Photon emission near superconducting bodies

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We study the photon emission due to a magnetic spin-flip transition of a two-level atom in the vicinity of a dielectric body such as a normal conducting metal or a superconductor. For temperatures below the transition temperature T_c of a superconductor, the corresponding spin-flip lifetime is boosted by several orders of magnitude as compared to the case of a normal conducting body. Numerical results of an exact formulation are also compared to a previously derived approximative analytical expression for the spin-flip lifetime, and we find excellent agreement. We present results on how the spin-flip lifetime depends on the temperature T of a superconducting body as well as its thickness H. Finally, we study how nonmagnetic impurities as well as possible Eliashberg strong-coupling effects influence the spin-flip rate. It is found that nonmagnetic impurities as well as strong-coupling effects have no dramatic impact on the spin-flip lifetime.

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I. INTRODUCTION

It is well known that the rate of spontaneous emission of atoms will be modified due to the presence of a dielectric body [1]. In current investigations of atom microtraps this issue is of fundamental importance since such decay processes have a direct bearing on the stability of, e.g., atom chips. In magnetic microtrap experiments, cold atoms are, e.g., trapped due to the presence of magnetic field gradients created by current carrying wires [2]. Such microscopic traps provide a powerful tool for the control and manipulation of cold neutral atoms over micrometer distances [3]. Unfortunately, this proximity of the cold atoms to a dielectric body introduces additional decay channels. Most importantly, Johnson-noise currents in the material give rise to electromagnetic field fluctuations. For dielectric bodies at room temperature made of normal conducting metals, these fluctuations may be strong enough to deplete the quantum state of the atom and, hence, expel the atom from the magnetic microtrap [4]. Reducing this disturbance from the surface is therefore strongly desired. In order to achieve this, the use of superconducting dielectric bodies instead of normal conducting metals has been proposed [5]. Some experimental work in this context has been done as well, e.g., by Nirrengarten *et al.* [6], where cold atoms were trapped near a superconducting surface.

In the present article we will consider the spin-flip rate when the electrodynamic properties of the superconducting body are described in terms of either a simple two-fluid model or in terms of the detailed microscopic Mattis-Bardeen [7] and Abrikosov-Gor'kov-Khalatnikov [8] theory of weak-coupling BCS superconductors. In addition, we will also study how nonmagnetic impurities, as well as strong coupling effects according to the low-frequency limit of the Eliashberg theory [9], will affect the spontaneous emission rate.

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II. GENERAL THEORY

Following Ref. [10] we consider an atom in an initial state $|i\rangle$ and trapped at position $\mathbf{r}_A=(0,0,z)$ in vacuum near a dielectric body. The rate Γ_B of spontaneous and thermally stimulated magnetic spin-flip transition into a final state $|f\rangle$ is then

$$\Gamma_B = \mu_0 \frac{2(\mu_B g_S)^2}{\hbar} \sum_{j,k} S_j S_k^* \mathrm{Im}[\boldsymbol{\nabla} \times \boldsymbol{\nabla} \times \boldsymbol{G}(\mathbf{r}_A,\mathbf{r}_A,\omega)]_{jk}(\bar{n}+1), \tag{1}$$

where we have introduced the dimensionless components $S_j \equiv \langle f|\hat{S}_j/\hbar|i\rangle$ of the electron spin operators $\hat{S}_j$ with $j=x,y,z$. Here $g_S \approx 2$ is the gyromagnetic factor of the electron and $\boldsymbol{G}(\mathbf{r},\mathbf{r}',\omega)$ is the dyadic Green tensor of Maxwell's theory. Equation (1) can be derived from Fermi's golden rule and the correlation spectrum of the electromagnetic field fluctuations [11,12]. It also follows from a consistent quantum-mechanical treatment of electromagnetic radiation in the presence of an absorbing body [13,14]. In this theory a local response is assumed; i.e., the characteristic skin depth should be larger than the mean free path of the electric charge carriers of the absorbing body. Thermal excitations of the electromagnetic field modes are accounted for by the factor $(\bar{n}+1)$, where $\bar{n}=1/(e^{\hbar\omega/k_B T}-1)$ and $\omega \equiv 2\pi\nu$ is the angular frequency of the spin-flip transition. Here T is the temperature of the dielectric body, which is assumed to be in thermal equilibrium with its surroundings. The dyadic Green tensor $\boldsymbol{G}(\mathbf{r},\mathbf{r}',\omega)$ is a unique solution to the Helmholtz equation

$$\boldsymbol{\nabla} \times \boldsymbol{\nabla} \times \boldsymbol{G}(\mathbf{r},\mathbf{r}',\omega) - k^2 \epsilon(\mathbf{r},\omega)\boldsymbol{G}(\mathbf{r},\mathbf{r}',\omega) = \delta(\mathbf{r}-\mathbf{r}')\mathbf{1}, \tag{2}$$

with appropriate boundary conditions. Here $k=\omega/c$ is the wave number in vacuum, c is the speed of light, and $\mathbf{1}$ is the unit dyad. The dyadic tensor $\boldsymbol{G}(\mathbf{r},\mathbf{r}',\omega)$ contains all relevant information about the geometry of the material and, through the relative electric permittivity $\epsilon(\mathbf{r},\omega)$, about its dielectric properties. The fluctuation-dissipation theorem is built into this theory [13,14].

The decay rate Γ_B^0 of a magnetic spin-flip transition for an atom in free-space is well known (see, e.g., Refs. [1,10]). This free-space decay rate is $\Gamma_B^0 = \Gamma_B S^2$, where $\Gamma_B = \mu_0(\mu_B g_S)^2 k^3/(3\pi\hbar)$ and where we have introduced the dimensionless spin factor $S^2 \equiv S_x^2 + S_y^2 + S_z^2$. The free-space lifetime corresponding to this magnetic spin-flip rate is $\tau_B^0 \equiv 1/\Gamma_B^0$. In the present paper we only consider ^{87}Rb atoms that are initially pumped into the $|5S_{1/2}, F=2, m_F=2\rangle \equiv |2,2\rangle$ state and assume as a rate-limiting transition $|2,2\rangle \rightarrow |2,1\rangle$ in correspondence with recent experiments [6,15–17]. The spin factor is $S^2 = 1/8$ (cf. Ref. [10]) and the frequency is $\nu = 560$ kHz. The numerical value of the free-space lifetime then is $\tau_B^0 = 1.14 \times 10^{25}$ s.

In the following we will consider a geometry where an atom is trapped at a distance z away from a dielectric slab with thickness H. Vacuum is on both sides of the slab, i.e., $\epsilon(\mathbf{r}, \omega) = 1$ for any position $\mathbf{r}$ outside the body. The slab can, e.g., be a superconductor or a normal conducting metal described by a dielectric function $\epsilon(\omega)$. The total transition rate for magnetic spontaneous emission,

$$\Gamma_B = (\Gamma_B^0 + \Gamma_B^{\text{slab}})(\bar{n} + 1), \tag{3}$$

can then be decomposed into a free part and a part purely due to the presence of the slab. The latter contribution for an arbitrary spin orientation is then given by

$$\Gamma_B^{\text{slab}} = 2\Gamma_B^0[(S_x^2 + S_y^2)I_\parallel + S_z^2 I_\perp], \tag{4}$$

with the atom-spin-orientation-dependent integrals

$$I_\parallel = \frac{3}{16kz}\text{Re}\left\{ \int_0^{2kz} dx\, e^{ix}\left[\mathcal{C}_N(x) - \left(\frac{x}{2kz}\right)^2 \mathcal{C}_M(x) \right] + \int_0^\infty dx\, e^{-x}\frac{1}{i}\left[\mathcal{C}_N(ix) + \left(\frac{x}{2kz}\right)^2 \mathcal{C}_M(ix) \right] \right\}, \tag{5}$$

$$I_\perp = \frac{3}{8kz}\text{Re}\left\{ \int_0^{2kz} dx\, e^{ix}\left[1 - \left(\frac{x}{2kz}\right)^2 \right] \mathcal{C}_M(x) + \int_0^\infty dx\, e^{-x}\frac{1}{i}\left[1 + \left(\frac{x}{2kz}\right)^2 \right] \mathcal{C}_M(ix) \right\}, \tag{6}$$

where the scattering coefficients are given by [18]

$$\mathcal{C}_N(x) = r_p(x)\frac{1 - e^{ixH/z}}{1 - r_p^2(x)e^{ixH/z}}, \tag{7}$$

$$\mathcal{C}_M(x) = r_s(x)\frac{1 - e^{ixH/z}}{1 - r_s^2(x)e^{ixH/z}}. \tag{8}$$

The electromagnetic-field-polarization-dependent Fresnel coefficients are

$$r_p(x) = \frac{\epsilon(\omega)x - \sqrt{(2kz)^2[\epsilon(\omega) - 1] + x^2}}{\epsilon(\omega)x + \sqrt{(2kz)^2[\epsilon(\omega) - 1] + x^2}}, \tag{9}$$

$$r_s(x) = \frac{x - \sqrt{(2kz)^2[\epsilon(\omega) - 1] + x^2}}{x + \sqrt{(2kz)^2[\epsilon(\omega) - 1] + x^2}}. \tag{10}$$

For the special case $H = \infty$, the integrals in Eqs. (5) and (6) are simply a convenient rewriting of Eqs. (8)–(12) in Ref. [19]. Note that $I_\perp \approx 2I_\parallel$ provided $kz \ll 1$. Throughout this article, we use the same spin-orientation as in Refs. [5,20], i.e., $S_y^2 = S_z^2$ and $S_x = 0$.

III. TWO FLUID- AND DRUDE-LIKE MODELS

As the total current density is assumed to respond linearly and locally to the electric field, the dielectric function $\epsilon(\omega)$ can be written in the form

$$\epsilon(\omega) = 1 - \frac{\sigma_2(T)}{\epsilon_0\omega} + i\frac{\sigma_1(T)}{\epsilon_0\omega}. \tag{11}$$

Here $\sigma(T) \equiv \sigma_1(T) + i\sigma_2(T)$ is the, in general, frequency-dependent, complex optical conductivity. We may now parametrize this complex conductivity in terms of the London penetration length $\lambda_L(T) \equiv \sqrt{1/\omega\mu_0\sigma_2(T)}$ and the skin depth $\delta(T) \equiv \sqrt{2/\omega\mu_0\sigma_1(T)}$. In this case, the dielectric function is $\epsilon(\omega) = 1 - 1/k^2\lambda_L^2(T) + i2/k^2\delta^2(T)$. If, in addition, we consider a nonzero and sufficiently small frequency in the range $0 < \omega \ll \omega_g \equiv 2\Delta(0)/\hbar$, where $\Delta(0)$ is the energy gap of the superconductor at zero temperature, the current density may be described in terms of a two-fluid model [21]. The London penetration length is $\lambda_L(T) = \lambda_L(0)/\sqrt{n_s(T)/n_0}$ and the skin depth is $\delta(T) = \delta(T_c)/\sqrt{n_n(T)/n_0}$. Here the electron density in the superconducting and normal state is $n_s(T)$ and $n_n(T)$, respectively, such that $n_s(T) + n_n(T) = n_0$ and $n_s(0) = n_n(T \geq T_c) = n_0$ [21]. A convenient summary of the two-fluid model is expressed by the relations

$$\sigma_1(T) = \sigma_n\frac{n_n(T)}{n_0}, \quad \sigma_2(T) = \sigma_L\frac{n_s(T)}{n_0}, \tag{12}$$

where $\sigma_n \equiv \sigma_1(T_c)$ and $\sigma_L \equiv 1/\omega\mu_0\lambda_L^2(0)$. Considering, in particular, the Gorter-Casimir temperature dependence [22] for the current densities, the electron density in the normal state is $n_n(T)/n_0 = (T/T_c)^4$. For niobium we use $\delta(T_c) = \sqrt{2/\omega\mu_0\sigma_n} \approx 150$ μm as $\sigma_n \approx 2 \times 10^7$ $(\Omega\text{ m})^{-1}$ and $\lambda_L(0) = 35$ nm according to Ref. [23]. In passing, we remark that the value of σ_n as obtained in Ref. [24] is two orders of magnitude larger than the corresponding value inferred from the data presented in Ref. [23].

The lifetime $\tau_B \equiv 1/\Gamma_B$ for photon emission as a function of T is shown in Fig. 1 for $H = 0.9$ μm [curve (iii)]. We confirm the observation in Ref. [20] that for temperatures below T_c and for $H = \infty$ [curve (ii)], the spin-flip lifetime is boosted by several orders of magnitude. In Ref. [20], the spin-flip lifetime was, however, calculated by making use of the approximative and analytical expression

$$\frac{\tau_0^B}{\tau^B} = (\bar{n} + 1)\left[1 + \left(\frac{3}{4}\right)^3 \sqrt{\epsilon_0\omega}\frac{\sigma_1(T)}{\sigma_2^{3/2}(T)}\frac{1}{(kz)^4} \right], \tag{13}$$

valid provided $\lambda_L(T) \ll \delta(T)$ and $\lambda_L(T) \ll z \ll \lambda$. Comparing this analytical expression with the numerical results as pre-

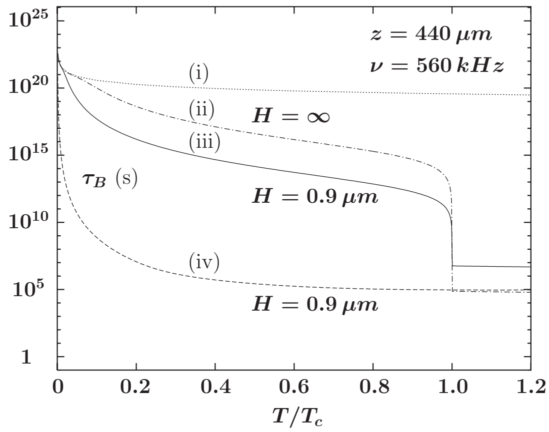

FIG. 1. τ_B of a trapped atom near a superconducting film as a function of the temperature T/T_c. Curve (i) shows the free-space lifetime $\tau_B^0/(\bar{n}+1)$, where $\tau_B^0=1.14\times 10^{25}$ s. Curves (ii) and (iii) correspond to the two-fluid mode with the Gorter-Casimir temperature dependence for $H=\infty$ and $H=0.9$ μm, respectively. We use $\lambda_L(0)=35$ nm and $\delta(T_c)\approx 150$ μm [23], corresponding to niobium. The critical temperature is $T_c=8.31$ K [23]. For $T/T_c\gtrsim 1$ we put $\sigma_2(T)\simeq 0$ but $\sigma_1(T)=2/\omega\mu_0\delta(T_c)^2$. Curve (iv) corresponds to a film made of gold described by the dielectric function given by Eq. (15).

sented in Fig. 1, based on the exact equations (3)–(10), we find excellent agreement. This observation remains true when $\sigma_1(T)$ and $\sigma_2(T)$ are obtained from more detailed and microscopic considerations to be discussed below. For temperatures $T/T_c>1$ we can neglect the $\sigma_2(T)$ dependence and, for $\delta(T)\ll z$, we confirm the result of Ref. [5], i.e.,

$$\frac{\tau_0^B}{\tau^B}=(\bar{n}+1)\left[1+\left(\frac{3}{4}\right)^3\sqrt{\frac{2\epsilon_0\omega}{\sigma_1(T)}\frac{1}{(kz)^4}}\right]. \qquad (14)$$

For $T\simeq T_c$ we have to resort to numerical investigations. Equations (13) and (14) can also be obtained from the asymptotic form of the various expressions in Ref. [12].

In contrast to the traditional Drude model, more realistic descriptions of a normal conducting metal in terms of a permittivity include a significant real contribution to the dielectric function in addition to an imaginary part. One such description is discussed in Ref. [25], where

$$\epsilon(\omega)=1-\frac{\omega_p^2}{\omega^2+\nu(T)^2}+i\frac{\nu(T)\omega_p^2}{\omega[\omega^2+\nu(T)^2]} \qquad (15)$$

and $\hbar\nu(T)=0.0847(T/\theta)^5\int_0^{\theta/T}dx\, x^5e^x/(e^x-1)^2$ eV using a Bloch-Grüneisen approximation. Here $\theta=175$ K for gold. The plasma frequency is $\hbar\omega_p=9$ eV. For $\omega=2\pi\times 560$ kHz and for temperatures $T\simeq 0.25T_c$, we observe that Eq. (15) leads to $\sigma_1(T)\simeq\sigma_2(T)$, and that for lower temperatures, $\sigma_2(T)$ will be the dominant contribution to the conductivity. For temperatures $T/T_c\gtrsim 1$, with use of Eq. (15) we can set $\sigma_2(T)\simeq 0$ when calculating the lifetime. For a bulk material of gold this leads to almost two orders of magnitude longer lifetime as compared to niobium since for gold $\delta(T_c)\approx 1$ μm, using the parameters corresponding to Fig. 1. This finding is in accordance with Eq. (14). As seen from Fig. 1, for a thin film and for $T/T_c\gtrsim 1$ we find the opposite and remarkable result: i.e., a decrease in conductivity can lead to a larger lifetime.

IV. MICROSCOPIC DESCRIPTIONS

A much more detailed and often used description of the electrodynamic properties of superconductors than the simple two-fluid model was developed by Mattis and Bardeen [7] and independently by Abrikosov, Gor'kov, and Khalatnikov [8], based on the weak-coupling BCS theory of superconductors. In the clean limit, i.e., $l\gg\xi_0$, where l is the electron mean free path and ξ_0 is the coherence length of a pure material, the complex conductivity, normalized to $\sigma_n\equiv\sigma_1(T_c)$, can be expressed in the form [26]

$$\frac{\sigma(T)}{\sigma_n}=\int_{\Delta(T)-\hbar\omega}^{\infty}\frac{dx}{\hbar\omega}\tanh\left(\frac{x+\hbar\omega}{2k_BT}\right)g(x)-\int_{\Delta(T)}^{\infty}\frac{dx}{\hbar\omega}\tanh\left(\frac{x}{2k_BT}\right)g(x), \qquad (16)$$

where $g(x)=[x^2+\Delta^2(T)+\hbar\omega x]/u_1u_2$ and $u_1=\sqrt{x^2-\Delta^2(T)}$, $u_2=\sqrt{(x+\hbar\omega)^2-\Delta^2(T)}$. Here, the well-known BCS temperature dependence for the superconducting energy gap $\Delta(T)$ is given by [27]

$$\ln\{[\hbar\omega_D+\sqrt{(\hbar\omega_D)^2+\Delta^2(0)}]/\Delta(0)\}=\int_0^{\hbar\omega_D}\frac{dx}{\sqrt{x^2+\Delta^2(T)}}\tanh\left[\frac{\sqrt{x^2+\Delta^2(T)}}{2k_BT}\right], \qquad (17)$$

where ω_D is the Debye frequency and $\Delta(0)=3.53k_BT_c/2$. For niobium, the Debye frequency is $\hbar\omega_D=25$ meV. According to a theorem of Anderson [31,32], the presence of nonmagnetic impurities, which we only consider in the present paper, will not modify the superconducting energy gap as given by Eq. (17). The complex conductivity will, however, in general be modified due to the presence of such impurities.

In the dirty limit where $l\ll\xi_0$, the complex conductivity has been examined within the framework of the microscopic BCS theory (see, e.g., Ref. [34]). In this case, the complex conductivity, now normalized to σ_L, can conveniently be written in the form

$$\frac{\sigma(T)}{\sigma_L}=\int_{\Delta(T)-\hbar\omega}^{\infty}\frac{dx}{2}\tanh\left(\frac{x+\hbar\omega}{2k_BT}\right)\left(\frac{g(x)+1}{u_2-u_1+i\hbar/\tau}-\frac{g(x)-1}{u_2+u_1-i\hbar/\tau}\right)-\int_{\Delta(T)}^{\infty}\frac{dx}{2}\tanh\left(\frac{x}{2k_BT}\right)\times\left(\frac{g(x)+1}{u_2-u_1+i\hbar/\tau}+\frac{g(x)-1}{u_2+u_1+i\hbar/\tau}\right). \qquad (18)$$

Here we choose τ such that $\hbar/\tau\Delta(0)=\pi\xi_0/l=13.61$, corresponding to the experimental coherence length $\xi_0=39$ nm and the mean free path $l(T\simeq 9K)=9$ nm. The normalization constant is $\sigma_L=1.85\times 10^{14}$ $(\Omega\text{ m})^{-1}$ corresponding to $\lambda_L(0)=35$ μm for niobium [23].

As the temperature decreases below T_c, Cooper pairs will be created. Despite a very small fraction of Cooper pairs for

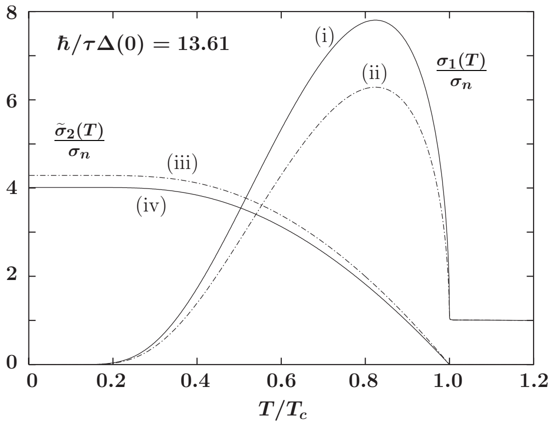

FIG. 2. The complex conductivity $\sigma(T) \equiv \sigma_1(T) + i\sigma_2(T)$ as a function of the temperature T/T_c with $\hbar/\tau\Delta(0) = 13.61$ [23]. Curves (i) and (ii) show $\sigma_1(T)/\sigma_n$ with σ_1 as given by Eqs. (18) and (16), respectively. Curves (iii) and (iv) show $\widetilde{\sigma_2}(T)/\sigma_n$ with $\widetilde{\sigma_2}(T) \equiv 0.25 \times 10^{-5}\sigma_2(T)$ and where $\sigma_2(T)$ is given by Eqs. (16) and (18), respectively.

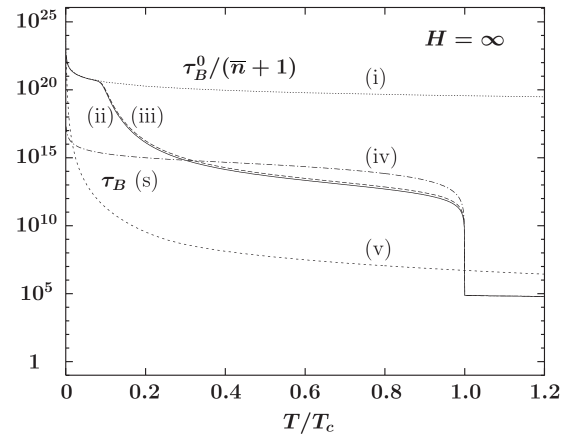

FIG. 3. τ_B of a trapped atom near a superconducting bulk as a function of temperature T/T_c. The other relevant parameters are the same as in Fig. 1. Curve (i) shows the free-space lifetime $\tau_B^0/(\bar{n}+1)$, with $\tau_B^0 = 1.14 \times 10^{25}$ s. Curve (ii) shows the lifetime τ_B using the microscopic BCS theory, i.e., Eq. (18). Curve (iii) corresponds to the Mattis-Bardeen theory, i.e., using Eq. (16). Curve (iv) shows the lifetime τ_B using the two-fluid model and Eq. (19), with $\hbar/\tau\Delta(0) = 13.61$. Curve (v) corresponds to a film made of gold described by the dielectric function given by Eq. (15).

temperatures just below T_c, the imaginary part of the conductivity as given by Eq. (18) exhibits a vast increase (cf. Fig. 2). Furthermore, due to the modification of the quasiparticle dispersion in the superconducting state, there is an increase in $\sigma_1(T)$ as well just below T_c. This is the well-known coherence Hebel-Slichter peak [28]. The importance of this coherence peak in this context was first pointed out in Ref. [29] and commented upon in Ref. [30]. In contrast to the simple Gorter-Casimir temperature dependence, both Eqs. (16) and (18) describe well the presence of the Hebel-Slichter peak, with a peak height less than $8\sigma_n$ for both cases (cf. Fig. 2), at least for the values of the physical parameters under consideration in the present paper. In the opposite temperature limit, i.e., $T \ll T_c$, numerical studies of Eq. (18) show that $\sigma_1(T)$ decreases exponentially fast. As seen in Fig. 2, the imaginary part of the conductivity, on the other hand, is more or less constant for such temperatures.

In passing we observe that there is only a minor difference in $\sigma_2(T)$ as obtained from Eqs. (16) and (18), respectively. For temperatures around the peak value of the Hebel-Slichter peak, $\sigma_1(T)$ obtained from Eq. (18) is, however, approximately 20% larger than $\sigma_1(T)$ as obtained from Eq. (16). This difference has, nevertheless a small effect on the lifetime τ_B. Hence, computing τ_B using Eq. (16) and (18) for the complex conductivity, we realize that the presence of nonmagnetic impurities has no dramatic impact on the lifetime for spontaneous emission (see Fig. 3). A comparison of the values of τ_B as obtained using the two-fluid model for $H = \infty$ as presented in Fig. 1 and the corresponding result as shown in Fig. 3 shows, for our set of physical parameters, that the two-fluid model overestimates τ_B by three orders of magnitude.

For finite values of the lifetime τ and for nonmagnetic impurities we can also investigate the validity of the two-fluid model approximation in terms of the lifetime τ_B for spontaneous emission processes. As we now will see, there are large deviations between the microscopic theory and the two-fluid model approximation, in particular for small temperatures. According to Abrikosov and Gor'kov (for an excellent account see, e.g., Ref. [33] and references cited therein), the density of superconducting electrons is given by

$$\frac{n_s(T)}{n_0} \approx \frac{\pi\tau}{\hbar}\Delta(T)\tanh\left(\frac{\Delta(T)}{2k_BT}\right), \tag{19}$$

provided that $\tau\Delta(0)/\hbar \ll 1$. We can now compute the dielectric function (11) using Eq. (12). We find that $\sigma_2(T)/\sigma_L$ obtained in this way agrees well with the corresponding quantity obtained from Eq. (18). There is, however, a considerable discrepancy between the two-fluid expression for $\sigma_1(T)/\sigma_L$ and the corresponding expressions obtained from the microscopic theory as given by Eq. (18). The numerical results for the lifetime in this case are illustrated in curve (iv) in Figs. 3 and 4.

Since we are considering low frequencies $0 < \omega \ll \omega_g \equiv 2\Delta(0)/\hbar$, strong-coupling effects can now be estimated by making use of the low-frequency limit of the Eliashberg theory [9] and its relation to the BCS theory (see, e.g., Ref. [35]). The so-called mass-renormalization factor Z_N, which in general is both frequency and temperature dependent, is then replaced by its zero-temperature limit, which for niobium has the value $Z_N \approx 2.1$ [35]. Using the strong-coupling expressions for the optical conductivity in a suitable form, such as, e.g., given in Ref. [26], we then find that the complex conductivity $\sigma(T)/\sigma_n$ is rescaled by $\sigma_n \to \sigma_n/Z_N$ with the lifetime of nonmagnetic impurities rescaled by $\tau \to \tau/Z_N$. The change in the lifetime for spontaneous emission can then, e.g., be inferred from the relation (13), and we find only a minor decrease of τ_B by the numerical factor $1/\sqrt{Z_N} \approx 0.69$, which also agrees well with more precise numerical evaluations.

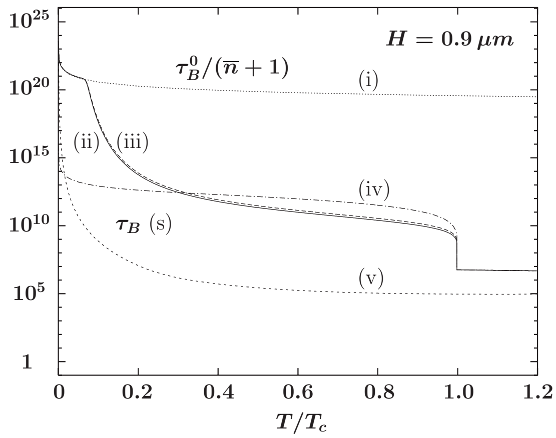

FIG. 4. τ_B of a trapped atom near a superconducting film as a function of the temperature T/T_c with $H=0.9$ μm. The other relevant parameters and labels are the same as in Fig. 3.

As observed in Refs. [5,36], the lifetime for spontaneous emission exhibits a minimum with respect to variation of the thickness H of a normal conducting film. This fact is also illustrated for a superconducting film in Fig. 5. Below the minimum at $H_{min} \approx 0.1$ μm, a decrease of the thickness H leads to an increase of lifetime in proportion to H^{-1}. This might be traced back to the fact that the region generating the noise is becoming thinner as it is limited by H, and not $\delta(T)$. Eventually, the lifetime reaches the free-space lifetime τ_B^0 as H tends to zero. On the other hand, for large H, i.e., $H \gg \delta(T)$,the lifetime is constant with respect to H, giving the same result as for an infinite thick slab. In the region between, i.e., $\lambda_L(T) \lesssim H \lesssim \delta(T)$, the lifetime is proportional to H. Numerical studies show that a nonzero $\sigma_2(T)$ is important for a well-pronounced minimum of τ_B as a function of H.

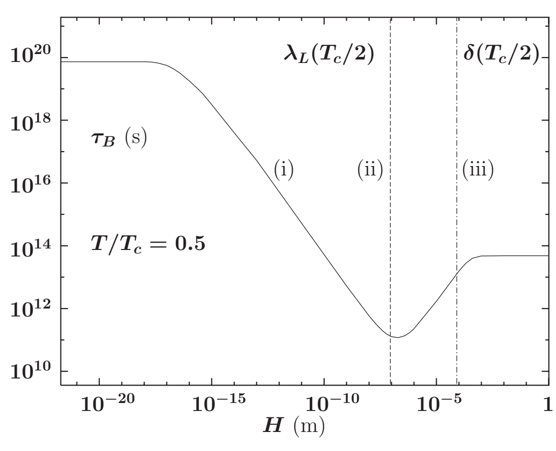

FIG. 5. Curve (i) shows τ_B as a function of the thickness H of the film. The complex conductivity is computed applying Eq. (18) with $\hbar/\tau\Delta(0)=13.61$. Other relevant parameters are the same as in curve (iii) in Fig. 1. In the limit $H=0$, i.e., no slab at all, the lifetime is simply $\tau_B^0/(n+1)=7.34\times10^{19}$ s for the parameters under consideration. Line (ii) corresponds to the London length $\lambda_L(T_c/2)$ $=89.2$ nm. Line (iii) corresponds to the skin depth $\delta(T_c/2)$ $=80.5$ μm.

V. FINAL REMARKS AND SUMMARY

Some experimental work has been done using a superconducting body: e.g., Nirrengarten *et al.* [6]. Here cold atoms were trapped near a superconducting surface. At the distance of 440 μm from the chip surface, the trap lifetime reaches 115 s at low atomic densities and with a temperature 40 μK of the chip. We believe the vast discrepancy between this experimental value and our theoretical calculations must rely on effects that we have not taken into account in our analysis.

The use of a thin superconducting film may lead to the presence of vortex motion and pinning effects (see, e.g., Refs. [37,38]). The presence of vortices will in general modify the dielectric properties of the dielectric body. If we, as an example, consider a vortex system in the liquid phase in a finite-slab geometry, one expects a strongly temperature-dependent $\sigma_1(T)$ with a peak value

$$\sigma_1(T) \simeq 1.3\times10^7\ (\Omega\ \mathrm{m})^{-1}/H^2[\mu\mathrm{m}]\ \ \nu[\mathrm{kHz}]$$

(see Ref. [37]). Close to this peak, $\sigma_1(T) \simeq \sigma_2(T)$. For $H[\mu\mathrm{m}] \simeq 0.9$ and $\nu[\mathrm{kHz}] \simeq 560$ we find, for $T/T_c \gtrsim 0.5$, a lifetime for spontaneous emission four orders of magnitude larger than a film made out of gold with the same geometry. It is an interesting possibility that spontaneous emission processes close to thin superconducting films could be used for an experimental study of the physics of vortex condensation. This possibility has also been noticed in a related consideration, which has appeared during the preparation of the present work [39]. There are also fabrication issues concerning the Nb-O chemistry [40] which may have an influence on the lifetime for spontaneous emission.

To summarize, we have studied the rate for photon emission, due to a magnetic spin-flip transition, of a two-level atom in the vicinity of a normal conducting metal or a superconductor. Our results confirms the conclusion in Ref. [20] namely, that the corresponding magnetic spin-flip lifetime will be boosted by several orders of magnitude by replacing a normal conducting film with a superconducting body. This conclusion holds when describing the electromagnetic properties of the superconductivity in terms of a simple two-fluid model as well as in terms of more detailed and precise microscopic Mattis-Bardeen and Abrikosov-Gor'kov-Khalatnikov theories. For the set of physical parameters as used in Ref. [20] it so happens, more or less by chance, that the two-fluid model results agree well with the results from the microscopic BCS theory. We have, however, seen that even though the two-fluid model gives a qualitatively correct physical picture for photon emission, it, nevertheless, leads in general to large quantitative deviations when compared to a detailed microscopic treatment. We therefore have to resort to the microscopic Mattis-Bardeen [7] and Abrikosov-Gor'kov-Khalatnikov [8] theories in order to obtain precise predictions. We have also shown that nonmagnetic impurities as well as strong-coupling effects have no dramatic impact on the rate for photon emission. Vortex condensation in thin superconducting films may, however, be of great importance. Finally, we stress the close dependence between the spin-flip rate for photon emission and the complex conductivity, which indicates a new method to experimentally study the electrodynamical properties of a superconductor or a normal conducting metal. In such a context the parameter dependence for a bulk material as given by Eq. (13) may be useful.

After the submission of the present paper, a related work appeared with some conclusions similar to ours [41].

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