillate synchronously.

There is nothing inherently quantum mechanical about evanescent waves. They are seen in water waves [3], acoustic waves [4], waves on a string [5], and, indeed, classical electromagnetic waves [6]. Occam's razor tells us that we should always seek the simplest consistent theory that explains all the observable facts [7]. Thus it has not been found necessary to quantize water waves in order to explain the generation and tunneling of evanescent ocean waves [3]. In like manner, since the number of photons per cubic wavelength in a microwave field of even 1 V/m greatly exceeds unity, the phenomenon of evanescence of electromagnetic waves in a waveguide must be described by classical electrodynamics unless one is dealing with quantum noise or vacuum fluctuations.

Evanescence is observed when a wave tries to propagate in a region where it cannot exist with a real propagation constant. The Klein-Gordon equation is a classical model for such waves and is easily derived from Maxwell's equations. For a y-polarized transverse electric field propagating along the z direction, the wave equation is

$$\frac{\partial^2 E_y}{\partial x^2} + \frac{\partial^2 E_y}{\partial y^2} + \frac{\partial^2 E_y}{\partial z^2} - \frac{1}{c^2}\frac{\partial^2 E_y}{\partial t^2} = 0. \tag{1}$$

By writing the field as the product

$$E_y(x,z,t) = E_y(x)\phi(z,t)$$

and using separation of variables, we obtain the Klein-Gordon equation

$$\frac{\partial^2 \phi}{\partial z^2} - \frac{1}{c^2}\frac{\partial^2 \phi}{\partial t^2} = \gamma^2 \phi \tag{2}$$

and the transverse mode equation

$$\frac{d^2 E_y}{dx^2} = -\gamma^2 E_y(x). \tag{3}$$

With boundary conditions $E_y(0)=E_y(b)=0$, where b is the width of the waveguide, we find

$$E_y(x) = A\sin(\pi x/b);$$

hence the eigenvalue $\gamma=\pi/b$ for the fundamental mode. For the Klein-Gordon equation, we assume a solution of the form

$$\phi(z,t) = e^{i(kz-\omega t)},$$

which leads to the dispersion relation

$$k^2 = \omega^2/c^2 - \gamma^2.$$

If $\omega < \gamma c \equiv \omega_c$, the cutoff frequency, then k becomes imaginary and we have

$$k = \pm i\kappa = \pm i(\omega_c^2 - \omega^2)^{1/2}/c. \tag{4}$$

Below the cutoff frequency, the solutions are exponential forms,

$$\phi(z,t) = (Ae^{-\kappa z} + Be^{\kappa z})e^{-i\omega t}. \tag{5}$$

For an infinite waveguide, as considered in Refs. [1,2], we must discard the growing term, and hence the field solution is

$$\phi(z,t) = Ae^{-\kappa z}e^{-i\omega t}. \tag{6}$$

This is a pure evanescent wave, the normal mode of an infinite waveguide excited below the cutoff frequency. It is not a propagating wave but an exponentially attenuated standing wave in which all spatial points oscillate in phase. The term "propagating evanescent wave" is thus an oxymoron. A pure evanescent wave does not propagate. Its energy is largely localized within a $1/e$ distance of $l_c=1/\kappa$. The fact that all points move up and down synchronously does not imply that anything has moved with superluminal velocity to connect those points. At exactly the cutoff frequency, there is no attenuation and every point oscillates with the same amplitude. When the waveguide is excited with an impulse, it will ring at the cutoff frequency [8]. As the excitation frequency is reduced from cutoff, the attenuation constant increases, becoming a maximum as $\omega \to 0$. The localization distance reaches its minimum of $1/\gamma$. It is also seen that the stored energy is localized within a distance of $1/\gamma$, independent of the length of the evanescent region. This saturation of the stored energy has been used to explain the Hartman effect, the saturation of group delay with barrier length [8–12]. The evolution of a pure evanescent electromagnetic wave is thus seen to be a purely classical phenomenon completely explained by Maxwell's equations. It does not involve propagation and hence cannot be superluminal. These arguments directly contradict the assertion of Wang *et al.* that "the superluminal behavior of photons through an undersized waveguide is due to a purely quantum-mechanical effect…" [2]. The evolution of evanescent electromagnetic modes is neither superluminal nor a manifestation of quantum mechanics.

If the evanescent wave is a nonpropagating entity, how does it get into the waveguide in the first place? The penetration of a wave into an evanescent region has been treated in detail in Ref. [8] When a wave is first turned on, there is a transient at the front which propagates at c because it contains high frequencies that lie above the cutoff. Behind this front a standing wave builds up. The evanescent wave in an infinite medium is a steady-state solution in which all the field points oscillate synchronously.

We turn now to the evidence supplied by the authors as proof of the superluminality of evanescent modes [1,2]. They consider the same Klein-Gordon equation but treat $\phi(x)$ as a quantum field operator whose vacuum state is $|0\rangle$. A one-quantum state occurring at the space-time point $x=(t,\mathbf{x})$ is represented by $\phi(x)|0\rangle$, while one occurring at $y=(t,\mathbf{y})$ is given by $\phi(y)|0\rangle$. The authors' entire case for the superluminality of evanescent modes is based on a calculation of the quantity

$$D(x-y) = \langle 0|\phi(x)\phi(y)|0\rangle, \tag{7}$$

which represents the amplitude for finding the particle at the point x given that it was initially at point y. For definiteness the authors take $x=(t,0,0,r)$ and $y=(0,0,0,0)$, where the third space coordinate r lies along the axis of the infinitely long waveguide. For a spacelike interval, $x^2=t^2-r^2<0$ and one can always find a reference frame in which $t=0$. In other words, there is an inertial frame in which the two events at x and y occur simultaneously. Because the two events have a spacelike interval, they cannot be causally linked: one event cannot cause the other. The result of the calculation, given in standard textbooks [13,14], is that asymptotically, for a pure space interval,

$$D(x-y) \to r^{-3/2}\exp(-\gamma r). \tag{8}$$

On the basis of this result the authors make the claim that "the propagation of photons along the undersized waveguide is superluminal." This interpretation is simply wrong. All Eq. (8) is saying is that one cannot localize the photon to within a distance shorter than $1/\gamma$ [13]. The states $\phi(x)|0\rangle$ and $\phi(y)|0\rangle$ are not orthogonal, which means that they do not represent states occurring with certainty at x and *not* occurring at y for any $x \neq y$. Because of the finite spatial extent of these states, the "photon" is spread out so that it is simultaneously at $\mathbf{x}$ and at $\mathbf{y}$. It does not mean the photon has propagated superluminally from $\mathbf{y}$ to $\mathbf{x}$. The test for superluminality is whether a measurement at x can influence a measurement at y over a spacelike interval. That would require a nonvanishing commutator between the two operators.

The impossibility of superluminal propagation of photons described by the Klein-Gordon field is easily seen by considering the commutator $[\phi(x),\phi(y)]$. A textbook calculation [14] shows that

$$[\phi(x),\phi(y)] = D(x-y) - D(y-x). \tag{9}$$

For a spacelike interval $(x^2-y^2)<0$, a Lorentz transformation of the second term takes $(x-y) \to -(x-y)$ and hence the two terms in the commutator are identical and cancel to give zero:

$$[\phi(x),\phi(y)] = 0. \tag{10}$$

As stated in Ref. [14] "no measurement in the Klein-Gordon theory can affect another measurement outside the light-cone." Superluminality is impossible for fields that satisfy the Klein-Gordon equation.

In Ref. [1], further erroneous statements are made regarding causality. The claim is made that the equal-time commutation relations do not vanish for two evanescent modes. This claim is supposedly in agreement with a conclusion of Carniglia and Mandel [15]. First, the commutators indeed vanish as I have pointed out above. Second, the authors missed an important point in the work they cite. In their paper, Carniglia and Mandel note that "an element of noncausality was introduced right at the beginning of our treatment, when we chose to ignore the high-frequency behavior of the refractive index, at frequencies in the region of anomalous dispersion and beyond. For this reason, the commutator given by Eq. 84 cannot be expected to be strictly causal…." Thus their model was noncausal from the beginning, whereas the Klein-Gordon model is strictly causal. Believing (erroneously) that the commutators do not vanish for evanescent modes, Wang *et al.* then try to explain this by asserting that "because evanescent modes inside an undersized waveguide are not observable, causality is preserved" [1]. Again, this cannot be true since experimenters routinely observe evanescent modes by translating a probe along a slotted waveguide [6].

In conclusion, the results and interpretations given in the two papers [1,2] are erroneous. The evolution of evanescent waves is a purely classical effect that does not require quantum field theory for its understanding. The purported evidence for superluminality of these modes is shown to be false. A pure evanescent wave is a standing wave which cannot propagate at all, much less superluminally.

[1] Z. Y. Wang, C. D. Xiong, and B. He, Phys. Rev. A **75**, 013813 (2007).
[2] Z. Y. Wang and C. D. Xiong, Phys. Rev. A **75**, 042105 (2007).
[3] J. Thomson, S. Elgar, and T. H. C. Herbers, Geophys. Res. Lett. **32**, L10602 (2005).
[4] S. Yang, J. H. Page, Z. Liu, M. L. Cowan, C. T. Chan, and P. Sheng, Phys. Rev. Lett. **88**, 104301 (2002).
[5] F. S. Crawford, *Waves*, Berkeley Physics Course, Vol. 3 (New York, McGraw-Hill, New York, 1968).
[6] J. J. Hupert and G. Ott, Am. J. Phys. **34**, 260 (1966).
[7] http://en.wikipedia.org/wiki/Occam's_Razor
[8] H. G. Winful, IEEE J. Sel. Top. Quantum Electron. **9**, 17 (2003).
[9] H. G. Winful, Opt. Express **10**, 1491 (2002).
[10] H. G. Winful, Phys. Rev. Lett. **90**, 023901 (2003).
[11] H. G. Winful, Phys. Rev. E **68**, 016615 (2003).
[12] H. G. Winful, Phys. Rep. **436**, 1 (2006).
[13] W. Greiner and J. Reinhardt, *Quantum Electrodynamics* (Springer-Verlag, Berlin, 1992), pp. 55–60.
[14] M. E. Peskin and D. V. Schroeder, *An Introduction to Quantum Field Theory* (Addison-Wesley, New York, 1995), p. 28.
[15] C. K. Carniglia and L. Mandel, Phys. Rev. D **3**, 280 (1971).