Implementation of a nonlocal N-qubit conditional phase gate by single-photon interference

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By virtue of single-photon interference, we present how to realize a nonlocal N-qubit conditional phase gate, which might be quite useful for the synthesis of arbitrary entangled quantum states of remote qubits required by distributed quantum information processing. Without considering photon loss, our scheme would work in a repeat-until-success fashion with an automatic feedback line added. Even by taking photon loss into consideration, only the success probability is affected, not the gate fidelity.

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In quantum computation, it is well known that any unitary operation can be decomposed into two kinds of elementary gates [1]—i.e., one-qubit rotations and two-qubit conditional gates. For some applications such as quantum Grover searches [2], quantum error corrections [3], and entangled-state preparation [4], the implementation of multiqubit conditional phase gates is frequently needed. Since the decomposition of the N-qubit gates becomes more and more complicated with the increase of qubit number N, it is much more preferable to realize an N-qubit gate directly. For this purpose, many schemes concerning the direct realization of multiqubit gates have been proposed [5,6].

On the other hand, generation of multiparty entanglement states is of vital importance, which enables the experimental implementation of many distributed quantum information processing tasks—e.g., multiparty quantum teleportation [7], quantum telecloning [8], quantum secret sharing [9], and distributed quantum computation [10]. According to Ref. [4], any arbitrary entangled states can be synthesized by using networks involving multiqubit conditional phase gates, whereas for states distributed among different nodes, nonlocal multiqubit conditional phase gates have to be used. In Ref. [11], the authors investigated the minimal resources required for the nonlocal gate if auxiliary entanglement pairs, local operations, and classical communication are permitted. For N-party nonlocal controlled-unitary gates, it needs $2(N-1)$ bits of classical communication and $N-1$ additional shared ebits. There are various proposals for two-qubit nonlocal gates [6,12–15]. By using cavity-assisted photon scattering [16], Ref. [12] showed explicitly how to get the nonlocal two-qubit controlled NOT gate in the spirit of Ref. [11], while Refs. [6,13] realized the two-qubit controlled phase gate simply by reflecting the same single photon by two cavities sequentially. In Ref. [14], the controlled phase gate between two remote atoms is obtained by using photon interference and detection. A direct connection between two spatially distant nodes by an optical fiber can also achieve the nonlocal controlled phase gate between them [15].

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In this work, we are aiming at directly realizing the nonlocal N-qubit conditional phase gate via the single-photon interference effect [17]. Compared with Ref. [12], our scheme also employs cavity-assisted photon scattering, but without the use of auxiliary entanglement pairs and classical communication. Unlike the local multiqubit gate in Refs. [5,6],our proposed nonlocal multiqubit conditional phase gate might be very useful for preparing arbitrary entangled states [4] of spatially different nodes, which are an indispensable ingredient for the distributed quantum information processing tasks mentioned above. Moreover, due to direct implementation of nonlocal multiqubit gates, the physical realization can be greatly simplified from those resorting to two-qubit nonlocal gates [6,12–15].

First, we briefly recall the controlled phase gate between a single photon and an atom inside a cavity [6,12,13,16], as shown in Fig. 1. The atom j has a three-level configuration—i.e., one excited level and two ground levels. Levels $|0\rangle_j$ and $|e\rangle_j$ are resonantly coupled by a cavity mode with h polarization, which can also be resonantly driven by the h component of the input photon. Suppose the input photon is of h polarization; it will have a resonant interaction with the cavity if the atom is in state $|1\rangle_j$. When $\kappa T \gg 1$ (where T is the duration of input photon pulse and κ is the cavity decay rate) is satisfied, the pulse will be reflected by the cavity with its pulse shape almost unchanged but its phase added by π. If the atom is in state $|0\rangle_j$, the coupling between the atom and the cavity will shift the cavity frequency. Thus the pulse will be reflected by an off-resonant cavity with both its shape and phase unchanged. When it is of v polarization, the input

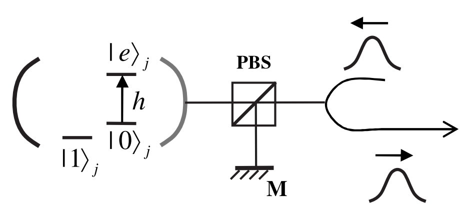

FIG. 1. Schematic setup for implementing the controlled phase gate between a single photon and an atom inside a cavity, where the level structure of the atom is depicted inside the cavity and the PBS is used to transmit the h component while to reflect the v component of the input single-photon pulse.

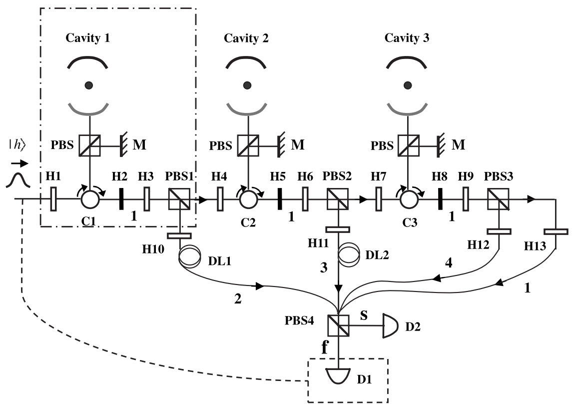

FIG. 2. Schematic setup for implementing the nonlocal three-qubit conditional phase gate, where $H1$, $H3$, $H4$, $H6$, $H7$, $H9$, $H10$, $H11$, $H12$, and $H13$ (open rectangles) are 22.5°-titled half-wave plates, $H2$, $H5$, and $H8$ (solid rectangles) are 45° titled half-wave plates, $C1$, $C2$, and $C3$ are circulators, and DL1 and DL2 are delay lines. The optical lengths of the four possible paths 1-2, 1-1-3, 1-1-1-4, and 1-1-1-1 should be well designed to have good interference at PBS4. The detector $D1$ would be also eliminated and replaced by a path directed to the initial photon input port.

photon will be reflected by the mirror M without any change. So by reflecting a polarization-encoded single photon, the controlled phase gate $U_j = \exp(i\pi|1\rangle_j\langle 1| \otimes |h\rangle\langle h|)$ is realizable.

The schematic setup for realizing the nonlocal three-qubit conditional phase gate is plotted in Fig. 2. Both 22.5°- and 45°-tilted half-wave plates (the titled angle means the angle between the axis of the half-wave plate and the horizontal direction) are needed, and we distinguish them by labeling with open and solid rectangles, respectively. The 22.5°-tilted half-wave plates perform a Hadamard gate on the photon polarization states—i.e., $|h\rangle \rightarrow (1/\sqrt{2})(|h\rangle + |v\rangle)$, $|v\rangle \rightarrow (1/\sqrt{2})(|h\rangle - |v\rangle)$—while the 45°-tilted half-wave plates rotate the photon polarization as $|h\rangle \leftrightarrow |v\rangle$. We assume that the input photon is initially in the state $|h\rangle$ and the three remote atoms are in an arbitrary state $\alpha_1|000\rangle + \alpha_2|001\rangle + \alpha_3|010\rangle + \alpha_4|011\rangle + \alpha_5|100\rangle + \alpha_6|101\rangle + \alpha_7|110\rangle + \alpha_8|111\rangle$, where $|klm\rangle$ denotes the state of atom 1 to atom 3 from left to right with k, l, m equal to 0 or 1. So after the photon passes through the half-wave plate $H1$, we get

$$\frac{1}{\sqrt{2}}(|h\rangle_1 + |v\rangle_1) \otimes (\alpha_1|000\rangle + \alpha_2|001\rangle + \alpha_3|010\rangle + \alpha_4|011\rangle + \alpha_5|100\rangle + \alpha_6|101\rangle + \alpha_7|110\rangle + \alpha_8|111\rangle). \tag{1}$$

Then the photon is reflected by cavity 1, and the state of the whole system is

$$\frac{1}{\sqrt{2}}(|h\rangle_1 + |v\rangle_1) \otimes (\alpha_1|000\rangle + \alpha_2|001\rangle + \alpha_3|010\rangle + \alpha_4|011\rangle) + \frac{1}{\sqrt{2}}(-|h\rangle_1 + |v\rangle_1) \otimes (\alpha_5|100\rangle + \alpha_6|101\rangle + \alpha_7|110\rangle + \alpha_8|111\rangle). \tag{2}$$

The reflected photon from cavity 1 then goes through half-wave plates $H2$ and $H3$ and polarization beam splitter PBS1. The photon-atom system now becomes

$$|h\rangle_1 \otimes (\alpha_1|000\rangle + \alpha_2|001\rangle + \alpha_3|010\rangle + \alpha_4|011\rangle) + |v\rangle_2 \otimes (\alpha_5|100\rangle + \alpha_6|101\rangle + \alpha_7|110\rangle + \alpha_8|111\rangle). \tag{3}$$

It is easy to see that the design inside the dash-dotted rectangle is used to decide whether to transmit or to reflect the photon pulse depending on the state of atom 1. If atom 1 is in state $|0\rangle_1$, the input photon $|h\rangle$ will pass through PBS1 without a change of the photon state, while the input photon will be turned into state $|v\rangle$ and reflected by PBS1, if atom 1 is in state $|1\rangle_1$. Similarly, we let the transmitted h component of the photon from PBS1 (PBS2) be transmitted or reflected by PBS2 (PBS3) depending on the state of the atom inside the second (third) cavity. Following the steps in Fig. 2, we have

$$\xrightarrow{H4,C2,H5,H6,\mathrm{PBS2}} |h\rangle_1 \otimes (\alpha_1|000\rangle + \alpha_2|001\rangle) + |v\rangle_3 \otimes (\alpha_3|010\rangle + \alpha_4|011\rangle) + |v\rangle_2 \otimes (\alpha_5|100\rangle + \alpha_6|101\rangle + \alpha_7|110\rangle + \alpha_8|111\rangle),$$

$$\xrightarrow{H7,C3,H8,H9,\mathrm{PBS3}} |h\rangle_1 \otimes \alpha_1|000\rangle + |v\rangle_4 \otimes \alpha_2|001\rangle + |v\rangle_3 \otimes (\alpha_3|010\rangle + \alpha_4|011\rangle) + |v\rangle_2 \otimes (\alpha_5|100\rangle + \alpha_6|101\rangle + \alpha_7|110\rangle + \alpha_8|111\rangle),$$

$$\xrightarrow{H10,H11,H12,H13,\mathrm{PBS4}} \frac{1}{\sqrt{2}}|h\rangle_f \otimes (\alpha_1|000\rangle + \alpha_2|001\rangle + \alpha_3|010\rangle + \alpha_4|011\rangle + \alpha_5|100\rangle + \alpha_6|101\rangle + \alpha_7|110\rangle + \alpha_8|111\rangle) - \frac{1}{\sqrt{2}}|v\rangle_s \otimes (-\alpha_1|000\rangle + \alpha_2|001\rangle + \alpha_3|010\rangle + \alpha_4|011\rangle\alpha_5|100\rangle + \alpha_6|101\rangle + \alpha_7|110\rangle + \alpha_8|111\rangle). \tag{4}$$

During the above process, the photon might take four possible paths from the initial photon input port to arrive at PBS4. We have to make sure that the optical lengths of all the four possible paths are equal. At the final output ports f and s, we make measurement on the photon state with photon detectors $D1$ and $D2$, respectively. Each detector has one-half probability to register a photon. If $D1$ clicks, nothing has happened to the three-atom state. If $D2$ clicks, a conditional phase gate $U_{cp}^3 = \exp(i\pi|000\rangle\langle 000|)$ has been applied to the three atoms. We could also replace $D1$ with a path directed to the input port of the photon pulse [17], so the h polarization photon can be automatically fed back to restart a new trial. In this way, all we need to do is to wait for a photon detection at $D2$, which confirms the accomplishment of the U_{cp}^3. Therefore, without considering the photon loss, we could implement U_{cp}^3 in a repeat-until-success fashion.

The generalization to the nonlocal N-qubit conditional phase gate is illustrated in Fig. 3, where S_i is for the ith cavity with atomic qubit inside, the same as the structure

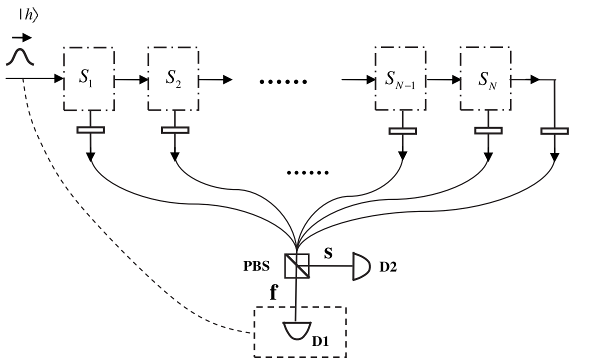

FIG. 3. Schematic setup for implementing the nonlocal N-qubit conditional phase gate, where S_1 denotes the design in the dash-dotted rectangle in Fig. 2 for cavity 1 and S_i is for the ith cavity having the same structure as S_1. The open rectangles, which denote the 22.5°-tilted half-wave plates, are used to perform Hadamard gate on the photon polarization states. The optical lengths of the $N+1$ possible paths should be well designed to have good interference at PBS.

plotted as the dash-dotted rectangle for cavity 1 in Fig. 2. The function of S_i could be known from Fig. 2; i.e., if the ith atom is in state $|0\rangle_i$, the input $|h\rangle$ photon will be transmitted to S_{i+1}, while the input photon will be turned into state $|v\rangle$ and directed to the PBS, if the ith atom is in state $|1\rangle_i$. Among the 2^N computational states $|X\rangle_{1,2,\ldots,N}$ with $X=0,1,\ldots,2^N-1$, it is obvious that only when the atoms are in state $|0\rangle_{1,2,\ldots,N}$ will the input photon be kept in the state $|h\rangle$. All the other computational states of the atoms will make the $|h\rangle$ photon convert to a $|v\rangle$ photon. Before reaching the PBS, we perform the Hadamard gate on the photon polarization states. The detection stage is the same as in the three-qubit case. With 50% probability, nothing has happened to the N-qubit atomic states if $D1$ gets clicked. With another 50% probability, $D2$ detects a photon and the N-qubit conditional phase gate $U_{cp}^N=\exp(i\pi|0\rangle_{1,2,\ldots,N}\langle 0|)$ is realized. Also like in Fig. 2, we would alternatively replace the detector $D1$ by a path directed to the initial input port of the photon.

The controlled phase gate U_j has been numerically simulated in Refs. [13,16]. When $\kappa T\gg 1$ and g is several times larger than κ and Γ (where g is the atom-cavity coupling strength and Γ is the spontaneous emission rate of the level $|e\rangle$) are satisfied, the gate fidelity of U_j is almost unity and the spontaneous emission probability is quite small. Considering a neutral atom trapped in a Fabry-Perot cavity with $(g,\kappa,\Gamma)/2\pi\approx(25,8,5.2)$ MHz, one may calculate the fidelity F to be 99.9% if $T=240/\kappa$ and the probability P_s=3.2% responsible for the gate error due to the spontaneous emission [16]. In the case of a rare-earth ion embedded in a silica-microsphere cavity with $(g,\kappa,\Gamma)/2\pi\approx(10^3,32,10^{-3})$ MHzand $T=3$ μs, F can reach 99.998% and P_s is about 10^{-8} [13], which are much better than in the former system. In our case, as the basic component of our scheme is from Refs. [13,16], if we simply suppose every single-photon polarization rotation to be perfectly carried out, we could roughly estimate the fidelity and the success probability of our proposed N-qubit conditional phase gate in one trial to be F^N and $(1-P_s)^N/2$, respectively. For $N=10$, we may obtain that the fidelity is $(0.99998)^{10}\approx 99.98\%$ and the success probability is $(1-10^{-8})^{10}/2\approx$ 50% for only one trial. Actually, there exist other factors leading to photon loss in realistic experiments, such as cavity absorption, cavity scattering, and absorption in the transmission lines.Photon loss in the middle way would affect the subsequent interactions. However, with detectors at the end of a design, the absence of a photon can signal those photon loss events out. If the photon loss has happened, the ideal repeated-until-success method fails and we have to restart the scheme. Thus, the photon loss events can be identified and discarded, which only affects the success probability, but does no damage to the gate fidelity. On the other hand, as single-photon multipath interference has been used, a significant challenge for carrying out our scheme is to maintain phase stability [18], which requires the path lengths to be stable at subwavelength levels. Anyway, with the rapid development of single-photonexperimental technology, we hope our scheme can be achieved in the near future.

In summary, by using single-photon interference, we have shown how to implement nonlocal N-qubit conditional phase gates with the aid of cavity-assisted photon scattering. We argue that our scheme might be very useful in quantum information processing distributed in different spatial nodes.

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[1] A. Barenco, C. H. Bennett, R. Cleve, D. P. DiVincenzo, N. Margolus, P. Shor, T. Sleator, J. A. Smolin, and H. Weinfurter, Phys. Rev. A **52**, 3457 (1995).
[2] L. K. Grover, Phys. Rev. Lett. **79**, 325 (1997).
[3] P. W. Shor, Phys. Rev. A **52**, R2493 (1995).
[4] M. Sasura and V. Buzek, Phys. Rev. A **64**, 012305 (2001).
[5] H. Goto and K. Ichimura, Phys. Rev. A **70**, 012305 (2004); A. Gábris and G. S. Agarwal, *ibid.* **71**, 052316 (2005); C.-Y. Chen, M. Feng, and K. L. Gao, *ibid.* **73**, 064304 (2006); Y.-F. Xiao, X.-B. Zou, and G.-C. Guo, *ibid.* **75**, 014302 (2007); **75**, 054303 (2007).
[6] L.-M. Duan, B. Wang, and H. J. Kimble, Phys. Rev. A **72**, 032333 (2005); X.-M. Lin, Z.-W. Zhou, M.-Y. Ye, Y.-F. Xiao, and G.-C. Guo, *ibid.* **73**, 012323 (2006).
[7] W. Dür and J. I. Cirac, J. Mod. Opt. **47**, 247 (2000); A. Karlsson and M. Bourennane, Phys. Rev. A **58**, 4394 (1998); J. Joo, Y.-J. Park, S. Oh, and J. Kim, New J. Phys. **5**, 136 (2003).
[8] D. Bruss, D. P. DiVincenzo, A. Ekert, C. A. Fuchs, C. Macchiavello, and J. A. Smolin, Phys. Rev. A **57**, 2368 (1998); M. Murao, D. Jonathan, M. B. Plenio, and V. Vedral, *ibid.* **59**, 156

(1999).

[9] M. Hillery, V. Buzek, and A. Berthiaume, Phys. Rev. A **59**, 1829 (1999); R. Cleve, D. Gottesman, and H.-K. Lo, Phys. Rev. Lett. **83**, 648 (1999).

[10] L. K. Grover, e-print arXiv:quant-ph/9607024; J. I. Cirac, A. K. Ekert, S. F. Huelga, and C. Macchiavello, Phys. Rev. A **59**, 4249 (1999).

[11] J. Eisert, K. Jacobs, P. Papadopoulos, and M. B. Plenio, Phys. Rev. A **62**, 052317 (2000).

[12] X.-F. Zhou, Y.-S. Zhang, and G.-C. Guo, Phys. Rev. A **71**, 064302 (2005).

[13] Y.-F. Xiao, X.-M. Lin, J. Gao, Y. Yang, Z.-F. Han, and G.-C. Guo, Phys. Rev. A **70**, 042314 (2004).

[14] X. B. Zou and W. Mathis, Phys. Rev. A **71**, 042334 (2005); Y. L. Lim, A. Beige, and L. C. Kwek, Phys. Rev. Lett. **95**, 030505 (2005).

[15] A. Serafini, S. Mancini, and S. Bose, Phys. Rev. Lett. **96**, 010503 (2006).

[16] L.-M. Duan and H. J. Kimble, Phys. Rev. Lett. **92**, 127902 (2004).

[17] J. Hong and H.-W. Lee, Phys. Rev. Lett. **89**, 237901 (2002); C.-H. Yuan and K.-D. Zhu, J. Phys. B **40**, 801 (2007).

[18] E. Knill, R. Laflamme, and G. J. Milburn, Nature (London) **409**, 46 (2001); J. L. O'Brien, G. J. Pryde, A. G. White, T. G. Ralph, and D. Branning, *ibid.* **426**, 264 (2003); D. E. Browne and T. Rudolph, Phys. Rev. Lett. **95**, 010501 (2005); N. K. Langford, T. J. Weinhold, R. Prevedel, A. Gilchrist, J. L. O'Brien, G. J. Pryde, and A. G. White, *ibid.* **95**, 210504 (2005).