Nonadiabatic geometric quantum computation

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A different way to realize nonadiabatic geometric quantum computation is proposed by varying parameters in the Hamiltonian for nuclear-magnetic resonance, where the dynamical and geometric phases are implemented separately without the usual operational process. Therefore the phase accumulated in the geometric gate is a pure geometric phase for any input state. In comparison with the conventional geometric gates by rotating operations, our approach simplifies experimental implementations making them robust to certain experimental errors. In contrast to the unconventional geometric gates, our approach distinguishes the total and geometric phases and offers a wide choice of the relations between the dynamical and geometric phases.

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I. INTRODUCTION

The wave function of a quantum system retains a memory of its motion in the form of a geometric phase factor [1–3] when it undergoes a closed evolution in the parameter space. Geometric phase has been observed in spin 1/2 systems through nuclear-magnetic-resonance (NMR) experiments [4] and with polarized photons using interferometers (PPI) [5]. Recently, geometric phase has been attracting increasing interest because of its importance for understanding and implementing quantum computation in real physical systems [6–9]. Geometric (holonomic) quantum computation is a scheme that is potentially intrinsically fault tolerant and therefore resilient to certain types of computational errors.

Up to now, there have been two approaches to obtain the geometric gate: (i) driving qubits to undergo appropriate adiabatic or nonadiabatic cyclic evolutions and (ii) displacing a harmonic oscillator along a closed path conditional on the state of the qubits. A gate obtained via the first approach is called a conventional geometric gate, where the dynamical phase is typically canceled by single-loop or multiloop rotating operations [6–12]. The latter is referred to as an unconventional geometric gate and it depends only on global geometric features in the rotating frame at the cavity frequency [13–17]. In comparison with the conventional geometric gates, the unconventional geometric gates do not require additional operations to cancel the dynamical phase. However, this approach does not distinguish between the total and the geometric phases.

It is difficult to experimentally realize quantum computation with adiabatic evolution [6] because the long operation time is required, especially for solid-state systems whose decoherence time is very short [10–12]. This is especially true given that the evolution has to be repeated several times to cancel the dynamical phase. It may be better therefore to construct geometric quantum gates by using the nonadiabatic geometric phase [18,19] since this allows for shortening gate times. In the established methods of geometric quantum computation, it is necessary to remove the dynamical component, such as by using dark states [8] and by rotating operations in so-called single-loop and multiloop schemes. The experimental errors are, obviously, increased because of the operational process. More worryingly, the dynamic phase accumulated in the gate operation is possibly nonzero and cannot be eliminated. It is interesting therefore to propose a different scheme to perform the geometric quantum computation, where the dynamical phase may either equal zero or has an expression with the corresponding geometric phase by controlling the Hamiltonian parameters.

II. SINGLE-QUBIT SYSTEM

Consider the Hamiltonian for a single-qubit system in nuclear magnetic resonance (NMR),

$$H(t) = -\frac{1}{2}\Omega_0(\sigma_x \sin\theta\cos\omega t + \sigma_y \sin\theta \sin\omega t) - \frac{1}{2}\Omega_1\sigma_z\cos\theta, \quad (1)$$

where $\Omega_i = g\mu B_i/\hbar$ with $g(\mu)$ are the gyromagnetic, B_i $(i=1,2)$ and θ act as an external controllable parameters and can be experimentally changed, and σ_i $(i=x,y,z)$ are the Pauli operators. For the initial time $t=0$ the magnetic field lies in the x-z plane. As the evolving time t increases the magnetic field rotates in the x-y plane. In the rotating frame, the effective Hamiltonian is $H' = -(\Omega/2)\exp(-i\theta'\sigma_y/2)\sigma_z \exp(i\theta'\sigma_y/2)$, where $\Omega = [\Omega_0^2 \sin^2\theta + (\Omega_1\cos\theta + \omega)^2]^{1/2}$ and $\theta' = \tan^{-1}\{\Omega_0 \sin\theta/(\Omega_1\cos\theta + \omega)\}$. Because the effective Hamiltonian H' is independent of the evolving time, the wave function $\psi'(t)$ in the rotating frame is exactly given by $\psi'(t) = \exp\{-itH'\}\psi'(0)$. Thus the wave function in NMR may be expressed as

$$\psi(t) = u(t)\psi(0) = \exp(-i\omega t\sigma_z/2)\exp(-itH')\psi(0), \quad (2)$$

where $u(t) = \exp(-i\omega t\sigma_z/2)\exp(-itH')$ and $\psi(0)$ is given by eigenfunctions of the Hamiltonian H', such as $H'\phi_k = -\frac{1}{2}\Omega k\phi_k$, where $k = \pm 1$. Thus the eigenfunctions may be written as $\phi_+ = \exp\{-\frac{i}{2}\theta'\sigma_y\}|0\rangle$ and $\phi_- = \exp\{-\frac{i}{2}\theta'\sigma_y\}|1\rangle$, where $|0\rangle$ and $|1\rangle$ constitute the computational basis for the qubit. For a cyclic motion, the total and dynamical phases are

$$\alpha_k = \pi k - \pi(\Omega/\omega)k, \quad \gamma_k^d = -\pi k(\Omega/\omega - \cos\theta'). \tag{3}$$

Thus the geometric phase is given by

$$\gamma_k^g = \alpha_k - \gamma_k^d = \pi k(1 - \cos\theta'). \tag{4}$$

From Eqs. (3) and (4), we find that, when the relation $(1+r)(\Omega_1 \cos\theta+\omega)=(\Omega/\omega+r)\Omega$ is satisfied, the total, dynamical and geometric phases satisfy

$$\gamma_k^d = r\gamma_k^g = rk\gamma, \quad \alpha_k = (1+r)\gamma_k^g = (1+r)k\gamma, \tag{5}$$

where $\gamma=\pi(1-\cos\theta')$ and the proportionality constant r may be either determined by

$$r = [(\Omega_1 \cos\theta+\omega) - \Omega^2/\omega]/[\Omega - (\Omega_1 \cos\theta+\omega)], \tag{6}$$

besides a trivial geometric phase, or used also as an arbitrary constant by adjusting the initial physical quantities, such as Ω_0, Ω_1, θ, and ω, according to requirements in experiment or in theory. We find

$$\Omega_0^2 \sin^2\theta = (1+r)\omega(\Omega_1 \cos\theta+\omega) - (\Omega_1 \cos\theta+\omega)^2 + \frac{1}{2}r^2\omega^2 \pm r\omega^{3/2}\sqrt{(1+r)(\Omega_1 \cos\theta+\omega) + \frac{1}{4}r^2\omega}, \tag{7}$$

which implies that there exist, indeed, some physically meaningful solutions. In other words, by adjusting the parameters Ω_1 and θ for a given r and frequency ω, we can find a positive value of magnetic field parameter Ω_0 in the x-y plane. For example, for $r=0$, if $\sin\theta>0$ and $\cos\theta<0$ are chosen, $\Omega_0=\sqrt{\omega(\Omega_1 \cos\theta+\omega)-(\Omega_1 \cos\theta+\omega)^2}/\sin\theta$ will be determined. If $\sin\theta<0$ and $\cos\theta<0$ are chosen, $\Omega_0=-\sqrt{\omega(\Omega_1 \cos\theta+\omega)-(\Omega_1 \cos\theta+\omega)^2}/\sin\theta$ will be determined. For $r=1$, similarly, by choosing $-(9/8)\omega<\Omega_1 \cos\theta<\sqrt{3/2}\omega$, we can get a group of positive Ω_0. These choices may easily be experimentally realized.

From Eq. (5), we find when $r=0$, $\gamma_k^d=0$ and $\alpha_k=\gamma_k^g$. In this case, the system is in a dark state and its dynamical phase disappears. The result is similar to the conventional geometric quantum computation, where the dynamical phase is canceled by rotating operations in single-loop or multiloop approaches. For $r\neq 0$, the dynamical and total phases are proportional to the conditional geometric phase. When $r=1$, especially, $\gamma_k^d=\gamma_k^g$ and $\alpha_k=2\gamma_k^g$. This result is similar to the unconventional geometric quantum computation, where one uses global geometric features in the rotating frame at the cavity frequency and does not distinguish between the total and geometric phases. In the two cases, however, our approach is to control the parameters in the Hamiltonian [see Eq. (7)] and does not need any operations, which may be a distinct advantage for experimentally implementing geometric quantum computation.

We see that our approach offers a wide choice of the relations between the dynamical and geometric phases. Thus a pair of orthogonal states $\phi_\pm$ can evolve cyclically according to $u(T)\phi_\pm=\exp[\mp i(1+r)\gamma]\phi_\pm$. An arbitrary initial state can be expressed as $\psi_i=a_+\phi_+ + a_-\phi_-$ with $a_\pm=(\phi_\pm,\psi_i)$ and the final state at time $T=2\pi/\omega$ is calculated as $\psi_f = b_+e^{-i(1+r)\gamma_+}\phi_+ + b_-e^{i(1+r)\gamma_-}\phi_-$. Under the computational basis $\{|0\rangle, |1\rangle\}$, the unitary transformation $u(r,\gamma,\theta')$, between the input and output states, can be written as

$$u(r,\gamma,\theta') = \begin{pmatrix} e^{-i(1+r)\gamma}\cos^2\frac{\theta'}{2} + e^{i(1+r)\gamma}\sin^2\frac{\theta'}{2} & \frac{1}{2}\sin\theta'(e^{-i(1+r)\gamma} - e^{i(1+r)\gamma}) \\ \frac{1}{2}\sin\theta'(e^{-i(1+r)\gamma} - e^{i(1+r)\gamma}) & e^{-i(1+r)\gamma}\sin^2\frac{\theta'}{2} + e^{i(1+r)\gamma}\cos^2\frac{\theta'}{2} \end{pmatrix}. \tag{8}$$

III. TWO-QUBIT SYSTEM

For a two-qubit system in NMR, the most natural two-qubit gate is the one generated directly by the spin-spin coupling Hamiltonian. The Hamiltonian is

$$H(t) = -\frac{1}{2}\Omega_0[(\sigma_{1x}+\sigma_{2x})\sin\theta\cos\omega t + (\sigma_{1y}+\sigma_{2y})\sin\theta\sin\omega t] - \frac{1}{2}\Omega_1(\sigma_{1z}+\sigma_{2z})\cos\theta + \frac{1}{4}\lambda\vec{\sigma}_1\cdot\vec{\sigma}_2, \tag{9}$$

where λ is the strength of the interaction between two qubits. By redefining $\vec{J}=\vec{\sigma}_1+\vec{\sigma}_2$, where $[J_m,J_n]=2i\epsilon_{mnl}J_l$ $(m,n,l=x,y,z)$ are satisfied, we can rewrite the Hamiltonian (9) according to J_x, J_y, J_z, and $\vec{J}^2$.

Similarly, an exact solution of the Schrödinger equation can be expressed as

$$\Psi(t) = \exp(-i\omega tJ_z/2)\exp(-itH'')\Psi(0), \tag{10}$$

where the effective Hamiltonian in the rotating frame is

$$H'' = -\frac{1}{2}\Omega e^{-(i/2)\theta' J_y}\left(J_z + \frac{1}{8}\lambda(\vec{J}^2 - 6)\right)e^{(i/2)\theta' J_y}. \tag{11}$$

The eigenequation can be written as $H''\phi_{Jk} = \{-\frac{1}{2}\Omega k + \frac{1}{2}[J(J+1)-\frac{3}{2}]\}\phi_{Jk}$, where $J=1$ and $k=1,0,-1$ or $J=0$ and $k=0$. The corresponding eigenfunctions for $J=1$ are expressed, respectively, as $\Phi_{1+1}=\exp\{-\frac{i}{2}\theta' J_y\}|00\rangle$, $\Phi_{10} = \exp\{-\frac{i}{2}\theta' J_y\}\frac{1}{\sqrt{2}}(|01\rangle+|10\rangle)$, and $\Phi_{1-1}=\exp\{-\frac{i}{2}\theta' J_y\}|11\rangle$.

For $J=0$, the eigenfunction is $\Phi_{00}=\exp\{-\frac{i}{2}\theta' J_y\}\frac{1}{\sqrt{2}}(|01\rangle - |10\rangle)$, while $\{|00\rangle, |01\rangle, |10\rangle$, and $|11\rangle\}$ are the computational basis for the two-qubit system.

The evolution of the wave function, under the cyclic condition with the period $T=2\pi/\omega$, is given by

$$\Psi(T) = e^{-i\pi k}e^{-i(\pi/\omega)[-k\Omega+\lambda(J(J+1)-(3/2))]}\Phi_{Jk}. \tag{12}$$

Thus the total, dynamical, and geometric phases are

$$\alpha_{Jk} = \pi k + \frac{\pi}{\omega}\{-k\Omega + \lambda[J(J+1) - 3/2]\}. \tag{13}$$

$$\gamma_{Jk}^{d} = \frac{\pi}{\omega}\{-\Omega k + \lambda[J(J+1) - 3/2]\} + k\pi\cos\theta', \tag{14}$$

$$\gamma_{Jk}^{g} = \alpha_{Jk} - \gamma_{Jk}^{d} = \pi k(1 - \cos\theta'). \tag{15}$$

Similarly, we find that, when Eqs. (6) or (7) are satisfied for the two-qubit system, the total phase, dynamical, and geometric phases have the following relations:

$$\gamma_{Jk}^{d} = \frac{\pi\lambda}{\omega}[J(J+1) - 3/2] + r\gamma_{Jk}^{g}, \tag{16}$$

$$\alpha_{Jk} = \frac{\pi\lambda}{\omega}[J(J+1) - 3/2] + (1+r)\gamma_{Jk}^{g}. \tag{17}$$

We see that our approach for the two-qubit system is neither different from the conventional nor unconventional approaches, where the total, dynamical, and geometric phases satisfy the expressions. There does not exist, especially, solution of the dark state because of the spin-spin interaction.

For the $J=1$ subsystem, the input states are Φ_{1+1}, Φ_{1+0}, and Φ_{1-1}. After a cyclic evolution, the output state is $\Psi_1(f)=\exp\{-i\frac{\pi\lambda}{\omega}[J(J+1)-\frac{3}{2}]\}[b_{1+1}\exp\{-i(1+r)\gamma\}\Phi_{1+1} + b_{10}\Phi_{10} + b_{1-1}\exp\{i(1+r)\gamma\}\Phi_{1-1}]$, where b_{Jk}, decided by the initial states, are constants independent of the evolving time. For the $J=0$ subsystem, similarly, the input state is Φ_{00} and the output is $\Psi_0(f)=b_{00}\exp\{i3\pi\lambda/2\omega\}\Phi_{00}$. Thus the phase factors $\exp\{-i(\pi\lambda/\omega)[J(J+1)-3/2]\}$ and $\exp\{i3\pi\lambda/2\omega\}$ can be regarded as overall phase factors for the spin $J=1$ subsystem and the spin $J=0$ subsystem, respectively, which are not important and may be dropped in the quantum computation under the condition that the control qubit is far away from the resonance condition for the operation of the target qubit so that the strength λ of the interaction between two qubits is very small.

In terms of the computational basis $\{|00\rangle, |01\rangle, |10\rangle, |11\rangle\}$, where the first (second) bit represents the state of the control (target) qubit, the unitary transformation $U(r,\gamma,\theta')$ up to a relative phase factor, between the input and output states, can be written as

$$U = \frac{1}{2}\begin{pmatrix} a_1 + a_2\cos\theta' + \sin^2\theta' & a_2\sin\theta' - \frac{1}{2}\sin 2\theta' & a_2\sin\theta' - \frac{1}{2}\sin 2\theta' & a_1 - a_2\cos\theta' - \sin^2\theta' \\ \Gamma_1(1+\cos\theta') - \frac{1}{2}\sin 2\theta' & \Gamma_2 + \cos^2\theta' + 1 & \Gamma_2 + \cos^2\theta' - 1 & \Gamma_1(1-\cos\theta') + \frac{1}{2}\sin 2\theta' \\ \Gamma_1(1+\cos\theta') - \frac{1}{2}\sin 2\theta' & \Gamma_2 + \cos^2\theta' - 1 & \Gamma_2 + \cos^2\theta' + 1 & \Gamma_1(1-\cos\theta') + \frac{1}{2}\sin 2\theta' \\ a_3 + a_4\cos\theta' - \sin^2\theta' & a_4\sin\theta' + \frac{1}{2}\sin 2\theta' & a_4\sin\theta' + \frac{1}{2}\sin 2\theta' & a_3 - a_4\cos\theta' + \sin^2\theta' \end{pmatrix}, \tag{18}$$

where

$$a_{1,2} = \exp\{-i(1+r)\gamma\}\cos^2(\theta'/2) \pm \exp\{i(1+r)\gamma\}\sin^2(\theta'/2),$$

$$a_{3,4} = \exp\{-i(1+r)\gamma\}\sin^2(\theta'/2) \pm \exp\{i(1+r)\gamma\}\cos^2(\theta'/2),$$

$$\Gamma_1 = -i\sin(1+r)\gamma\sin\theta',$$

and

$$\Gamma_2 = -i\sin(1+r)\gamma\sin^2\theta'.$$

Thus we achieve the entangling universal quantum gates based entirely on purely geometric operations. As an example, we choose the parameters as $\{\cos\theta=-\omega/\Omega_1$, $\sin\theta = \sqrt{1-\omega^2/\Omega_1^2}$, $\cos\theta'=0$, $\sin\theta'=1$, $r=1$, $\gamma=\pi$, $\gamma_k^d=-\pi k\Omega/\omega\}$, Eq. (18) may be written as

$$U = \frac{1}{2}\begin{pmatrix} 2 & 0 & 0 & 0 \\ 0 & 1 & -1 & 0 \\ 0 & -1 & 1 & 0 \\ 0 & 0 & 0 & 2 \end{pmatrix}, \tag{19}$$

which is a nontrivial conditional geometric phase gate.

In the following, we briefly discuss a modified spin-spin coupling Hamiltonian, where the interaction term of Eq. (9) is replaced by $\frac{1}{4}\lambda\sigma_{1z}\sigma_{2z}$, which is often applied to superconducting flux qubits interacting via mutual inductance or change qubits interacting via a capacitor. Similarly, the total and dynamical phases for the system are given by

$$\alpha_{Jk} = \pi k + \frac{\pi}{\omega}[-k\Omega + \lambda(k^2 - 1/2)], \tag{20}$$

$$\gamma_{Jk}^{d}=\frac{\pi}{\omega}[-\Omega k+\lambda(k^2-1/2)]+k\pi\cos\theta', \tag{21}$$

and the geometric phase is the same as Eq. (15). It is noted that the total and dynamical phases are not relative to J. This implies that two eigenstates Φ_{10} and Φ_{00} are degenerate. Moreover, they include both k- and k^2-terms. Therefore the dynamical phase is difficult to be canceled by a single-loop rotating operation proposed by Zhang *et al.* [12]. If Eqs. (6) and (7) are satisfied, the dynamical and total phases are written as

$$\gamma_{Jk}^{d}=\frac{\lambda\pi}{\omega}\left(k^2-\frac{1}{2}\right)+rk\gamma, \tag{22}$$

$$\alpha_{Jk}=\frac{\lambda\pi}{\omega}\left(k^2-\frac{1}{2}\right)+(1+r)k\gamma. \tag{23}$$

We know that there exist two independent subspaces with discrete and degenerate eigenstates, respectively. Thus the initial states can be constructed in terms of the subspaces, such as $\Psi_1(i)=a_{1+1}\Phi_{1+1}+a_{1-1}\Phi_{1-1}$ and $\Psi_2(i)=a_{10}\Phi_{10}+a_{00}\Phi_{00}$ with $(\Psi_1(i),\Psi_2(i))=0$. After a cyclic evolution, the output states are $\Psi_1(f)=\exp\{-i\frac{\lambda\pi}{2\omega}\}[b_{1+1}\exp\{-i(1+r)\gamma\}\Phi_{1+1}+b_{1-1}\exp\{i(1+r)\gamma\}\Phi_{-1}]$ and $\Psi_2(f)=\exp\{i\frac{\lambda\pi}{2\omega}\}(b_{10}\Phi_{10}+b_{00}\Phi_{00})$ with $(\Psi_1(f),\Psi_2(f))=0$. Thus $\exp\{-i\frac{\lambda\pi}{2\omega}\}$ and $\exp\{i\frac{\lambda\pi}{2\omega}\}$ are overall phase factors for $\Psi_1(f)$ and $\Psi_2(f)$, respectively. Thus the universal quantum gates, based entirely on geometric operations, can be described by Eq. (18).

IV. DISCUSSION AND CONCLUSION

In conclusion, we have proposed a way to realize the nonadiabatic geometric computation. By varying some Hamiltonian parameters and letting them satisfy Eq. (7), we find that the total and dynamical phases have the expressions (5) for the single-qubit system and Eqs. (16) and (17) for the two-qubit system with the geometric phase. Thus the phase accumulated in the geometric gates is a pure geometric phase. In comparison with the conventional geometric gates our approach does not need any such process, which leads to a possible reduction in experimental errors as well as gate timing.

In the unconventional approach, the geometric gates are executed by using global geometric features in the rotating frame at the cavity frequency [13,14]. Therefore there exists a direct proportionality, which is independent on all parameters of the system, between the total, geometric, and dynamical phases, respectively. If the proportionality between the geometric and dynamical phases is 0, the dynamical phase is 0. Moreover, the total phase vanishes because the total phase is also proportional to the dynamical phase in the unconventional approach. Furthermore, the geometric phase disappears. A similar information is for the proportionality with −1. In this case, the total phase is 0 so that the geometric phase vanishes because the geometric phase is proportional to the total phase. In the laboratory frame, especially, the total phase is not geometric [see Eq. (2.67) of Ref. [20]].

In contrast to the unconventional geometric gates, we calculate the total, dynamical, and geometric phases in the laboratory frame for the cyclic case and offer a wide choice of the relations between the dynamical phase and geometric phase. The proportionality constant includes all possible values in the physical region. Therefore our approach is more general compared to the approach based on the unconventional geometric gates. It is known that the errors are mainly from the proportionality r and the geometric phase because of fluctuations of the parameters in the Hamiltonian. In our conditional geometric gates, the proportionality r is constant, which is similar to the unconventional geometric gates [13,14]. Therefore our scheme is tolerant against an error just like the unconventional approach. In the rotating frame for the NMR system with single or two qubits, the dynamical phase is equal to the total phase and the geometric phase disappears. This means that the unconventional scheme does not adapt the NMR system because the proportionality is zero [14]. The same cannot be performed with the geometric gates for the noncyclic case in the NMR system.

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