Enhancement of entanglement for two-mode fields generated from four-wave mixing with the help of the auxiliary atomic transition

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(Received 4 May 2007; published 20 November 2007)

The entanglement properties of two mode fields generated from four-wave mixing are discussed in the system of an ensemble of *V*-type three-level atoms embedded in a two-mode cavity, in which the atomic transitions from the two excited states to the ground state are driven by one strong and one relative weak laser field, respectively. The nondegenerate four-wave mixing process occurs in the strong laser-driven transition, which is also coupled by the two cavity modes. With the help of the auxiliary atomic transition driven by the weak laser field and adjusting the frequency difference of the two modes, highly squeezed and entangled light with high intensity can be generated.

DOI: 10.1103/PhysRevA.76.053827 PACS number(s): 42.50.Dv, 42.50.Gy

I. INTRODUCTION

How to generate Gaussian-entangled light is one of the energetic research fields in quantum optics, because it is widely applied in continuous variable (CV) quantum information processing such as quantum teleportation [1], quantum telecloning [2], and quantum dense coding [3]. It is well known that the two-mode squeezed electromagnetic state is capable of exhibiting Einstein-Podolsky-Rosen (EPR) correlations [4] and now becomes a basic resource of CV quantum information processing [5]. The optical nondegenerate parametric down-conversion (NPD) which has long been envisaged as a source of two-mode squeezed states is proved to be one of the effective ways to produce two-mode entanglement. The experimental observation of the two-mode entanglement produced from NPD with nonlinear optical processes and linear elements has been reported in Refs. [6]. In order to improve the strength of the NPD, engineering the NPD Hamiltonian within cavity QED has also attracted much attention [7]. For example, Prado *et al.* recently showed how to construct the NPD Hamiltonian through the interaction of a single driven two-level atom with two cavity modes [8]. Guzman *et al.* proposed to generate such a NPD Hamiltonian with a Λ-type three-level atomic ensemble in a two-mode cavity [9].

Besides NPD, it is proved theoretically and experimentally that nondegenerate four-wave mixing (NFWM) is another effective way for the generation of the two-mode squeezed field [10–12]. Comparing to the broadband paired photons (squeezed and entangled) from NPD, the narrowband entangled beams can be produced via NFWM, which may have potential applications in long-distance communication [13]. It is well known that four-wave mixing (FWM) in a strongly driven two-level system can result in four-wave parametric interactions to produce correlated photons [14,15]. However, it is experimentally difficult to obtain a large amount of squeezing and high generation rate by use of the FWM in atomic vapors [12,16]. This is because in the strongly driven two-level system, there are two FWM channels which destructively contribute to the third-order nonlinear susceptibility $\chi^{(3)}$, proportional to the product of the population difference between the two dressed states and $1/\sqrt{1+(\Delta_3/2\Omega_3)^2}$, where Δ_3 is the frequency detuning between the driving laser field and the two-level atom and Ω_3 the corresponding Rabi frequency [17]. Therefore, only when the two-level atom ensemble is interacted off-resonantly with the strong driving laser can the correlated photons be obtained with low generation rate and with a small amount of squeezing [18], as proved experimentally by Slusher *et al.* [19]. In the limit of a large pumping detuning Δ_3, the third-order nonlinear susceptibility $\chi^{(3)}$ is proportional to $1/\Delta_3^3$. In order to increase the paired-photon generation rate, Du *et al.* [12] recently proposed and proved a new scheme by using a single retroreflected pump beam. In this scheme, the two FWM processes occur with different time ordering so that their associated third-order nonlinear susceptibility is proportional to $1/\Delta_3^2$. Here, we focus on the generation of bright two-mode entangled fields in the NFWM system. We present a different scheme to increase $\chi^{(3)}$ in the NFWM system with the help of an auxiliary atomic transition. Our idea is that by use of the auxiliary atomic transition, the atom is selectively trapped into one of the two dressed states which are related to the NFWM processes, so that one of the two NFWM channels is closed and only one NFWM channel survives. Consequently the pumping field for the NFWM can be resonant to the relevant atomic transition which can greatly increase the third-order nonlinear susceptibility. Thus the two-mode squeezing and the entanglement can be enhanced significantly as compared with those obtained in the two-level NFWM system.

In this paper, we consider an ensemble of *V*-type three-level atoms embedded in a two-mode cavity, in which the two excited states are coupled to the ground state by one strong and one relative weak laser field, respectively. Meanwhile the strong laser-driven transition is also coupled by the two cavity modes, so that the NFWM process occurs. By adjusting the frequency of the laser field which drives the atomic transition between the another excited state and the ground state, the steady dressed-state population difference

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between the two dressed states which are relevant to the NFWM processes can be approximately achieved to unity even when the atom-pumping interactions are in resonance. This means that one of two NFWM channels, which interfere each other destructively, is closed. Only one channel is open for the NFWM processes so that no destructive interference happen and the third-order nonlinear susceptibility can be thus increased. This causes the degree of the squeezing and entanglement of the two-mode cavity field to be improved greatly. Meanwhile, through adjusting the frequency difference of the two cavity modes, we can effectively eliminate the ac Stark shifts, which causes the total photon number of the two-mode cavity field to be large. The intracavity entanglement in our modified NFWM system can exceed the entanglement limit of an intracavity NPD at its threshold. This paper is organized as follows: In Sec. II, the model is introduced and the master equation of the cavity field is derived. The variance matrix of the Wigner characteristic function of the two-mode cavity field is given in Sec. III. Section IV is devoted to a discussion of the two-mode squeezing and entanglement of the cavity field. In the last section, we give a summary.

II. MASTER EQUATION OF THE TWO-CAVITY FIELD

We consider an ensemble of V-type three-level atoms with two excited states $|3\rangle$ and $|1\rangle$ and one ground state $|2\rangle$ [see Fig. 1(a)] embedded in a two-mode optical cavity. The dipole-allowed transition $|3\rangle \leftrightarrow |2\rangle$ is pumped by a strong laser field 3 with frequency ν_3 and Rabi frequency $2\Omega_3$. This transition is also coupled to two quantified cavity modes denoted by annihilation operators a_1 and a_3 with frequencies ω_1 and ω_3. The NFWM process occurs in this two-level driven transition. An auxiliary transition $|1\rangle \leftrightarrow |2\rangle$ is driven by a relative weak laser field 1 with frequency ν_1 and Rabi frequency $2\Omega_1 \ll 2\Omega_3$.The interaction Hamiltonian of the system is given by

$$V_{int} = V_1 + V_2, \tag{1}$$

where

$$V_1 = \Delta_1\sigma_{11} + \Delta_3\sigma_{33} - \Omega_1(\sigma_{12} + \sigma_{21}) - \Omega_3(\sigma_{23} + \sigma_{32}), \tag{2}$$

$$V_2 = (g_1 a_1 e^{i\delta_1 t} + g_3 a_3 e^{-i\delta_3 t})\sigma_{32} + \text{H.c.} \tag{3}$$

Here σ_{mn} $(m,n=1,2,3)$ are the atomic population operators for $m=n$ and transition operators for $m \neq n$. The detunings $\Delta_l = \omega_{l2} - \nu_l$ $(l=1,3)$, $\delta_1 = \nu_3 - \omega_1$, and $\delta_3 = \omega_3 - \nu_3$, where ω_{l2} being the atomic transition frequencies from the excited states $|l\rangle$ to ground state $|2\rangle$. g_l are the coupling constants between the cavity fields and the atoms. The density operator ρ of the atom-field system is governed by the following master equation:

$$\frac{d}{dt}\rho = -i[V_{int},\rho] + \hat{L}_a\rho + \hat{L}_f\rho, \tag{4}$$

with

$$\hat{L}_a\rho = \sum_{l=1,3} \gamma_l(2\sigma_{2l}\rho\sigma_{l2} - \sigma_{l2}\sigma_{2l}\rho - \rho\sigma_{l2}\sigma_{2l}),$$

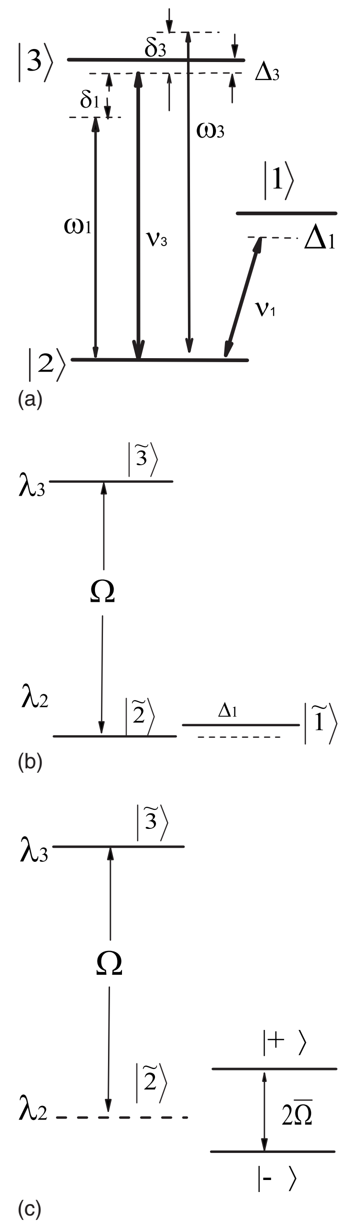

FIG. 1. (a) The atomic configuration. The two-level transitions $|2\rangle$-$|3\rangle$ are coupled to a strong pumping field with Rabi frequency $2\Omega_3$ and two quantum fields with frequencies ω_1 and ω_3, respectively. The auxiliary transition $|2\rangle$-$|1\rangle$ is driven by a weak pumping field with Rabi frequency $2\Omega_1$. (b) The configuration of the dressed levels in the representation of the strong driven atomic transition. (c) The configuration of the doubly dressed levels.

$$\hat{L}_f\rho_{af} = \sum_{l=1,3} \kappa_l(2a_l\rho a_l^\dagger - a_l^\dagger a_l\rho - \rho a_l^\dagger a_l), \tag{5}$$

where γ_l are the damping rates of the atomic excited levels $|l\rangle$ to the ground state and κ_l represent the dissipation rates of the cavity modes. By tracing out the atomic variables, one can get the dynamics of the cavity field:

$$\frac{d}{dt}\rho_c = \frac{d}{dt}\sum_{l=1,2,3} \langle l|\rho|l\rangle. \tag{6}$$

For convenience, we will derive the master equation (6) in the picture of the dressed states which are the eigenstates of the Hamiltonian $H_{a3}=\Delta_3\sigma_{33}-\Omega_3(\sigma_{23}+\sigma_{32})$. The pair of the eigenstates are

$$|\tilde{2}\rangle = \cos\phi|2\rangle + \sin\phi|3\rangle, \quad |\tilde{3}\rangle = \sin\phi|2\rangle - \cos\phi|3\rangle, \quad (7)$$

with

$$\cos\phi = \sqrt{\frac{1}{2}+\frac{\delta}{2\sqrt{\delta^2+1}}}, \quad \sin\phi = \sqrt{\frac{1}{2}-\frac{\delta}{2\sqrt{\delta^2+1}}}, \quad (8)$$

and $\delta=\Delta_3/2\Omega_3$. The dressed states $|\tilde{2}\rangle$ and $|\tilde{3}\rangle$ have the corresponding eigenvalues $\lambda_{2,3}=(\Delta_3\mp\Omega)/2$ with $\Omega=\sqrt{\Delta_3^2+4\Omega_3^2}$. The bare state $|1\rangle$ remains unchanged, and it is denoted by $|\tilde{1}\rangle$. In the dressed-state representation, the master equation (6) reduces to

$$\frac{d}{dt}\rho_c = -i[V_{\tilde{2}\tilde{2}},\rho_{\tilde{2}\tilde{2}}-\rho_{\tilde{3}\tilde{3}}] - i[V_{\tilde{2}\tilde{3}},\rho_{\tilde{3}\tilde{2}}] - i[V_{\tilde{3}\tilde{2}},\rho_{\tilde{2}\tilde{3}}] + \hat{L}_f\rho_c, \quad (9)$$

where

$$V_{\tilde{2}\tilde{2}} = g\sin\phi\cos\phi(a_1e^{i\delta_1 t}+a_3e^{-i\delta_3 t}+a_1^\dagger e^{-i\delta_1 t}+a_3^\dagger e^{i\delta_3 t}),$$

$$V_{\tilde{2}\tilde{3}} = g\sin^2\phi(a_1e^{i\delta_1 t}+a_3e^{-i\delta_3 t}) - g\cos^2\phi(a_1^\dagger e^{-i\delta_1 t}+a_3^\dagger e^{i\delta_3 t}). \quad (10)$$

Here we have taken $g_1=g_3=g$ for simplicity. We assume that the driving field is very strong so that $|\lambda_2-\lambda_3|\gg\gamma_1,\gamma_3$; therefore, the secular approximation can be adopted [20]. Without loss of generality, we assume that the state $|\tilde{1}\rangle$ is far detuned with the dressed state $|\tilde{3}\rangle$ but may be resonant with $|\tilde{2}\rangle$ as shown in Fig. 1(b). Following the standard procedure in the theory of the NFWM [14,15], under the condition of $|\delta_l|\gg|\delta_1-\delta_3|$, we can obtain the following master equation governing the dynamics of the two-mode cavity field as

$$\begin{aligned}\frac{d}{dt}\rho_c = &-\sum_{j=1,3}A_j(\rho_c a_j a_j^\dagger - a_j^\dagger\rho_c a_j) - \sum_{j=1,3}(B_j+\kappa_j)(a_j^\dagger a_j\rho_c \\ &- a_j\rho_c a_j^\dagger) + C_1(a_1^\dagger a_3^\dagger\rho_c - a_3^\dagger\rho_c a_1^\dagger) + D_1(\rho_c a_3^\dagger a_1^\dagger \\ &- a_1^\dagger\rho_c a_3^\dagger) + C_3(a_3^\dagger a_1^\dagger\rho_c - a_1^\dagger\rho_c a_3^\dagger) + D_3(\rho_c a_1^\dagger a_3^\dagger \\ &- a_3^\dagger\rho_c a_1^\dagger) - i\delta_{13}(a_1^\dagger a_1\rho_c + a_3^\dagger a_3\rho_c) + \text{H.c.},\end{aligned} \quad (11)$$

where $\delta_{13}=(\delta_1-\delta_3)/2$ and

$$A_1 = 4Ng^2[\sin^4\phi R(-\delta_1)+\cos^4\phi T(-\delta_1)]/D(-\delta_1) - Ng^2\sin^2\phi\cos^2\phi F^*(-\delta_1),$$

$$B_1 = 4Ng^2[\cos^4\phi R^*(\delta_1)+\sin^4\phi T(-\delta_1)]/D(-\delta_1) - Ng^2\sin^2\phi\cos^2\phi F(\delta_1),$$

$$C_1 = 4Ng^2\sin^2\phi\cos^2\phi\{[R^*(-\delta_1)+T^*(-\delta_1)]/D^*(-\delta_1)+F(-\delta_1)\},$$

$$D_1 = 4Ng^2\sin^2\phi\cos^2\phi\{[R(\delta_1)+T^*(-\delta_1)]/D^*(-\delta_1) + F^*(\delta_1)\}. \quad (12)$$

The parameters A_3, B_3, C_3, and D_3 have the same expressions as A_1, B_1, C_1, and D_1 by the replacement of δ_1 with $-\delta_3$. The parameter N is the total number of atoms, and the four functions $D(x)$, $R(x)$, $T(x)$, and $F(x)$ are defined as

$$D(x) = [b_1+b_2-2i(x+\bar{\Omega})][b_1+b_2-2i(x-\bar{\Omega})],$$

$$R(x) = b_1\rho^0_{\tilde{2}\tilde{2}} + i(\Omega_1\cos\phi\,\rho^0_{1\tilde{2}} - x\rho^0_{\tilde{2}\tilde{2}}),$$

$$T(x) = \rho^0_{\tilde{3}\tilde{3}}(b_1-ix),$$

$$F(x) = -\rho^0_{\tilde{2}\tilde{2}}f_{21}(x) - (\rho^0_{\tilde{2}\tilde{2}}+\rho^0_{\tilde{3}\tilde{3}})f_{22}(x) + i\rho^0_{\tilde{2}\tilde{1}}f_{23}(x) - \rho^0_{\tilde{2}\tilde{1}}f_{24}(x), \quad (13)$$

where

$$f_{21}(x) = 2\mu_3^2(\mu_6+ix) + \mu_4[(\mu_6+ix)^2+(\Delta_1-\lambda_2)^2]/U(x),$$

$$f_{22}(x) = 4\mu_3^2(\mu_6+ix) + (\mu_1+ix)[(\mu_6+ix)^2+(\Delta_1-\lambda_2)^2]/U(x),$$

$$f_{23}(x) = -(\mu_6+ix)(\mu_1\mu_3-2\mu_3\mu_4+i\mu_3x)/U(x),$$

$$f_{24}(x) = (\Delta_1-\lambda_2)(\mu_1\mu_3-2\mu_3\mu_4+i\mu_3x)/U(x),$$

$$U(x) = [(\Delta_1-\lambda_2)^2+(\mu_6+ix)^2][(x-i\mu_1)(x-i\mu_5)+\mu_2\mu_4] + 2\mu_3^2(\mu_6+ix)(2\mu_5-\mu_2+2ix),$$

$$\mu_1 = 2[2\gamma_1(1+\cos^2\phi)-\gamma_3\cos(2\phi)]/3,$$

$$\mu_2 = 2[-\gamma_1(1+\cos^2\phi)+\gamma_3(\sin^4\phi+2)]/3,$$

$$\mu_3 = -\Omega_1\cos\phi, \quad \mu_4 = 4(\gamma_1-\gamma_3)\cos(2\phi)/3,$$

$$\mu_5 = 2[-\gamma_1\cos(2\phi)+2\gamma_3(\sin^4\phi+2\cos^4\phi)]/3,$$

$$\mu_6 = \gamma_1+\gamma_3[4\sin^4\phi+\gamma_3\sin^2(2\phi)]/4,$$

$$b_1 = \mu_6 + i(\Delta_1-\lambda_3),$$

$$b_2 = \gamma_3[\sin^4\phi+\cos^4\phi+\sin^2(2\phi)] + i(\lambda_2-\lambda_3), \quad (14)$$

in which $\bar{\Omega}=\sqrt{\Omega_1\cos^2\phi-(b_1-b_2)^2/4}$ and $\rho^0_{\tilde{2}\tilde{2}}$, $\rho^0_{\tilde{3}\tilde{3}}$, and $\rho^0_{1\tilde{2}}$ are the steady solutions of the elements of atomic density operator in the dressed-state picture in the absence of the interaction between the atom and the quantized fields (i.e., $g=0$), which can be easily obatined [21–24]. The master equation (11) can be solved by use of the characteristic function method in the Wigner representation [20].

III. WIGNER CHARACTERISTIC FUNCTION

Assuming the cavity modes initially in vacuum, the state of cavity field governed by Eq. (11) in phase space should be a two-mode Gaussian state since the master equation (11) only contains the quadratic terms of the bosonic operators a_j and $a_j^\dagger$ $(j=1,3)$. For a two-mode Gaussian state, the quantum statistics properties of the two-mode field are completely determined by the covariance matrix of its Wigner characteristic function which is defined as [5]

$$M_{ij} = \mathrm{Tr}[\rho(\Delta\hat{\xi}_l\Delta\hat{\xi}_{l'} + \Delta\hat{\xi}_{l'}\Delta\hat{\xi}_l)/2] = \langle(\hat{\xi}_l\hat{\xi}_{l'} + \hat{\xi}_{l'}\hat{\xi}_l)/2\rangle, \tag{15}$$

where $\hat{\xi}=(\hat{X}_1,\hat{Y}_1,\hat{X}_3,\hat{Y}_3)$. The quadrature operators are defined as $\hat{X}_l=(a_le^{-i\theta_l}+a_l^\dagger e^{i\theta_l})/\sqrt{2}$ and $\hat{Y}_l=-i(a_le^{-i\theta_l}-a_l^\dagger e^{i\theta_l})/\sqrt{2}$ with θ_l being the phase angles of the modes. By applying local phase rotations to eliminate the dependence of θ_l, which do not change the entanglement and the maximal two-mode squeezing of the two-mode field, the covariance matrix of the cavity field can be obtained as

$$M = \begin{pmatrix} n & 0 & c & 0 \\ 0 & n & 0 & -c \\ c & 0 & m & 0 \\ 0 & -c & 0 & m \end{pmatrix}, \tag{16}$$

where $n=\langle a_1^\dagger a_1\rangle+1/2$, $m=\langle a_3^\dagger a_3\rangle+1/2$, and $c=|\langle a_1a_3\rangle|$. From the master equation (11) we have[26]

$$\frac{d}{dt}\langle a_1^\dagger a_1\rangle = \frac{1}{2}(x_1+x_1^*)\langle a_1^\dagger a_1\rangle + y_1\langle a_1^\dagger a_3^\dagger\rangle + e_1/2 + \text{c.c},$$

$$\frac{d}{dt}\langle a_3^\dagger a_3\rangle = \frac{1}{2}(x_3+x_3^*)\langle a_3^\dagger a_3\rangle + y_3\langle a_1^\dagger a_3^\dagger\rangle + e_3/2 + \text{c.c.},$$

$$\frac{d}{dt}\langle a_1a_3\rangle = y_3\langle a_1^\dagger a_1\rangle + y_1\langle a_3^\dagger a_3\rangle + (x_1+x_3)\langle a_1a_3\rangle + e_2, \tag{17}$$

where $x_l=A_l-B_l-\kappa_l-i\delta_{13}$, $y_l=C_l-D_l$, $e_l=A_l+A_l^*$ $(l=1,3)$, and $e_2=C_1+C_3$. Since the expressions of the time-dependent solutions of the above equations are too lengthy, here we only present the steady-state solution. It is easily found that when the system obeys the condition $\mathrm{Re}[x_1+x_2+\sqrt{(x_1-x_3^*)^2+4y_1y_3^*}]<0$, the system can approach its steady state in the long-time limit. The steady-state solutions of the above equations are obtained as

$$\langle a_1^\dagger a_1\rangle D = -e_1x_3|x_1+x_3|^2 + (x_1+x_3)[y_1e_2^*(x_3+x_3^*) + y_1^*(y_3e_1 - y_1e_3)] + y_1y_3^*(y_1^*e_2 - y_1e_2^*) + \text{c.c.}, \tag{18}$$

$$\langle a_1a_3\rangle D = (x_3+x_3^*)\{(x_1^*+x_3^*)[(x_1+x_1^*)/2 - y_1e_3] + y_1(y_3e_2^* - y_3^*e_2)\} - y_1e_3(y_1y_3^* - y_1^*y_3) + [1\leftrightarrow 3], \tag{19}$$

where the denominator D is

$$D = \{(x_1+x_1^*)(x_1+x_3)[(x_3+x_3^*)(x_1^*+x_3^*)/4 - y_1y_3^*] + y_1^*y_3(y_1^*y_3 - y_1y_3^*)/2 + [1\leftrightarrow 3]\} + \text{c.c.}$$

Here the quantity $\langle a_3^\dagger a_3\rangle$ is given by Eq. (18) with 1 interchanged with 3.

After obtaining the characteristic function of the system, we can discuss the squeezing and entanglement of the two-mode cavity field.

IV. ENTANGLEMENT PROPERTY OF THE SYSTEM

The separability of a two-mode Gaussian state is well established by Duan *et al.* [25]. Defining the operators $\hat{u}=a\hat{X}_1-\frac{1}{a}\hat{X}_2$ and $\hat{v}=a\hat{Y}_1+\frac{1}{a}\hat{Y}_2$ and the sum of the variance $\Sigma=\langle(\Delta\hat{u})^2\rangle+\langle(\Delta\hat{v})^2\rangle$, we can find

$$\Sigma = 2na^2 + 2m/a^2 - 4|c|, \tag{20}$$

where a is a state-dependent real number. In fact, if we choose $a=1$, then the sum of the variances reduces to the variance characterizing the normal two-mode squeezing:

$$V = \langle(\hat{X}_1-\hat{X}_2)^2\rangle = \langle(\hat{Y}_1+\hat{Y}_2)^2\rangle = n+m-2|c|. \tag{21}$$

The variance $V<1$ indicates that the cavity field exhibits two-mode squeezing. According to the criterion Duan *et al.* [25], the cavity field is entangled if and only if the quantity Σ meets

$$\Sigma < a^2 + \frac{1}{a^2}. \tag{22}$$

Then, the entanglement condition of the cavity field can be given by

$$\Upsilon = \Sigma - a^2 - \frac{1}{a^2} = 2na^2 + 2m/a^2 - 4|c| - a^2 - \frac{1}{a^2} < 0, \tag{23}$$

where $a^2=\sqrt{(2m-1)/(2n-1)}$. It is evident that the entanglement condition, Eq. (23), reduces to the following inequality:

$$\sqrt{\langle a_1^\dagger a_1\rangle\langle a_3^\dagger a_3\rangle} - |\langle a_1a_3\rangle| < 0, \tag{24}$$

which indicates that, for the appearance of the entanglement of the cavity field, the nonclassical correlation should be established between the two cavity modes. It should be noted that the two-mode field with $\Upsilon=-2$ (or $V=0$) corresponds to the original EPR entanglement.

In order to gain insight into how the auxiliary transition $|1\rangle\leftrightarrow|2\rangle$ improves the entanglement of the two-mode field generated from the NWFM processes, we first consider the case of turning of the auxiliary transition $|1\rangle\leftrightarrow|2\rangle$ (i.e., $\Omega_1=0$). In this case, the system reduces to the two-level NFWM system and the parameters A_l, B_1, C_3, and D_3 in Eq. (11) become

$$A_1 = Ng^2\left[\frac{\sin^2\phi\cos^2\phi}{\Gamma_-+i\delta_1} + \frac{\rho_{\tilde{2}\tilde{2}}^0\sin^4\phi}{\Gamma_+-i(\Omega-\delta_1)} + \frac{\rho_{\tilde{3}\tilde{3}}^0\cos^4\phi}{\Gamma_++i(\Omega+\delta_1)}\right],$$

$$B_1 = Ng^2\left[\frac{\sin^2\phi\cos^2\phi}{\Gamma_-+i\delta_1} + \frac{\rho_{\tilde{2}\tilde{2}}^0\cos^4\phi}{\Gamma_++i(\Omega+\delta_1)} + \frac{\rho_{\tilde{3}\tilde{3}}^0\sin^4\phi}{\Gamma_+-i(\Omega-\delta_1)}\right],$$

$$C_3 = Ng^2 \sin^2\phi \cos^2\phi\left[-\frac{1}{\Gamma_- - i\delta_3} + \frac{\rho^0_{\tilde{2}\tilde{2}}}{\Gamma_+ + i(\Omega - \delta_3)} + \frac{\rho^0_{\tilde{3}\tilde{3}}}{\Gamma_+ - i(\Omega + \delta_3)}\right],$$

$$D_3 = Ng^2 \sin^2\phi \cos^2\phi\left[-\frac{1}{\Gamma_- - i\delta_3} + \frac{\rho^0_{\tilde{2}\tilde{2}}}{\Gamma_+ - i(\Omega + \delta_3)} + \frac{\rho^0_{\tilde{3}\tilde{3}}}{\Gamma_+ + i(\Omega - \delta_3)}\right], \tag{25}$$

where $\Gamma_+ = \gamma_3[\sin^4\phi + \cos^4\phi + \sin^2(2\phi)]$ and $\Gamma_- = 2\gamma_3(\sin^4\phi + \cos^4\phi)$. Now if assuming $|\delta_{1(3)}| \gg |\delta_{13}|, \gamma_3$ and $|\Omega \pm \delta_{1(3)}| \gg \gamma_3$—that is, the coupling between the two quantized fields and the dressed atom is far from resonance—the master equation (11) is approximately reduced as

$$\frac{d}{dt}\rho_c \approx -i\left\{\delta_{13} + \frac{\Omega g^2(\cos^4\phi + \sin^4\phi)}{\Omega^2 - \delta_1^2}(\rho^0_{\tilde{3}\tilde{3}} - \rho^0_{\tilde{2}\tilde{2}})\right\}[a_1^\dagger a_1 + a_3^\dagger a_3, \rho_c] + \frac{2i\Omega g^2 \cos^2\phi \sin^2\phi(\rho^0_{\tilde{3}\tilde{3}} - \rho^0_{\tilde{2}\tilde{2}})}{\Omega^2 - \delta_1^2}[a_1^\dagger a_3^\dagger + a_1 a_3, \rho_c] + L_f\rho_c. \tag{26}$$

The above equation shows that the NFWM system can be effectively treated as a nondegenerate parametric amplifier with detuning. This detuning arises from the ac Stark shifts of the two cavity modes due to the coupling between the dressed atom and the two cavity fields being far off resonance. This detuning can be canceled by choosing the appropriate detuning difference δ_{13} because it decreases the two-mode squeezing. Hence the system reduces to a nondegenerate parametric amplifier and its Hamiltonian is

$$V_{eff} = -\frac{2\Omega g^2 \cos^2\phi \sin^2\phi(\rho^0_{\tilde{3}\tilde{3}} - \rho^0_{\tilde{2}\tilde{2}})}{\Omega^2 - \delta_1^2}(a_1^\dagger a_3^\dagger + a_1 a_3). \tag{27}$$

We can see that the strength of the nondegenerate parametric amplifier resulting from the NFWM processes is proportional to the zeroth-order population difference of the two dressed states $|\tilde{3}\rangle$ and $|\tilde{2}\rangle$. This is because there may exist two channels to realize these processes. For example, the dressed atom starting from $|\tilde{3}\rangle$ ($|\tilde{2}\rangle$) with a weighting factor $\rho^0_{\tilde{3}\tilde{3}}$ ($\rho^0_{\tilde{2}\tilde{2}}$) emits one photon in mode 1 (or in mode 3) jumping to $|\tilde{2}\rangle$ ($|\tilde{3}\rangle$); then, because these one-photon processes are far off resonance, the dressed atom immediately emits another photon in mode 3 (or in mode 1) to $|\tilde{3}\rangle$ ($|\tilde{2}\rangle$). In these cascaded emission processes, two laser photons are absorbed. Because the nondegenerate two-photon cascaded processes for the two cavity modes obey two-photon resonant condition and one-photon far-off-resonance condition, these cascaded two-photon processes for the two cavity modes resulting from the NFWM mixing processes are dominant. Unfortunately, these two channels interfere destructively with each other so that the strength of the nondegenerate parametric amplifier in Eq. (27) is proportional to the population difference between two dressed states. It is well known that for the case of a two-level atom resonantly driven by a very strong field, the atom is nearly balanced in its two dressed states—i.e., $\rho^0_{\tilde{3}\tilde{3}} \approx \rho^0_{\tilde{2}\tilde{2}}$. Therefore, due to the NFWM signal vanishing as a result of the destructive interference between two channels [17], the nondegenerate parametric amplification processes disappear. As a result, the large detuning between the two-level atom and the pumping field is necessary for obtaining the population difference between two dressed states. However, the detuning Δ_3 will in turn decrease the value of the factor $\sin^2\phi \cos^2\phi$ containing in the strength of the nondegenerate parametric amplification processes, which takes its maximal at $\Delta_3 = 0$—i.e., the resonant interaction between the two-level transition $|3\rangle \leftrightarrow |2\rangle$ and the relevant pumping field. Therefore, in the two-level NFWM system, the strength of the parametric amplification processes could not be large, which resulting from the correlated photons can be obtained with low generation rate and with a small amount of squeezing, as proved experimentally by Slusher *et al.* [19].

However, if the atom can be set into one of the two dressed states when $\Delta_3 = 0$, the coupling strength of the nondegenerate parametric amplification can be increased greatly as shown in Eq. (27); consequently, the two-mode squeezing and the entanglement can be enhanced significantly. This is because one of the two NFWM channels is closed and only one channel is open for the NFWM processes so that no destructive interference happens. Therefore the pumping field for the NFWM can be resonant to the relevant atomic transition which can greatly increase the third-order nonlinear susceptibility. In the following we will prove that by applying the auxiliary transition $|1\rangle \leftrightarrow |2\rangle$ driven by laser field 1, the atom can be selectively populated in one of the two dressed states so that the population difference $|\rho^0_{\tilde{3}\tilde{3}} - \rho^0_{\tilde{2}\tilde{2}}| \approx 1$ even when the atomic transition $|3\rangle \leftrightarrow |2\rangle$ is resonantly driven by the pumping field—i.e., $\Delta_3 = 0$. Therefore, the highly squeezed and entangled cavity field can be obtained in the present system, compared to the result given in the standard two-level NFWM system [15,18].

In Fig. 2 we plot the dependence of the steady-state variance V and the function Υ which, respectively, characterize the steady two-mode squeezing and entanglement of the intracavity field on the detuning Δ_1 of the auxiliary atomic transition $|1\rangle \leftrightarrow |2\rangle$ driven by the laser field 1. From the inset we can see that the dressed-state populations $\rho^0_{\tilde{3}\tilde{3}}$ and $\rho^0_{\tilde{2}\tilde{2}}$ approach about 0.997 and 0.0026, respectively, at the value $\Delta_1 = \lambda_2$ and for $\Omega \gg \Omega_1, \gamma_1 \gg \gamma_3$. Physically, when $\gamma_1 \gg \gamma_3$ [for example, in atomic barium [23] in which the states $6s6p\ ^3P_1$, $6s\ ^21^S_0$, and $6s6p\ ^1P_1$ are denoted by the states $|3\rangle$, $|2\rangle$, and $|1\rangle$ as shown in Fig. 1(a), $\gamma_3/\gamma_1 = 1/400$] and the $\Delta_3 = 0$, the decay rates γ_{32} and γ_{23} for the dressed state decay $|\tilde{2}\rangle \leftrightarrow |\tilde{3}\rangle$ are equal to each other. The decay rates γ_{32} and γ_{23} are much smaller than γ_{12} and γ_{13}. So the atom is hardly populates the state $\tilde{1}\rangle$. When $\Delta_1 = \lambda_2$ [22,23], the coherent coupling between states $|\tilde{1}\rangle$ and $|\tilde{2}\rangle$ driven by laser field 1 is resonant and the coupling between $|\tilde{1}\rangle$ and $|\tilde{3}\rangle$ is far from

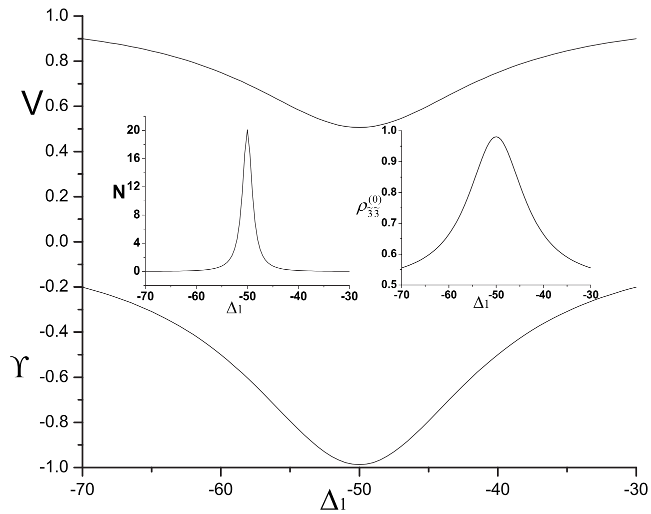

FIG. 2. The steady variances V and the quantity Υ characterizing the entanglement vs Δ_1 for $\gamma_1=1.0$, $\gamma_3=1/100$, $\Omega_1=1.0$, $\Omega_3=50.0$, $g=10.0$, $\delta_1\approx\delta_3=50.0$, $\Delta_3=0.0$, and $\kappa_1=\kappa_3=0.67$ (all these parameters are scaled by γ_1). The insets are the total mean photon number N of the cavity modes and the dressed-state population $\rho^0_{\tilde{3}\tilde{3}}$.

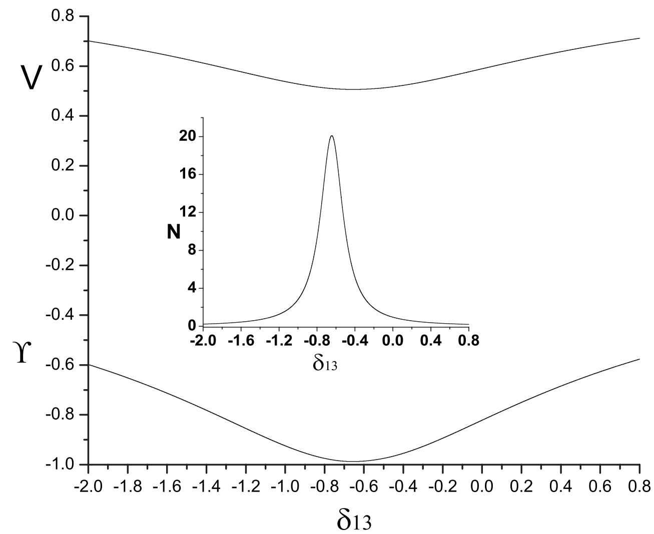

FIG. 3. The steady variances V and the quantity Υ characterizing the entanglement vs Δ_{13} for $\Delta_1=-50.0$. The other parameters are the same as in Fig. 2. The insets is the total mean photon number N of the cavity modes.

resonance, so the last coupling can be neglected as compared with the first one. Therefore, an additional two-step channel from $|\tilde{2}\rangle$ to $|\tilde{1}\rangle$ through resonance interaction and from $|\tilde{1}\rangle$ to $|\tilde{3}\rangle$ through spontaneous decay results in that the atom being trapped in the dressed state $|\tilde{3}\rangle$. Therefore, the population difference $\rho^0_{\tilde{3}\tilde{3}}-\rho^0_{\tilde{2}\tilde{2}}\approx 1$. If we choose $\Delta_1=\lambda_3$, the coherent coupling between the states $|\tilde{1}\rangle$ and $|\tilde{3}\rangle$ driven by laser field 1 is resonant and the coupling between $|\tilde{1}\rangle$ and $|\tilde{3}\rangle$ is far from resonance, which can be neglected. In this case, only the additional two-step channel from $|\tilde{3}\rangle$ to $|\tilde{1}\rangle$ through resonance interaction and from $|\tilde{1}\rangle$ to $|\tilde{2}\rangle$ through spontaneous decay results in that the atom being trapped in the dressed state $|\tilde{2}\rangle$ (here the results in this case are similar to those for $\Delta_1=\lambda_2$, which are not shown in here). As a result, together with the resonant condition $\Delta_3=0$, we can have the maximal strength for the effective nondegenerate parametric amplifying processes as described in Eq. (27), which leads to maximal squeezing and entanglement of the steady cavity field at $\Delta_1=\lambda_2$ (shown in Fig. 2). Additionally, the equivalent ac Stark shifts, which are caused by terms proportional to the imaginary parts of A_l and B_l in Eq. (11) and harmful to the squeezing and entanglement, can be eliminated by adjusting the frequency difference δ_{13} of the two cavity modes. In Fig. 2 the optimal δ_{13} has been chosen for the different detuning values of Δ_1. By using γ_1 as unit and, henceforth, for a special detuning $\Delta_1=\lambda_2=50.0$ we can obtain the optimal $\delta_{13}=0.644$. Evidently, here if δ_{13} departs from this optimal value, the squeezing and entanglement and the photon number of the cavity field will decrease, as shown in Fig. 3. As shown in Fig. 2 or 3, a highly squeezed and entangled field with $V\approx 0.49$ and $\Upsilon\approx -0.99$ can be generated. The generated paired-photon number approaches easily more than 20. It is well known that for a nondegenerate optical parametric oscillator in a cavity, the two-mode squeezing and entanglement of the field produced in the cavity near threshold approach $V=0.5$ and $\Upsilon=-1.0$ [27]. In Figs. 2 and 3 the cooperativity parameter is chosen as $c=Ng^2/\kappa\gamma_3\approx 1.5\times 10^4$, which can be experimentally achievable. In fact, the cooperativity $c=1.2\times 10^4$ has been demonstrated in a recent experiment [28].

Now we proceed to study the squeezing and entanglement properties when the detuning $\delta_{1(3)}$ is in the vicinity of the Rabi frequency Ω. That is, the two cavity modes are nearly in resonance with the left and right sidebands of the well-known Mollow triplet [29] in the resonance fluorescence spectrum from the strongly driven two-level atom system (i.e., $\Omega_1=0$), respectively. Because the quantity δ_{13} which is used to cancel the ac Stark shifts for the two cavity modes is much smaller than the detuning $\delta_{1(3)}$—i.e., $|\delta_{13}|\ll\delta_{1(3)}$—without loss of generality we neglect the difference between δ_1 and δ_3 in the parameters A_i, B_i, C_i, and D_i in Eq. (11) to simplify the calculation. With $\Delta_1=\lambda_2$ and $\Delta_3\ll 2\Omega_3$, the atom is selectively nearly trapped in the dressed state $|\tilde{3}\rangle$ with the aid of the appropriate auxiliary transition $|1\rangle\leftrightarrow|2\rangle$ driven by the relative weak laser field 1. That is, the zeroth-order populations in the two dressed states $|\tilde{2}\rangle$ and $|\tilde{3}\rangle$ obey $\rho^0_{\tilde{2}\tilde{2}}\ll\rho^0_{\tilde{3}\tilde{3}}$. In this case, the master equation (11) approximately reduces to

$$\frac{d\rho_c}{dt}=-(B_1^{(R)}+\kappa_1)(a_1^\dagger a_1\rho_c-a_1\rho_c a_1^\dagger)+A_3^{(R)}(\rho_c a_3 a_3^\dagger-a_3^\dagger\rho_c a_3)-\kappa_3(a_3^\dagger a_3\rho_c-a_3\rho a_3^\dagger)+C_1^{(R)}(a_1^\dagger a_3^\dagger\rho_c+\rho_c a_1^\dagger a_3^\dagger-2a_1^\dagger\rho_c a_3^\dagger)+iC_1^{(I)}(a_1^\dagger a_3^\dagger+a_1a_3)\rho_c+\text{H.c.},\quad(28)$$

where $A_3=\frac{4N\rho_{\tilde{3}\tilde{3}}(b_1+i\delta_1)g^2\cos^4\phi}{\Xi}$, $B_1=\frac{4N\rho_{\tilde{3}\tilde{3}}(b_1+i\delta_1)g^2\sin^4\phi}{\Xi}$, and $C_3=\frac{4N\rho_{\tilde{3}\tilde{3}}(b_1+i\delta_1)g^2\sin^2\phi\cos^2\phi}{\Xi}$ with $\Xi=[b_1+b_2+2i(\delta_1+\bar{\Omega})][b_1+b_2$

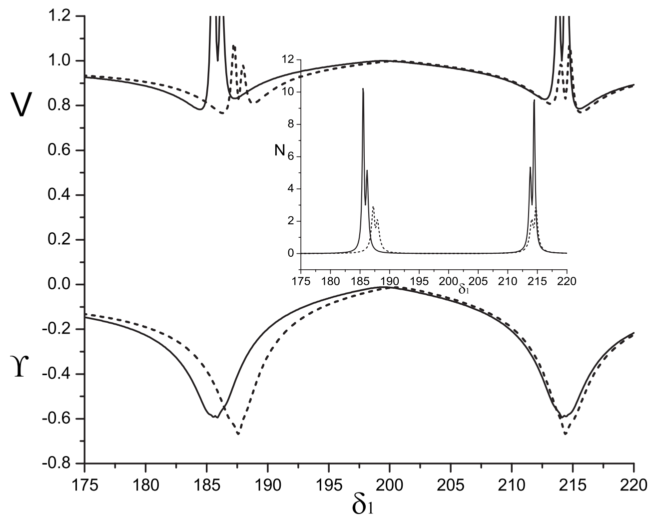

FIG. 4. The quantities V and Υ vs δ_1 with $\gamma_1=1.0$, $\gamma_3=1/400$, $\Omega_1=20.0$, $\Omega_3=100.0$, $g=1.0$, and $\kappa_1=\kappa_3=0.20$ for $\Delta_3=0.0$ (solid line) and $\Delta_3=20.0$ (dashed line). The inset is the total mean photon number N of the cavity modes.

$+2i(\delta_1-\bar{\Omega})]$. Here the markers R and I denote the real and imaginary parts, respectively. Note that in the above master equation the equivalent ac Stark shifts have been eliminated with the choice of the frequency difference δ_{13}. When $\Omega_1 \ll \Omega_3$, due to the resonant interaction of pumping field 1 with the dressed-state transition $|\tilde{1}\rangle \leftrightarrow |\tilde{2}\rangle$—i.e., $\Delta_1=\lambda_2$—both states $|\tilde{1}\rangle$ and $|\tilde{2}\rangle$ are split into a doublet of doubly dressed states $|\pm\rangle$ [30] in the superposition of $|\tilde{2}\rangle$ and $|\tilde{1}\rangle$, separated approximately by $\bar{\Omega}$ for $\Omega_1 > \gamma_1$ as shown in Fig. 1(c). This gives rise to the two peaks of the radiated field (the two-mode cavity field) around the frequency $\pm\Omega$, as shown in Fig. 4. Figure 4 displays the steady-state squeezing and entanglement versus the detuning δ_1. Under the condition $\delta_1=\Omega\pm\bar{\Omega}$, the cavity fields resonantly interact with the transitions between the $|\tilde{3}\rangle$ and $|\pm\rangle$ and hence two dips appear at $\delta_1=\Omega\pm\bar{\Omega}$. If pumping field 1 meets $\Omega_3 \gg \Omega_1 \gg \gamma_1$, we find that the parameter $C_1^{(I)}$ is negligible, compared to the parameters $B_1^{(R)}\left(\approx\frac{Ng^2\cos^4\phi}{\gamma_1}\right)$, $A_3^{(R)}\left(\approx\frac{Ng^2\sin^4\phi}{\gamma_1}\right)$, and $C_1^{(R)} \times (=\sqrt{A_3^{(R)}B_1^{(R)}})$. Immediately, we can find that the above master equation (28) is identical to the one of a nondegenerate three-level cascade laser with the injected atom prepared in the coherent superposition of its lower and upper states [31]. For $\Delta_3=0$ we have $\sin\phi=\cos\phi$ and $B_1^{(R)}=A_3^{(R)}$, which is the same as the case in Ref. [31] of the atoms with maximal coherence. In this case, at the two dips $\delta_1\approx 187.587$ and 214.133 as displayed in Fig. 4, the cavity field exhibits entanglement but no squeezing. For $\Delta_3<0$ (in Fig. 4 we set $\Delta_3=-20.0$; in this case, $\rho^0_{\tilde{2}\tilde{2}} \ll \rho^0_{\tilde{3}\tilde{3}}$ still holds) we have $\sin\phi > \cos\phi$ and hence $B_1^{(R)} > A_3^{(R)}$. We can find that squeezing and entanglement can be achieved at the two dips, which are shifted to $\delta_1\approx 185.867$ and 214.408, coincident with $\Omega\pm\bar{\Omega}$. More interestingly, shown in Table I, near the resonant peaks $\delta_1\approx 185.867$ and 214.408, by adjusting the appropriate the cooperative parameter c, the quantity Υ can be smaller than -1.0, which means that the entanglement produced in our system can exceed the entanglement limit of an intracavity nondegenerate parametric amplifier at threshold with $\Upsilon=-1.0$. Additionally, from Table I one can find that near the peaks, the quantity V characterizing the two-mode squeezing of the cavity field is still larger than 0.5. This is because the two-mode squeezing is sensitive to the difference of the mean photon number of the two cavity modes [32], which can be seen from the equivalent expression of the two-mode squeezing condition $V<1$ as

TABLE I. The variances V and Υ and the mean photon number of the two-mode cavity near the resonant peaks in Fig. 4 (dashed line) with different cooperativity parameter c.

δ_1	c	V	Υ	N
187.5700	1.14×10^3	1.0035	−1.0812	4.4850
187.5767	2.00×10^3	0.9243	−1.1164	4.5310
187.5867	0.89×10^3	0.8367	−1.1961	5.100
187.6000	0.80×10^3	0.6697	−1.3515	6.1567
214.4000	1.43×10^3	0.7875	−1.3514	7.3860
214.4083	1.33×10^3	0.9562	−1.2627	7.1634
214.4100	1.00×10^3	0.9648	−1.2425	6.8426
214.4200	2.00×10^3	1.0574	−1.1541	5.9826

$$|\langle a_1a_3\rangle| > \sqrt{\frac{1}{4}[\langle a_1^\dagger a_1\rangle - \langle a_3^\dagger a_3\rangle]^2 + \langle a_1^\dagger a_1\rangle\langle a_3^\dagger a_3\rangle}. \quad (29)$$

Because in Eq. (11) the gain and the absorption of the two modes are asymmetric near the two resonant dips, the mean photon numbers in two cavity modes are unequal. The mean photon difference $\langle a_1^\dagger a_1\rangle - \langle a_3^\dagger a_3\rangle$ damages the two-mode squeezing so that near the resonant dips we have only small squeezing. Nevertheless, we can still have high entanglement with tens of photons since the entanglement is independent of the mean photon difference, according to the entanglement condition, Eq. (24).

Finally, we can see, from Fig. 4, that the variance is split into two peaks near each dip. This is because when the two cavity are detuned far away from the resonant dips, the system behaves as a nondegenerate parametric amplifier as described in Eq. (27). When the two cavities are exactly detuned at the two resonant dips, the behavior of the system is similar to that of a nondegenerate three-level cascade laser with injected atomic coherence [31] below threshold. Near the two resonant dips, these two processes contribute to the two-mode squeezing simultaneously. Unfortunately, the two-mode squeezing directions from these two processes are not coincident with each other; this leads to a destructive interference for the squeezing as pointed out by Agarwal [33] who revealed this quantum interference effect in the squeezed spectrum for a degenerate optical parametric oscillation in a cavity which is coupled to a broadband squeezed bath. As shown in Eq. (28), although the strength $C_1^{(I)}$ for the nondegenerate parametric amplifying processes is very weak as compared with $C_1^{(R)}$ when the detuning δ_1 departs from the resonant peaks, the effect of the nondegenerate parametric amplifying processes could not be neglected and play the role of degrading the squeezing of the system. Therefore two peaks which reflected the deterioration of the two-mode squeezing appear near each resonant dip.

V. CONCLUSIONS

The entanglement properties of two mode fields generated from four-wave mixing are investigated in the system of an ensemble of *V*-type three-level atoms embedded in a two-mode cavity, in which the atomic transitions from the two excited states to the ground state are driven by one strong and one relative weak laser field, respectively. Meanwhile, the strong laser-driven transition is also coupled by the two cavity modes, so that the NFWM processes occur. Through the appropriate auxiliary transition, the steady population difference of the two dressed state of the two-level driven atom even in the presence of the atom-pumping resonance can be improved to be unity, which leads the degree of squeezing and entanglement of the two-mode cavity field to be improved greatly. Meanwhile, by adjusting the frequency difference of the two modes, the equivalent of Stark shifts for the two cavity modes can be eliminated, which leads to the total photon number of the two-mode cavity field increasing significantly. Consequently, a strong two-mode cavity field, which is highly entangled, can be produced. We also find that when the cavity modes are tuned to nearly resonantly interact with the dressed atoms, the entanglement of the two cavity modes can exceed the entanglement limit of an intracavity nondegenerate parametric oscillator at its threshold.

ACKNOWLEDGMENTS

M.M. would like to thank Huazhong Normal University for hospitality during his visiting. This work is supported by the National Natural Science Foundation of China (under Grant Nos. 60478049 and 10674052), the National Basic Research Project of China (2005 CB724508), and the Ministry of Education under project NCET (under Grant No. NCET-06-0671).

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