Simulation of the superradiant quantum phase transition in the superconducting charge qubits inside a cavity

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In this paper, we propose an experimentally feasible scheme that the superconducting quantum interference devices are coupled with a high-quality cavity supporting a single-mode photon, to realize an effective Dicke model. By using several hundred artificial two-level atoms, the strong coupling regime can be successfully achieved. Moreover, in our proposal the superradiant phase transition for this model can be well controlled by the frequency of external magnetic flux when the superconducting charge qubits work at their optimal point, in which the qubits can be mostly immune from charge noise produced by uncontrollable charge fluctuations. Finally, we propose to observe this phase transition by detecting the intracavity intensity in terms of a heterodyne detector out of the cavity.

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In quantum optics the well-known Dicke model describes N two-level natural atoms interacting with a single-mode photon and has been regarded as an essential model to explore the fundamental quantum phenomena [1]. In the unit of $\hbar$, the Hamiltonian without rotating-wave approximation can be written as $H_D=\omega_0 a^\dagger a+\Sigma_{j=1}^{N}[\varepsilon\sigma_z^j+\lambda(a^\dagger+a)\sigma_x^j/\sqrt{N}]$, where a and $a^\dagger$ are the photon annihilation and creation operators with the frequency ω_0, σ_l $(l=x,z)$ is the lth component of the Pauli matrices, ε is the frequency of the splitting between the atomic levels, and λ is the atom-field interaction strength. It has been demonstrated that for the large N limit this model can exhibit a second-order phase transition at the critical point $\lambda_c=\sqrt{\omega_0\varepsilon}/2$ [2] and the corresponding critical exponent for the order parameter vanishes as $N^{-2/3}$ at this transition point [3]. As an important development in quantum information and quantum computing, this phase transition has some relations, to the quantum entanglement [4] and to the Berry phase [5], as well as to the quantum chaos [6]. Since in the traditional Dicke model realized in the natural two-level atomic ensembles, the frequencies ω_0 and ε typically exceed the atom-field coupling strength λ by many orders of magnitude, the quantum dissipation due to atomic spontaneous emission and cavity loss is usually unavoidable. Moreover, in standard current technique the atom-field coupling strength λ is difficult to be controlled. Therefore, this predicted phase transition has been never observed, which remains a challenge to provide a practical physical system to exhibit this interesting behavior [7].

In this paper we propose an experimentally feasible scheme that the superconducting quantum interference device (SQUID) are coupled with a high-quality cavity supporting a single-mode photon, to realize an effective Dicke model. It has been demonstrated in both theory and experiment that this SQUID can act as the essential role of an artificial two-level atom near the degenerate point [8]. It should be noted that the natural atom is driven by using microwave photons that excite electrons from one state to other, whereas this artificial atom is mainly manipulated by currents, voltages, and magnetic flux, which can be controlled easily in experimental setups. Recently, the SQUID inside a high-quality cavity has been recognized as a promising macroscopic device to explore various quantum phenomena, process quantum information, and implement quantum computing [9]. One of the advantages is that the cavity can effectively protect the qubits from the environment, which is important for a useful operation of qubits especially in the scaling up of the solid-state devices. The other is that an architecture using one-dimensional transmission line resonators to arrive at the strong coupling regions between the SQUID and photon field have been achieved [10].

With a proper choice of parameters for the SQUID, the realized Dicke model in our proposal can be arrived at the strong coupling regime by using several hundred artificial atoms, which has never been achieved in other quantum systems. It is also shown that the superradiant phase transition for this model can be well controlled by the frequency of external magnetic flux since in experiments, it is much easier to produce precise frequency shifts as opposed to changing the amplitude of the dc signal [11]. Moreover, in this case the SQUIDs can work at their optimal point, in which the qubits can be mostly immune from charge noise produced by uncontrollable charge fluctuations. Finally, we propose to observe this phase transition by detecting the intracavity intensity in terms of a heterodyne detector out of the cavity [12].

Figure 1(a) shows the ith SQUID constructed by two identically Josephson junctions with lower capacitance C_J^i and Josephson energy E_J^i. If we choose a material that the superconducting energy gap is larger than the single-electron charging energy, which can suppress the quasiparticle tunneling at low temperatures, the Hamiltonian for Fig. 1(a) can be written as [8]

$$H_i=4E_C^i(n_i-\bar{n}_i)^2-E_J^i(\cos\gamma_1+\cos\gamma_2), \tag{1}$$

where $E_C^i=e^2/[2(C_g^i+2C_J^i)$ is the charging energy with C_g^i being the gate capacitance, n_i is the number of the excess

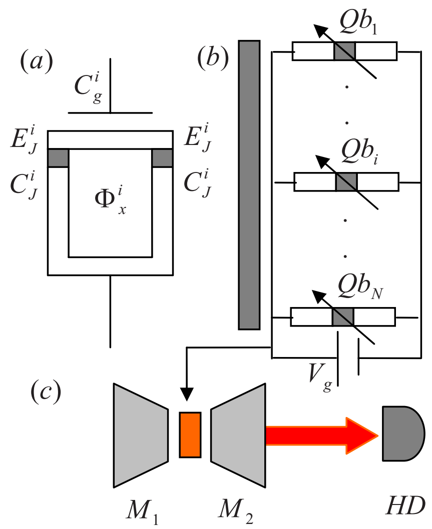

FIG. 1. (Color online) Scheme of our proposed experimental setups. (a) A qubit, (b) a series of qubits monitored by a unified gate voltage, (c) the total quantum device where the quantum network shown in (b) is loaded into the cavity formed by the reflective surfaces of mirrors (M_1, M_2). The HD out of the cavity can allow the real-time determination of intracavity intensity $I \propto |\langle a \rangle|^2$.

Cooper pair on the island, $\bar{n}_i = C_g^i V_g/(2e)$ is the dimensionless gate charge with V_g being the unified tunable gate voltage, $\gamma_m (m=1,2)$ is the gauge-invariant phase difference between points on opposite sides of the mth junction. If this SQUID locates within a single-mode cavity with frequency $\omega_0/2\pi$ shown in Fig. 1(b), the gauge-invariant phase difference γ_m satisfies the condition such that $\gamma_1 + \gamma_2 = 2\theta$ and $\gamma_1 - \gamma_2 = 2\pi\Phi_x^i/\Phi_0 + 2g(a+a^\dagger)$, where θ conjugates to the Cooper pair number n_i, a, and $a^\dagger$ are the annihilation and creation operators for the single-mode photon, Φ_x^i is the adjustable external magnetic flux, and $\Phi_0 = \pi\hbar/e$ is the flux quantum. In the Coulomb gauge, the coupling strength $g = e\hat{\kappa}\cdot\hat{\mathbf{l}}/\sqrt{2\kappa\omega_0 V\hbar}$, where $\hat{\kappa}$ is the unit polarization vector of the cavity mode, l is the thickness of the insulating layer in the junction, and V is the volume of the cavity, has a range from 10^{-5} to 10^{-2} for the typical parameters. Generally speaking, the effective flux $g\langle a^\dagger + a\rangle$ induced by the cavity mode is much less than $\tilde{\Phi}_i = \pi\Phi_x^i/\Phi_0$ and therefore the Lamb-Dicke limit $(g\sqrt{\langle a^\dagger a\rangle + 1} \ll 1)$ can be well satisfied.

When the charging energy E_C^i is much larger than the Josephson energy E_J^i, the relevant physics for the ith SQUID is captured by taking into account only two charge eigenstates $n_i = 0,1$, which constitute the basic vectors $\{|0\rangle, |1\rangle\}$ of the computational Hilbert space of the qubit. Hence, the whole system can be similar to an artificial two-level atom interacting with the quantum harmonic oscillator, and the corresponding Hamiltonian for the ith qubit can be written as $H_i = \omega_0 a^\dagger a + \varepsilon_i \sigma_z^i - E_J^i \cos[\tilde{\Phi}_i + g(a^\dagger + a)]\sigma_x^i$, where $\varepsilon_i = 2E_C^i(2\bar{n}_i - 1)$ and the Pauli matrices are defined as $\sigma_z^i = |0\rangle^i\,{}^i\langle 0| - |1\rangle^i\,{}^i\langle 1|$ and $\sigma_x^i = |0\rangle^i\,{}^i\langle 1| + |1\rangle^i\,{}^i\langle 0|$.

If we consider the identical SQUIDs with the same external magnetic flux and do not introduce the interaction between the ith and jth qubits, the Hamiltonian for Fig. 1(b) is given by $H = \omega_0 a^\dagger a + \varepsilon J_z - E_J \cos[\tilde{\Phi} + g(a^\dagger + a)]J_x$, where $J_l = \Sigma_{i=1}^N \sigma_l^i$ with $j = N/2$ are the collective artificial atomic operators satisfying SU(2) commutation relations. In order to show our scheme, we set $\pi\Phi_x/\Phi_0 = \omega t$, $\varepsilon = 0$, to the rewritten collective Hamiltonian as

$$H(t) = \omega_0 a^\dagger a - \frac{E_J}{2}(e^{i[\omega t + g(a^\dagger + a)]} + \text{H.c.})J_x. \tag{2}$$

Expanding the Hamiltonian (2) to the first order of g in the Lamb-Dicke limit, we have $H(t) = \omega_0 a^\dagger a - \frac{iE_J g}{2}(e^{i\omega t} - e^{-i\omega t})(a^\dagger + a)J_x$ [13]. Since a unitary transformation does not change the eigenvalues of the system, in the rotating reference frame through a unitary transformation $R(t) = e^{-i\omega t J_z}$ and under rotating-wave approximation, the Hamiltonian is equivalently transferred to an effective time-independent Hamiltonian $H_e = R^\dagger(t)H(t)R(t) - iR^\dagger(t)dR(t)/dt = \omega_0 a^\dagger a - \omega J_z - \frac{E_J g}{2}(a^\dagger + a)J_y$. Finally, in terms of a proper coordinate transformation, an effective Dicke-like Hamiltonian can be obtained by

$$H_e = \omega_0 a^\dagger a + \omega J_z + \frac{\lambda}{\sqrt{N}}(a^\dagger + a)J_x, \tag{3}$$

where $\lambda = gE_J\sqrt{N}/2$. It can be seen clearly that the Hamiltonian (3) is identical to that of the standard Dicke model in quantum optics. However, in our proposal the frequency ω can be well controlled since in experiments, it is much easier to produce precise frequency shifts as opposed to changing the amplitude of the dc signal.

For the current experimental parameters with $C_J = 79$ aF, $C_g = 0.50$ aF, $V_g = 0.325$ V, and $E_J \approx 100$ μeV, the charging energy E_C can be evaluated by $E_C \approx 1000$ μeV. For the strong interaction between the SQUID and cavity, the coupling strength g is considered by $g \approx 10^{-2}$. For typical cavity with the length of the order $\sim$cm and the SQUID with the loop dimension of the order $\sim\mu$m, the number of the SQUID is of the order $\sim 10^3$. If the number of the artificial atom is chosen as $N = 100$, the atom-field coupling strength can be obtained by $\lambda = gE_J\sqrt{N}/2 = 5$ μeV. Comparing with the photonic energy $\hbar\omega_0 \approx 30$ μeV [14], it is seen clearly that the strong coupling regime can be successfully achieved. Therefore, in our scheme, the quantum dissipation arising from atomic spontaneous emission and cavity loss can be neglectable. Moreover, the Lamb-Dicke and strong coupling limits can be satisfied simultaneously.

Following the procedure of Ref. [6] the ground-state properties for the Hamiltonian (3) can be evaluated by means of Holstein-Primakoff transformation [15], which is defined as $J_+ = b^\dagger\sqrt{N - b^\dagger b}$, $J_- = \sqrt{N - b^\dagger b}\,b$ and $J_z = (b^\dagger b - N/2)$ with $[b, b^\dagger] = 1$, and boson expansion method [16]. However, in order to describe the collective behavior for the Hamiltonian (3) we should introduce shifting boson operators $c^\dagger$ and $d^\dagger$ with properly scaled auxiliary parameters α and β such as $c^\dagger = a^\dagger + \sqrt{N}\alpha$ and $d^\dagger = b^\dagger - \sqrt{N}\beta$. Thus, we have

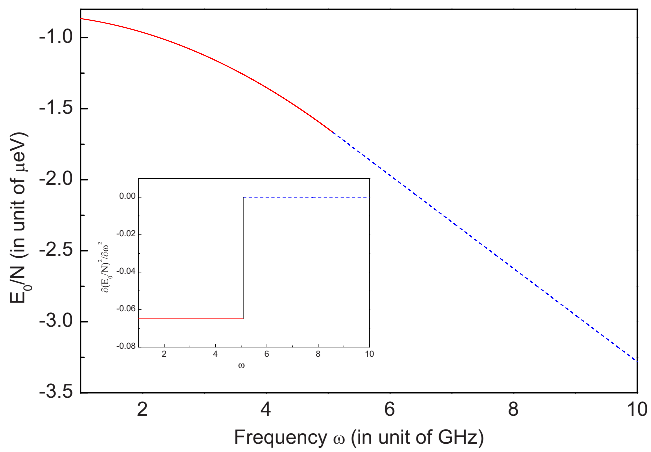

FIG. 2. (Color online) The scaled ground-state energy E_0/N versus the frequency ω with the parameters $E_C \approx 1000$ μeV (C_J=80 aF, C_g=0.50 aF), $E_J \approx 100$ μeV, $g \approx 10^{-2}$, N=100, and $\hbar\omega_0 \approx 30$ μeV. The critical frequency is evaluated by ω_c=5.08 GHz. Inset: the second-order derivative of E_0/N with respect to ω versus ω.

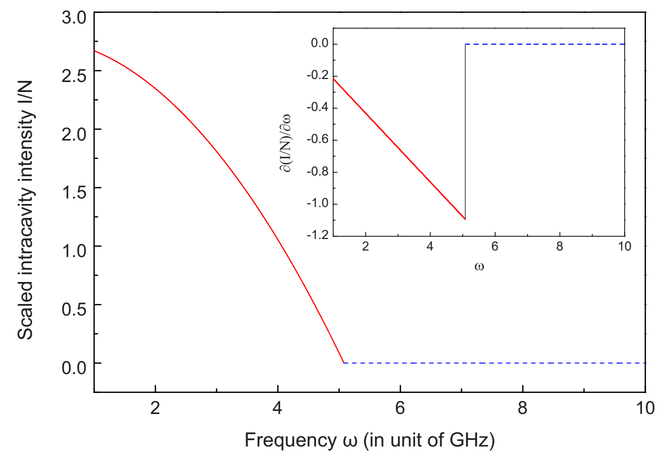

FIG. 3. (Color online) The scaled intracavity intensity I/N versus the frequency ω with the same parameters as those in Fig. 2. Inset: The first-order derivative of I/N with respect to ω versus ω.

$$
\begin{aligned}
H_e &= NH_0 + N^{1/2}H_1 + \cdots \\
&= N[\omega_0\alpha^2 + \omega(\beta^2 - 1/2) - 4\tilde{\lambda}\alpha\beta\sqrt{k}] \\
&\quad + N^{1/2}\{(-\omega_0\alpha + 2\tilde{\lambda}\beta\sqrt{k})(c^\dagger + c) \\
&\quad + [\omega\beta - 2\tilde{\lambda}\alpha(\sqrt{k} - \beta^2/\sqrt{k})](d^\dagger + d)\} + \cdots,
\end{aligned}
$$

where $k=1-\beta^2$.

The critical frequency of external magnetic flux can be derived from $N^{1/2}H_1=0$ by [6]

$$\omega_c = \frac{Ng^2E_J^2}{\omega_0}. \tag{4}$$

With the above-mentioned parameters, this critical frequency is evaluated by ω_c=5.08 GHz, which can be achieved experimentally. For $\omega < \omega_c$, the effective potential $U[=\lambda(a^\dagger + a)J_x/\sqrt{N}]$ plays the essential role for the whole Hamiltonian H_e and the macroscopic excitations can occur. The corresponding phase is called the superradiant phase. With the increasing of ω, the contribution of this effective potential U is cut back. When ω exceeds ω_c, the rest Hamiltonian $H_0(=\omega_0 a^\dagger a + \omega J_z)$ dominates in the Hamiltonian H_e and the system is only microscopically excited. The corresponding phase becomes the normal phase. The ground state energies for the superradiant and normal phases are given by $E_0 = -N[\delta\omega/2 + \lambda^2(1-\delta^2)/\omega_0]$ with $\delta = \omega\omega_0/Ng^2E_J^2$ and $E_0 = -N\omega/2$, respectively. Figure 2 shows the scaled ground state energy E_0/N and its second-order derivative with respect to ω as a function of ω, which clearly illustrates the nature of the second-order phase transition. The corresponding auxiliary parameters α and β are given by $\alpha = \lambda\sqrt{1-\delta^2}/\omega_0$ and $\beta = \sqrt{(1-\delta)/2}$ and $\alpha=\beta=0$, respectively.

In the rest of this paper we give a scheme of how to observe this superradiant phase transition in the strong coupling regime. Figure 1(c) shows the total quantum device where the SQUID's quantum network is loaded into an optical cavity formed by the reflective surfaces of mirrors (M_1, M_2). It should be noted that in such quantum system that the many-body quantum pseudospin state is not accessible to observe, here we propose to detect the direct and striking signatures of the photon field by a heterodyne detector (HD) out of the cavity. In the conventional quantum optics the well-defined intracavity intensity $I \propto |\langle a \rangle|^2$ can be conveniently observed by means of the quantum nondemolition measurement [12]. In terms of the inverse transformation $a^\dagger = c^\dagger - \sqrt{N}\alpha$ and the ground state properties, the scaled intracavity intensity I/N can be evaluated by

$$\frac{I}{N} \propto \begin{cases} N\lambda^2(1-\delta^2)/\omega_0^2, & \omega < \omega_c, \\ 0, & \omega > \omega_c. \end{cases} \tag{5}$$

Figure 3 shows the scaled intracavity intensity I/N and its first-order derivative with respect to ω as a function of ω. In the normal phase the scaled intracavity intensity I/N vanishes to zero whereas it depends on the relative parameters of both the SQUID and the cavity in the superradiant phase, which also illustrates a macroscopic collective excitation phenomenon. It is interesting that this superradiant phase transition characterized by the nonanalyticity of the scaled intracavity intensity I/N is remarkably of the first order. When the frequency ω approaches the critical value ω_c, the scaled intracavity intensity I/N vanishes as $(I/N) \propto |\omega - \omega_c|$. Since the diverging characteristic length scale is $\zeta \sim |\omega - \omega_c|^{-v}$ with $v=1/2$, the critical exponent for the scaled intracavity intensity I/N can be derived from $I/N \propto |\omega - \omega_c|^{zv}$ by $z=2$, which shows the universality principle of quantum phase transition [17].

Comparing with the existing scheme to observe the superradiant phase transition, our proposed scheme processes likely the following advantages. (1) The SQUID and the high-quality cavity can be highly fabricated and the SQUID's quantum network can be scaled to many qubits. (2) The superradiant phase transition can be well controlled by the frequency of external magnetic flux since in experiments, it is much easier to produce precise frequency shifts as opposed to changing the amplitude of the dc signal. (3) The SQUIDs can work at their optimal point ($\varepsilon=0$), in which the qubits can be mostly immune from charge noise produced by uncontrollable charge fluctuations. (4) The intracavity intensity $I \propto |\langle a \rangle|^2$, which is remarkably of the first order, is a good witness to observe this superradiant phase transition in the current experimental setups.

In conclusion, we have proposed an experimentally feasible scheme to realize an effective Dicke model with the strong coupling regime. The interesting collective behavior of this model, the superradiant phase transition, which has never been observed, has been also suggested to be realized. Before ending this paper, we should make one remark. The no-go theory tell us that this phase transition cannot occur if in the atomic system three conditions including minimal coupling, classical radiation field, and dipole approximation are satisfied simultaneously [18]. However, in our proposal the quantized field (or photon) is considered. Moreover, the corresponding Hamiltonian is described by the system of spins interacting with radiation. Therefore, we argue that this proposed solid-state system is a perfect system to study the superradiant phase transition, which awaits the experimental validation.

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