Photonic module: An on-demand resource for photonic entanglement

Simon J. Devitt,[1] Andrew D. Greentree,[1] Radu Ionicioiu,[2] Jeremy L. O'Brien,[3] William J. Munro,[2,4] and Lloyd C. L. Hollenberg[1]

[1]*Centre for Quantum Computing Technology, School of Physics, University of Melbourne, Victoria 3010, Australia*

[2]*Hewlett-Packard Laboratories, Filton Road, Stoke Gifford, Bristol BS34 8QZ, United Kingdom*

[3]*Centre for Quantum Photonics, H. H. Wills Physics Laboratory and Department of Electrical and Electronic Engineering, University of Bristol, Merchant Venturers Building, Woodland Road, Bristol BS8 1UB, United Kingdom*

[4]*National Institute of Informatics, 2-1-2 Hitotsubashi, Chiyoda-ku, Tokyo 101-8430, Japan*

(Received 14 September 2007; published 16 November 2007)

Photonic entanglement has a wide range of applications in quantum computation and communication. Here we introduce a device: the photonic module, which allows for the rapid, deterministic preparation of a large class of entangled photon states. The module is an application independent, "plug and play" device, with sufficient flexibility to prepare entanglement for all major quantum computation and communication applications in a completely deterministic fashion without number-discriminated photon detection. We present two alternative constructions for the module, one using free-space components and one in a photonic band-gap structure. The natural operation of the module is to generate states within the stabilizer formalism and we present an analysis on the cavity requirements to experimentally realize this device.

DOI: 10.1103/PhysRevA.76.052312 PACS number(s): 03.67.Mn, 42.50.Dv

I. INTRODUCTION

Multipartite entanglement is the most important resource needed when attempting to perform quantum processing. Entanglement forms the basis of quantum algorithms [1,2], secure cryptographic protocols and secret sharing [3,4], Heisenberg limited optical lithography [5], and even a generic resource for a universal quantum computer [6,7]. However, it has proven to be a difficult challenge to efficiently generate useful multiqubit entangled states that can be used as a resource for all these disparate applications. Here we illustrate the construction of the photonic module (Fig. 1). A single atom–cavity system which leads to an extremely versatile device that can be used as a static resource for preparing entangled photonic states for computation and/or communication quickly, and with complete determinism.

Multipartite entanglement can be prepared in systems such as trapped ions [8] and solid state qubits [9–14], but as resources for quantum cryptography and communications they are problematic due to a high sensitivity to environmental decoherence, qubit immobility and the inevitable incorporation of quantum bus protocols [15–17], and/or fusion methods [18–21] to solve problems related to information transport. In contrast, photonic qubits are extremely easy to move and are robust against decoherence. However, performing appropriate gate operations to prepare photonic entanglement is extremely difficult, with experimental implementations of optical computing generally utilizing down-converted sources [22,23]. Recently, more viable methods for the preparation of photonic entanglement have been developed [24–29] and utilized [30,31] based on the measurement induced nonlinearities proposed by Knill, Laflamme, and Milburn [32]. However, each of these measurement based methods result in probabilistic interactions and generally require number-discriminated photon detection, the result of which is that large multiphoton entangled states need to be probabilistically grown. Probabilistic state preparation and slow photon detection results in long preparation times for large entangled multiphoton states, requiring significant resources for photon storage and limiting the applicability of on-chip devices as entanglement resources.

Although utilizing a single atomic qubit to mediate the preparation of photonic entanglement is not a new concept [33] the nature of the interaction exhibited by this scheme leads to a simple and versatile plug and play device. A single module, or multiple connected modules, can be constructed and with classical routing prepares entanglement without the downsides of probabilistic interactions or single-photon detection. Atom–cavity mediated entanglement is well understood and there exists several schemes which can be adapted for use in the modules. These include the cavity-assisted interaction proposed by Duan, Wang, and Kimble [34,35] who utilized a similar network to achieve gate based photonic quantum computation and recent experimental schemes from Schuster *et al.* [36] which demonstrated a photon nondemolition interaction using a Cooper-pair box qubit and microwave cavity photons.

We describe photonic modules for which the natural operation allows for the preparation of any N-photon entangled state which can be described via the stabilizer formalism of Gottesman [37]. As such, the internal construction of each module is independent of the state being prepared, no single-photon (or multiphoton) detection is needed and the coherence time required for the atomic qubit is limited only by the time required to measure specific stabilizers describing the state (which can be small, even for large, highly entangled, multiphoton states). Therefore, the module is a completely generic resource, which can be applied to a vast variety of quantum applications.

The possible uses for these modules are extensive. The ability to prepare any stabilizer state allows for the deterministic preparation of *any* geometric graph state, including

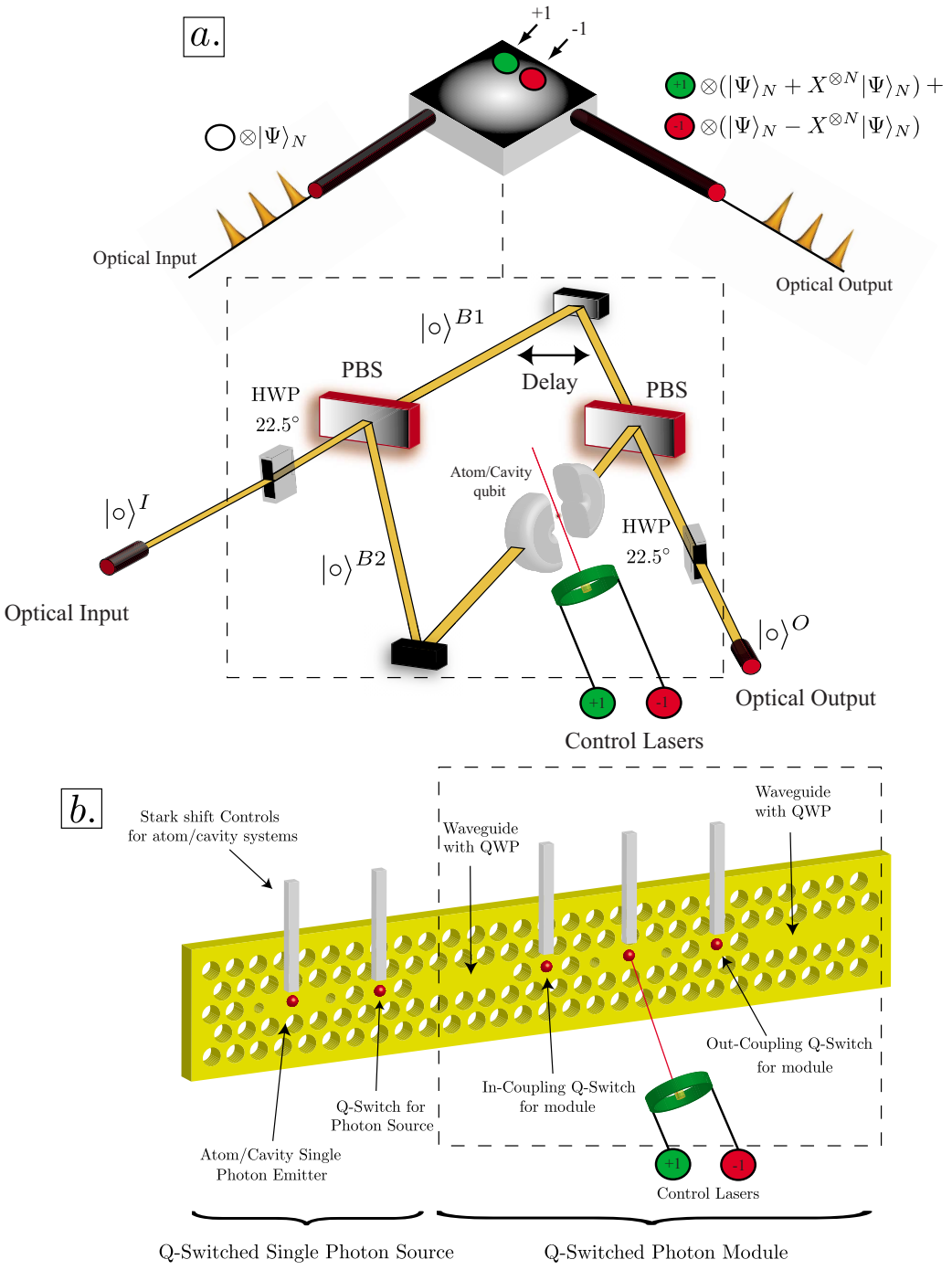

FIG. 1. (Color online) Schematics showing the basic design of a photonic module (components within the dashed boxes) in free space and photonic crystals. (a) Photonic module design for a polarization-independent atom-photon interaction, requiring two HWP and two PBS in free space. The polarization-dependent interferometer ensures that only the vertical component of the single photon interacts with the atom–cavity qubit. (b) A photonic band-gap structure for a polarization-dependent atom-photon version of the photonic module, required to prepare two-photon Bell states and higher order GHZ states. The initial two cavities represent a Q-switched single-photon source [45]. Single photons are adiabatically switched from the source to the first Q-switch cavity and then switched into the waveguide containing a quarter wave plate (QWP) which rotates $|\pm\rangle \rightarrow |\sigma_\pm\rangle$. The second Q-switch cavity is then used to adiabatically load the photon into the module cavity which contains the atomic system with a differential coupling between $|\sigma_\pm\rangle$ photons. The final Q-switch cavity is then used to out-couple the photon back into the waveguide mode once the interaction is complete, where a second QWP rotates $|\sigma_\pm\rangle \rightarrow |\pm\rangle$. In both schematics, the atom–cavity qubit has appropriate laser control such that it can be initialized and read out. The measurement result of the atom qubit (denoted via the green and red readout channels of the external control lasers) determines which eigenstate of $X^{\otimes N}$ the exiting photon train is projected to. A green measurement outcome corresponds to the photons projected into a +1 eigenstate while a red measurement outcome corresponds to projecting to the −1 eigenstate.

states appropriate for optical cluster state computation [6,7,38]. Bell state analyzers and factories are useful resources for quantum cryptographic protocols [3], quantum dense coding [39,40], purification protocols, and quantum repeaters [41,42]. Additionally, quick and deterministic preparation of N-photon Greenberger-Horne-Zeilinger (GHZ) states can be utilized in loss protection schemes for optical quantum computing [43] and secret sharing protocols [4]. We begin the discussion with the module shown in Fig. 1(a).

The basic operation of the module is best understood if we choose to use it as a factory for two-photon Bell states, defined through polarization as

$$|\Phi^+\rangle = \frac{|H\rangle|H\rangle + |V\rangle|V\rangle}{\sqrt{2}}. \tag{1}$$

Given an appropriate single-photon source, which can produce a train of single-photon pulses of known polarization, separated by an interval Δt, a two-photon train is prepared in the product state $|H\rangle_2^I|H\rangle_1^I$, and sequentially sent through the module. The indices, $\{1,2\}=\{0,\Delta t\}$, represent the temporal mode of each single-photon pulse, I the spatial mode (in this case the optical input), and Δt is predefined and must be greater than the total time a single photon is present within the network.

For a single photon passing through the module, the natural operation of the module, M, is given by

$$M|+\rangle^I|\phi\rangle \rightarrow |+\rangle^O|\phi\rangle,$$

$$M|-\rangle^I|\phi\rangle \rightarrow |-\rangle^O|\phi'\rangle, \tag{2}$$

where $|\pm\rangle=(|H\rangle\pm|V\rangle)/\sqrt{2}$, $|\phi\rangle=\alpha|0\rangle+\beta|1\rangle$ is the state of the atomic qubit, $|\phi'\rangle=X|\phi\rangle=\alpha|1\rangle+\beta|0\rangle$, and the indices $\{I,O\}$ represent the input and output optical modes.

The atom–cavity system is positioned such that the cavity mode is coupled to the spatial mode $|\circ\rangle^{B2}$, where $\circ$ denotes the photon polarization and cavity Q switching (which allows for the adiabatic loading of a single photon into a cavity) is employed before and after the atom-photon interaction to ensure appropriate in and out coupling to and from the cavity. The mode $|\circ\rangle^{B1}$ contains an optical delay equal to the time required for the photon-atom interaction which is specific. A single photon passing through the atom–cavity system must induce a photonic nondemolition bit flip on the two-level atom, releasing the photon again into $|\circ\rangle^{B2}$ once the interaction is complete.

If the photonic state is $|+\rangle$, the initial half-wave plate (HWP) will rotate the state to $|H\rangle$ after which it will continue into the mode $|\circ\rangle^{B1}$ and not interact with the atom. The second polarizing beam splitter (PBS) and HWP will then couple $|\circ\rangle^{B1}$ to the output mode and rotate $|H\rangle$ back to $|+\rangle$. If the initial photonic state is $|-\rangle$, the HWP will rotate the state to $|V\rangle$ and the PBS will reflect the photon into the $|\circ\rangle^{B2}$ mode, where it flips the state of the atomic qubit. The photon is then released back into $|\circ\rangle^{B2}$ where the second PBS and HWP will reflect the photon into the output mode and rotate it from $|V\rangle$ to $|-\rangle$. Therefore, the two basis states, $|\pm\rangle$, of a single photon passing through the module will enact the transformation M shown in Eq. (2).

For a two-photon train, polarized in the state $|H\rangle_2^I|H\rangle_1^I=(|+\rangle_2^I+|-\rangle_2^I)(|+\rangle_1^I+|-\rangle_1^I)/2$, we are able to enact the same transformations on the photon-atom interaction, giving

$$\begin{aligned} M_{2,1}|H\rangle_2^I|H\rangle_1^I|\phi\rangle &= \frac{1}{\sqrt{2}}M_2(|H\rangle_2^I|+\rangle_1^O|\phi\rangle + |H\rangle_2^I|-\rangle_1^O|\phi'\rangle) \\ &= \frac{1}{2}(|-\rangle_2^O|+\rangle_1^O + |+\rangle_2^O|-\rangle_1^O)|\phi'\rangle \\ &\quad + \frac{1}{2}|-\rangle_2^O|-\rangle_1^O|\phi''\rangle + \frac{1}{2}|+\rangle_2^O|+\rangle_1^O|\phi\rangle, \end{aligned} \tag{3}$$

where $|\phi''\rangle=X|\phi'\rangle=X^2|\phi\rangle$. Since $X^2=I$, we can expand out the $|\pm\rangle$ states to give

$$\begin{aligned} M_{2,1}|H\rangle_2^I|H\rangle_1^I|\phi\rangle &= \frac{1}{2}(|H\rangle_2^O|H\rangle_1^O + |V\rangle_2^O|V\rangle_1^O)|\phi\rangle \\ &\quad + \frac{1}{2}(|H\rangle_2^O|H\rangle_1^O - |V\rangle_2^O|V\rangle_1^O)|\phi'\rangle. \end{aligned} \tag{4}$$

After both photons have passed through the module the final step is to measure the state of the atom–cavity qubit. If prior to the interactions, the atomic qubit is initialized in the state $|\phi\rangle=|0\rangle$ and the subsequent measurement is also $|0\rangle$, the photons are projected to the state

$$\frac{1}{\sqrt{2}}(|H\rangle_2^O|H\rangle_1^O + |V\rangle_2^O|V\rangle_1^O), \tag{5}$$

which represents an even parity Bell state. If the atom is measured in the state $|1\rangle$, the photons are projected to

$$\frac{1}{\sqrt{2}}(|H\rangle_2^O|H\rangle_1^O - |V\rangle_2^O|V\rangle_1^O), \tag{6}$$

which is an odd parity Bell state. The output pulse consists of the original two-photon train which is now polarization entangled into a two-photon Bell state. Unlike other schemes, the measurement result of the atom–cavity system never collapses the photons to unentangled states. In fact, since the odd and even parity Bell states differ through local phase flips, either result is acceptable and a positive parity state can be prepared by applying a local, classically controlled phase flip on any photon once the atom–cavity qubit is measured.

The preparation of the Bell state is therefore completely deterministic, with the classical result only giving parity information of the photon state. Additionally, since positive and negative parity states are interchangeable through local Clifford gates, correction can be fed forward to the end of subsequent operations on the photonic state. Although we explicitly considered the case when the induced operation was a bit flip, any Hermitian unitary operation, $U^2=I$, is acceptable provided it transforms the state of the atomic qubit between two orthogonal states.

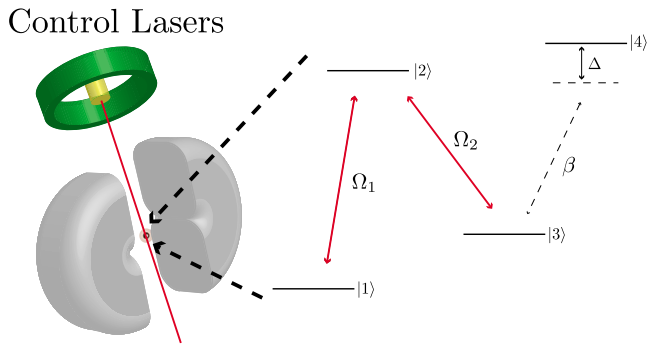

FIG. 2. (Color online) Four level atomic system required for the photonic module. The atomic system is initialized in the $|1\rangle$ state and classical pumping fields are used to take $|1\rangle \rightarrow (|1\rangle - |3\rangle)/\sqrt{2}$. The single-photon pulse is introduced to the cavity in a controlled manner using Q-switched cavities. The photon will induce a light shift on state $|3\rangle$ through the transition $|3\rangle \leftrightarrow |4\rangle$ (which may be polarization dependent), with strength $-\beta^2/\Delta$. Provided the photon remains in the cavity long enough, a π phase shift can be induced on $|3\rangle$ without destroying the photon, taking $(|1\rangle - |3\rangle)/\sqrt{2} \rightarrow (|1\rangle + |3\rangle)/\sqrt{2}$. After all atom-photon interactions have occurred, the classical fields are again applied and the atom readout is in the $\{|1\rangle, |3\rangle\}$ basis.

The transformation, M, shown in Eq. (2) is also exhibited by the module illustrated in Fig. 1(b) for a polarization-dependent interaction. For appropriate atomic systems it is well known that there exists states with a differential dipole coupling between σ_+ and σ_- polarized photons, e.g., NV^- diamond [44] or rubidium. If the atomic coupling to the cavity mode is chosen such that only σ_- polarized photons interact with the atomic qubit, we are able to eliminate the interferometer shown in Fig. 1(a). Instead, single-photon wave plates are used to rotate $|\pm\rangle \leftrightarrow |\sigma_\pm\rangle$ before and after the atom–cavity interaction. As the atom-photon interaction is polarization dependent, the transformation, M, for this modified version of the module still holds.

The engineering of the module when a polarization-dependent interaction is available is beneficial. The lack of the interferometer implies that this structure can be directly fabricated in systems such as photonic band-gap crystals, with cavity Q-switching protocols fabricated on-chip to control the in and out coupling of the single-photon pulses [Fig. 1(b)].

II. ATOM–CAVITY INTERACTION WITH PHOTON PULSE

The required atom–cavity interaction, crucial to the operation of the modules, has already been demonstrated at microwave frequencies. Schuster *et al.* [36] has demonstrated the nondestructive interaction we require where a single microwave cavity photon produces an effective Stark shift on a classically driven transition on a Cooper-pair box. These results were presented from the perspective of individual photon number detection, but the scheme can be inverted and used as the primary resource for a microwave version of the photonic module.

In the optical regime, we can consider two separate schemes. The proposal of Duan, Wang, and Kimble [34,35] considers a single atom–cavity system, such that a single photon reflecting from the cavity will produce a π phase shift *only* if the atomic qubit is in the state $|0\rangle$. Hence, if the atomic qubit is initially placed in the state $|+\rangle = (|0\rangle + |1\rangle)/\sqrt{2}$, subsequent photonic reflections will cause the atom to oscillate between the $\{|+\rangle, |-\rangle\}$ states. Performing readout in the $\{|0\rangle, |1\rangle\}$ basis corresponds to the (photonic) nondemolition, X operation on the atom, as required.

A second method employs a four-level atom in the N configuration, shown in Fig. 2, which can be utilized for both the polarization-dependent and polarization-independent modules illustrated in Fig. 1. The general principle is to induce a phase shift $Z \equiv \sigma_z$ on the atom, conditional on the presence or absence of a photon in the cavity mode.

Initialization. The atomic system is initialized in the ground state; classical fields (Ω_1, Ω_2) are then used to take $|1\rangle \rightarrow (|1\rangle - |3\rangle)/\sqrt{2}$.

Interaction. We employ the idea of cavity Q switching [45] in order to control the input-output pulse into the cavity system. A single photon is adiabatically switched into the cavity, where it is off-resonant with the $|3\rangle \rightarrow |4\rangle$ transition (which may be polarization dependent), inducing a light shift on the state $|3\rangle$. The magnitude of the shift is well known and is given by $\delta = -\beta^2/\Delta$ for $\Delta \gg \beta$. Therefore, to induce a phase shift of γ, the photon must be present in the cavity for a time given by $t = (\gamma\Delta)/\beta^2$. Consistent with the analysis in [45], this implies that the photon storage time, $\kappa = 1/t$, must be $\kappa = \beta^2/(\gamma\Delta)$. Taking $\gamma = \pi$ ensures that the state $(|1\rangle - |3\rangle)/\sqrt{2} \leftrightarrow (|1\rangle + |3\rangle)\sqrt{2}$ performs the required Z operation on the atomic system. The photon is then Q switched out of the cavity back into the optical mode using appropriate shaping techniques [45,46].

Readout. Readout is achieved by performing a second transform on the atomic state using the classical fields, Ω_1 and Ω_2, in the same way as for initialization. This takes the state $(|1\rangle - |3\rangle)/\sqrt{2} \rightarrow |1\rangle$ and $(|1\rangle + |3\rangle)/\sqrt{2} \rightarrow |3\rangle$. Computational readout can be performed by classically pumping the transition $|3\rangle \leftrightarrow |4\rangle$ and observing photoluminescence. Although the atom-photon interaction induces a Z gate on the basis states $\{(|1\rangle \pm |3\rangle)/\sqrt{2}\}$, the atomic rotations and readout in the $\{|1\rangle, |3\rangle\}$ basis ensures that a bit flip is performed on the atomic state.

Operational time. For this specific method, we can examine the transition time for a single photon in the module and compare our cavity requirements with systems currently in existence. In general, we wish to maintain single-photon absorption probabilities on the $|3\rangle \leftrightarrow |4\rangle$ transition of less than $\zeta \ll 1$, hence $\Delta \geq \beta/\sqrt{\zeta}$. For a π phase shift, and choosing the equality, this corresponds to $t = (\kappa)^{-1} = \pi/(\beta\sqrt{\zeta})$. For a proof of concept device, we assume $\zeta = 0.1$, which corresponds to an average of one in 10 photons being absorbed. Consequently, we can examine t as a function of some of the current experimental values for β and κ (Table I), The last column in Table I specifies the atomic decay rates, Γ, for the atomic systems used in each cavity (cesium, rubidium, and NV diamond). The coherence time of the atomic system will dictate the maximum parity weight which can be measured

TABLE I. Estimates on cavity requirements for various systems. The cavity from Boozer *et al.* [47] has been experimentally demonstrated, while the atom chip cavity of Trupke *et al.* [48] and the photonic band-gap cavity of Song *et al.* [49] have yet to couple the atomic qubit. Hence we use their theoretical estimates for the coupling, β, and atomic decay rates, Γ, for rubidium and NV^- [13,45] qubits. The first column quotes the atom–cavity coupling while the second column estimates the required photon storage time in the cavity to invoke a π phase shift in the atom with a single-photon absorption probability of 10%. The final column quotes the current photon storage time which has been experimentally demonstrated (estimated) for each cavity system. For both the Boozer and Trupke cavities, approximately an order of magnitude improvement in either the coupling constant or cavity lifetime is required. Current estimates suggest that the photonic band-gap cavity will be able to exhibit the interaction with the fastest operational time of all the systems and is also more amenable to current cavity Q-switching protocols [45]. The last column details estimates on atomic decay rates for the systems considered for each cavity, the ratio of the required photon storage time to the coherence time of the atomic system dictates the maximum parity weight a single module can measure in any one step.

Cavity	β (MHz)	t at $\zeta=10^{-1}$	Expt. $t=\kappa^{-1}$	Γ (MHz)
Cs [47]	34	0.29 μs	0.24 μs	2.6
Rb [48]	366	27 ns	1.7 ns	6.3
NV^- [49]	$\approx 10^4$	$\approx$1 ns	$\approx$3.4 ns	83

in any one step using the module. Hence taking the ratio of the required photon storage time to Γ, the Cs cavity of Boozer *et al.* [47] falls slightly short of being able to perform two-photon parity measurements (which are sufficient to prepare Bell states, GHZ states, and linear cluster states [17]). The microcavity of Trupke *et al.* [48], using Rb, theoretically has sufficient coherence to allow a parity check over six photons (sufficient for universal cluster states), while NV^- in photonic band-gap cavities could allow for parity weights up to 12 [13,49], allowing for a huge amount of flexibility in preparing highly entangled graph states very quickly.

III. ARBITRARY ENTANGLED STATE PREPARATION

The potential of these modules goes far beyond the preparation of Bell states. In fact, the unit can be augmented with appropriate single-photon routing and local operations to prepare *any* entangled photon state that can be expressed in terms of stabilizers [37]. These include codeword states for quantum error correction, Bell states, GHZ states, and arbitrary graph states (of which cluster states are a specific topological subset).

A remarkable property of the module is that the number of entangled photons that are prepared depends only on the number sent through the module; no internal structure of the module needs to be altered to entangle more photons.

To illustrate, consider an N-photon train, with each single-photon pulse separated by Δt. Each basis element, $|\psi\rangle$, of the state, $|\Psi\rangle$, can be written in the form $|\psi\rangle=\otimes_{a=0}^{N-1}|c_a\rangle_a=\otimes_{a=0}^{N-1}[|+\rangle_a+(-1)^{c_a}|-\rangle_a]$, where $|c_a\rangle=\{|H\equiv 0\rangle,|V\equiv 1\rangle\}$ and $|\pm\rangle=(|H\rangle\pm|V\rangle)/\sqrt{2}$, with each single-photon pulse centered at time $t=a\Delta t$. As we have shown, the transformations performed by the module are given by $M|+\rangle^I|\phi\rangle_q=|+\rangle^O|\phi\rangle_q$ and $M|-\rangle^I|\phi\rangle_q=|-\rangle^O|\phi'\rangle_q$, where $|\phi\rangle_q$ is the state of the atomic qubit, $|\phi'\rangle_q=X|\phi\rangle_q$, I and O are the input-output modes of the module and, for clarity, we have omitted the time index for the pulse. Assuming that the atomic system is initialized in the state $|\phi\rangle_q=|0\rangle$, an arbitrary basis state of $|\Psi\rangle_N$ transforms as

$$\begin{aligned} M_{N,\ldots,1}|\psi\rangle|0\rangle_q &= M_{N,\ldots,1}\bigotimes_{a=0}^{N-1}[|+\rangle+(-1)^{c_a}|-\rangle]|0\rangle_q \\ &= |0\rangle_q\bigotimes_{a=0}^{N-1}[|+\rangle+(-1)^{c_a}|-\rangle]_{Ev|-\rangle} \\ &\quad + |1\rangle_q\bigotimes_{a=0}^{N-1}[|+\rangle+(-1)^{c_a}|-\rangle]_{Od|-\rangle}, \end{aligned} \tag{7}$$

where the first term represents all states of the tensor product formed with an even number of $|-\rangle$ states and the second term represents all tensor products formed with an odd number of $|-\rangle$ states. The even and odd components of the basis terms, $|\psi\rangle$, can be written in the following manner:

$$\begin{aligned} \bigotimes_{a=0}^{N-1}[|+\rangle+(-1)^{c_a}|-\rangle]_{Ev|-\rangle} &= \frac{1}{2}\bigotimes_{a=0}^{N-1}[|+\rangle+(-1)^{c_a}|-\rangle] \\ &\quad + \frac{1}{2}\bigotimes_{a=0}^{N-1}[|+\rangle+(-1)^{c_a+1}|-\rangle], \\ \bigotimes_{a=0}^{N-1}[|+\rangle+(-1)^{c_a}|-\rangle]_{Od|-\rangle} &= \frac{1}{2}\bigotimes_{a=0}^{N-1}[|+\rangle+(-1)^{c_a}|-\rangle] \\ &\quad - \frac{1}{2}\bigotimes_{a=0}^{N-1}[|+\rangle+(-1)^{c_a+1}|-\rangle]. \end{aligned} \tag{8}$$

Noting that the second term in each equation is simply the state $X^{\otimes N}|\psi\rangle$, each basis term, $|\psi\rangle$, transforms as

$$M_{N,\ldots,1}|\psi\rangle|0\rangle_q=\frac{1}{2}(|\psi\rangle+X^{\otimes N}|\psi\rangle)|0\rangle_q+\frac{1}{2}(|\psi\rangle-X^{\otimes N}|\psi\rangle)|1\rangle_q, \tag{9}$$

and consequently, the total state, $|\Psi\rangle=\Sigma_j\beta_j|\psi\rangle_j$, transforms as

$$\begin{aligned} M_{N,\ldots,1}|\Psi\rangle|0\rangle_q &= \frac{1}{2}(|\Psi\rangle+X^{\otimes N}|\Psi\rangle)|0\rangle_q \\ &\quad + \frac{1}{2}(|\Psi\rangle-X^{\otimes N}|\Psi\rangle)|1\rangle_q. \end{aligned} \tag{10}$$

Therefore, the natural operation of the module is to project the train of photons into a ± 1 eigenstate of the $X^{\otimes N}$ operator, i.e., any arbitrary N-photon state will be transformed to

$$M_{N,\ldots,1}|\Psi\rangle_N|0\rangle = \frac{1}{2}(|\Psi\rangle_N + X^{\otimes N}|\Psi\rangle_N)|0\rangle + \frac{1}{2}(|\Psi\rangle_N - X^{\otimes N}|\Psi\rangle_N)|1\rangle, \quad (11)$$

where $\{|0\rangle,|1\rangle\}$ are the states of the atom–cavity qubit and all N photons have been passed through the module. The measurement outcome of the atomic system will determine which eigenstate is projected, with local operations applied to switch between eigenstates. The stabilizer formalism for describing large entangled states is extremely useful in this discussion as they are linked very closely to the concept of parity measurements.

To prepare *any* N-photon stabilized state, a parity check is performed on the N stabilizers which describe the state. As each of the stabilizers for an arbitrary N-photon state are described via an N-fold tensor product of the operators $\{I,X,Y,Z\}$, the ability to perform a parity check of the operator $X^{\otimes N'}$ for $N' \leq N$ and apply local operations is sufficient to stabilize an arbitrary state with respect to any operator of this form. Therefore, if we assume that we can selectively route photons within the train (which is possible, as each pulse is temporally tagged) and apply local operations to any photon, the parity measurement performed by the module is sufficient to prepare any stabilizer state.

For a general N-photon state, N parity checks are required. This can either be done by constructing and utilizing N separate modules, or it can be done by sequentially utilizing only one. If multiple modules are available, many parity checks can be done in parallel without waiting for atomic readout, potentially speeding up state preparation.

As the stabilizer structure of the desired state dictates the number of photons passed through the module for each parity check, the coherence time of the atom–cavity system does not depend on the total number of photons in the entangled state. Instead, the atomic system must only maintain coherence until the parity of a specific stabilizer operator is measured, which is extremely beneficial. The number of nonidentity operators in any given stabilizer operator (which we denote the "parity weight") dictates the number of photons passed through the module in any one step and therefore the coherence time required for the atom–cavity system.

For example, an N-photon cluster state appropriate for quantum computation has a well-known stabilizer structure [6], with a maximum parity weight of five. Hence, regardless of the total size of the cluster, the atomic system only needs to maintain coherence long enough for five photons to pass through the module between initialization and measurement.

Conversely, if the coherence time of the atomic system is short compared to $P_m \Delta t$, where P_m is the maximum parity weight of the state and Δt is the time required for a single photon to pass through the module, then fusion methods [19,21,38] can be employed to prepare states with large P_m. For example, N-photon GHZ states have $P_m = N$, corresponding to the stabilizer, $K = X^{\otimes N}$. If the coherence time of the atomic system only allows for $N' < N$-dimensional parity checks to be performed at any one time, then multiple N'-GHz states can be prepared and fused together via two-photon ZZ parity measurements.

IV. CONCLUSIONS

We have detailed the construction of a photonic module which, given a steady source of single photons, can deterministically prepare a large class of useful photonic entangled states. The construction of each module is generic and independent of which entangled state is being prepared, and the stabilizer nature of the entangled states implies that the coherence time of the atomic system only needs to be long compared with the maximum parity weight (P_m) $\times$ pulse separation (Δt) of the desired state (which can be small, even for large multiphoton entangled states).

The practical uses of these modules is quite extensive. Multiphoton entangled states can be utilized for quantum computation, quantum cryptography, quantum dense coding, and quantum repeaters.

As the internal design of the photonic module is completely independent of the state being prepared, multiple modules, combined with an appropriate single-photon source, optical wave plates, and classical routing can be used to construct a static, on-chip system, tailored for fast preparation of specific entangled states. For example, pumping out large cluster states for computation, or multiple Bell pairs, in succession, for communications and cryptography.

The engineering of an appropriate atom-photon interaction is still something that needs to be experimentally investigated. Cavity experiments reported in [36,47–50] show exceptional promise at both optical and microwave frequencies. Ideally, once the required interaction has been experimentally demonstrated, additional engineering to realize the photonic module should be straightforward.

ACKNOWLEDGMENTS

The authors thank J. H. Cole, Z. Evans, D. Jamieson, R. van Meter, G. J. Milburn, K. Nemoto, D. K. L. Oi, S. Prawer, R. Scholten, A. M. Stephens, and C.-H. Su for helpful discussions. One of the authors (S.J.D.) acknowledges the support of the David Hay Foundation. This work was supported, in part, by the Australian Research Council, the Australian Government, the U.S. National Security Agency (NSA), Advanced Research and Development Activity (ARDA), Army Research Office (ARO) under Contracts Nos. W911NF-04-1-0290 and W911NF-05-1-0284, QAP, and MEXT.

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