Quantum storage on subradiant states in an extended atomic ensemble

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A scheme for coherent manipulation of collective atomic states is developed such that total subradiant states, in which spontaneous emission is suppressed into all directions due to destructive interference between neighbor atoms, can be created in an extended atomic ensemble. The optimal conditions for creation of such states and suitability of them for quantum storage are discussed. It is shown that in order to achieve the maximum signal-to-noise ratio the shape of a light pulse to be stored and reconstructed using a homogeneously broadened absorption line of an atomic system should be a time-reversed regular part of the response function of the system. In the limit of high optical density, such pulses allow one to prepare collective subradiant atomic states with near flat spatial distribution of the atomic excitation in the medium.

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I. INTRODUCTION

The use of photons as quantum-information carriers involves the elaboration of effective quantum memory devices which are able to write, store, and reconstruct single photon quantum states of the electromagnetic field. A promising approach to optical quantum state storage uses the interaction of single photons with optically dense media. The current activities focus on electromagnetically induced transparency (EIT) [1–5], stimulated Raman absorption [6], and photon echo [7–13] phenomena. Besides, there is a successful experimental demonstration of quantum memory for multiphoton quantum states using off-resonant interaction of light with spin polarized atomic ensembles [14]. In [15,16] a scheme of coherent manipulation of collective atomic states was developed such that super-radiant states of the atomic system can be converted into subradiant ones and vice versa and possible applications of such a scheme for optical quantum-state storage were discussed. The advantage of the scheme is that storage and retrieval of a single-photon state may in principle be implemented by means of phase modulators only [16], i.e., without any additional control fields or pulses acting on the atomic system. On the other hand, the rate of collective spontaneous emission in this case can be suppressed only for a few collective modes, which means that storage time is limited by incoherent spontaneous emission into other modes. In this paper the scheme is developed such that total subradiant states, in which spontaneous emission into all directions is suppressed, can be created in an extended atomic system. Such an approach involves using homogeneously broadened absorption lines for storage and retrieval of information in an optically dense medium. In [16] it was pointed out that for high efficiency of such a quantum memory the time shape of a single-photon wave packet to be stored should be equal to the time-reversed response function of the optically dense medium. Then the shape of the emitted photon proves to be a time-reversed replica of the initial one. In the present paper this statement is discussed quantitatively and it is shown that such pulses are optimal in the context of the signal-to-noise ratio. It should be noted here that the connection between optimal photon storage and time reversal has been made in [8,10] in considering storage on inhomogeneously broadened transitions. A comprehensive analysis of the question was presented recently in [17,18], where optimal pulse shapes were derived providing the maximum efficiency for different approaches to pulse storage. The results obtained here do not contradict those presented in [17,18], which will be discussed below in detail, but involve another point of view, namely the maximization of the signal-to-noise ratio. Besides, instead of a general iteration procedure which was used by authors of these papers we use a more direct approach to the problem, where the main features of the proposed scheme are explicitly taken into account: the absence of a control field, homogeneous broadening of an absorption line, and forward retrieval. Our approach is based on the theory of matched filters [19], which allows us to write down the explicit expressions for the optimal pulse shape and total efficiency of quantum memory in a straightforward manner.

The paper is organized as follows. In Sec. II, we present a scheme for coherent manipulation of collective atomic states that enables the creation of total subradiant states in an extended atomic system. In Sec. III, the basic equations describing propagation of single-photon wave packets in an optically dense atomic medium are introduced and the pulse shape which maximizes the signal-to-noise ratio upon read out is determined.

II. QUANTUM STORAGE ON SUBRADIANT STATES

In addition to the implementation schemes proposed in [16] we consider here another simple procedure for writing and reconstructing single-photon states of light using subradiant states. Consider an extended system of identical three-level atoms forming an optically dense resonant medium (see Fig. 1). We assume that the atoms are not moving as, for example, impurities embedded in a solid state material. In this case we may consider them to be distributed regularly in space with an interatomic distance a along some axis, say x. Moreover, since the parity of energy states in such a system is usually indefinite (due to the low symmetry of impurity

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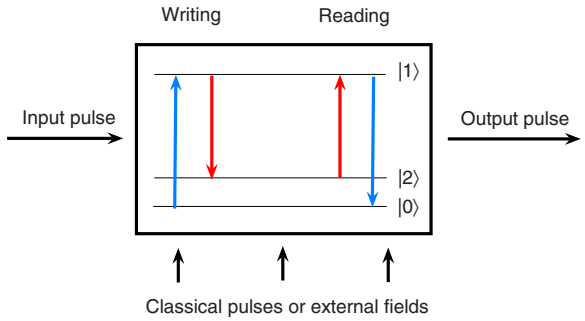

FIG. 1. (Color online) General scheme of quantum memory device based on a three-level extended atomic ensemble, collective states of which are controlled by an external electric field or classical pulses.

sites), we suppose that all transitions are dipole allowed. Assume that a single-photon wave packet which is resonant to the transition $|0\rangle \rightarrow |1\rangle$ propagates through the medium, which has a phase relaxation time much longer than the duration of the photon. At some moment of time the probability of finding the medium in the excited state and the field in the vacuum state is maximum and at this moment of time the atomic system can be subjected to a short coherent π pulse at the frequency of the transition $|1\rangle \rightarrow |2\rangle$, which corresponds to the writing of information. This step is typical of different quantum storage techniques involving a Λ-type medium with inhomogeneously broadened transitions [8] as well as homogeneously broadened ones [17]. Now, the spatial distribution of phase of the atomic state $|2\rangle$ in the medium that results from the excitation is described by a wave vector $\mathbf{k}-\mathbf{k}_w$, where $\mathbf{k}$ and $\mathbf{k}_w$ are wave vectors of the incoming photon and π pulse (writing pulse), respectively. If we put $|\mathbf{k}-\mathbf{k}_w| = \pi/a$, then the excited states $|2\rangle$ of adjacent atoms along the axis x will be opposite in phase, provided that the vector $\mathbf{k}-\mathbf{k}_w$ is directed along x. Consequently, if the wavelength of the transition $|2\rangle \rightarrow |0\rangle$, λ_{20}, satisfies the condition $2\pi/\lambda_{20} < \frac{1}{2}|\mathbf{k}-\mathbf{k}_w|$, i.e., $\lambda_{20} > 4a$, then at least $2^3=8$ atoms prove to be located in the volume $(\lambda_{20}/2)^3$, forming two equal groups with opposite phases of the state $|2\rangle$. As a result, a subradiant state is created, the rate of spontaneous emission of photons from which is suppressed for all directions. In this state the quantum storage is possible during times which may be much longer than the state-$|2\rangle$ radiative lifetime of a single atom. In order to read out the information it is necessary to apply a short π pulse (reading pulse) with the wave vector $\mathbf{k}_r = \pm \mathbf{k}_w$ to the transition $|2\rangle \rightarrow |1\rangle$. The signs + and − correspond to forward and backward retrieval.

Clearly the most promising materials for the creation of the subradiant states are those in which homogeneous linewidth, Γ_h, of the transition $|2\rangle \rightarrow |0\rangle$ is mainly determined by the spontaneous relaxation of the excited state. Linewidths approaching the limit $(\pi\Gamma_h)^{-1} \equiv T_2 = 2T_1$, where T_2 is the phase memory time and T_1 is the population lifetime, can be seen in materials where all other dephasing processes have been minimized, such as in rare-earth-metal-doped Y_2SiO_5 [20], where a T_2 of several ms has been observed. The difference $1/T_2' = 1/T_2 - 1/2T_1$, corresponding to such dephasing processes, determines the lifetime of the subradiant state, which may be an order larger than T_1 at cryogenic temperatures. For example, for the $^1D_2(1)$-$^3H_4(1)$ transition in $Pr^{3+}:Y_2SiO_5$ we have $T_1 = 0.222$ ms and $T_2 = 0.377$ ms [21], therefore $T_2' = 2.5$ ms. It should be noted that in such materials at low temperatures the phase memory of the hyperfine transitions, which are usually assumed to be used for storage in a Λ-type medium, is limited by the same processes that cause decoherence on the optical transitions, most notably the magnetic interaction with spins in the host material. Therefore, one can expect that the subradiant state lifetime is of the order of the hyperfine coherence time. Increasing the latter by applying, for example, a static magnetic field to the crystal implies increasing the former to the same extent. On the other hand, the frequency of the transition $|2\rangle \rightarrow |0\rangle$ may be much larger than $10^1 - 10^2$ MHz typical of hyperfine transitions. Taking the interatomic distance, $a = 1.5\lambda_{10}$, we obtain $\lambda_{20} > 6\lambda_{10}$, which corresponds to the frequency $\omega_{20} < \omega_{10}/6$.

As for homogeneous absorption lines, the technique of preparing of narrow absorbing peaks on a nonabsorbing background, i.e., isolated spectral features corresponding to a group of ions absorbing at a specific frequency, in rare-earth-metal-ion-doped crystals [22–27] can be very useful. Such specific structures can be created as follows. First, spectral pits, i.e., wide frequency intervals within the inhomogeneous absorption profile that are completely empty of all absorption, are created using hole-burning techniques. Then narrow peaks of absorption are created by pumping ions absorbing within a narrow spectral interval back into the emptied region. The peaks can have a width of the order of the homogeneous linewidth, if a laser with a sufficiently narrow linewidth is used for the preparation. Moreover, using two noncollinear laser pulses instead of a single one it is possible to prepare a periodic structure with a necessary spatial period a, since the atoms will be pumped mainly within antinodes of the laser field.

Finally, it should be noted that the total subradiant states in a macroscopic atomic ensemble can be created in principle by significantly changing the refractive index of the medium. There are many proposals aimed at the enhancement of the refractive index with vanishing absorption based on quantum interference effects which trace back to the works by Scully and colleagues (see [28] and references therein). Bearing them in mind we can consider the following procedure. Let the initial value of the refractive index of the host material be enhanced, so that the wavelength λ_{10} satisfies the condition $a = 3\lambda_{10}/2$. Then rather than apply a short coherent π pulse, we can reduce the refractive index at least by a factor of 6, so that λ_{10} becomes larger than $4a$ as in the previous case. As a result, a total subradiant state is created. By combining both techniques (applying the π pulse and changing the refractive index) it is possible to lift the restriction $\omega_{20} < \omega_{10}/6$.

III. OPTIMIZATION OF SIGNAL-TO-NOISE RATIO

The efficiency of the scheme considered above as well as of those considered in [16] depends strongly on the possibility of full (at some moment of time) photon absorption in the medium with a homogeneously broadened resonant transition and subsequent emission of the photon in the same di-

rection. This is possible only for a specific (optimal) time shape of the pulses to be stored. The optimal pulse shape which leads to the maximum efficiency of quantum storage may be found numerically using an iteration optimization procedure [17,18]. Here we consider the pulse shape which leads to the maximum peak value of the retrieved signal, given the energy of the pulse. Such pulses may be referred to as optimal in respect to the signal-to-noise ratio, the criterion generally employed in communications. It will be shown below that such pulses create almost uniform distribution of atomic excitation in the medium at some moment of time, corresponding to the end of the incoming pulse. This moment is optimal for application of the short writing π pulse creating a subradiant state. On the other hand, upon read out the emitted field proves to be the time-reversed replica of the initial field, which is the characteristic feature of the optimization [17,18]. Such a regime may be useful, for example, for a long-distance quantum communication using quantum repeaters [29], when the qubits are only stored and recalled once before being measured. Assuming, for example, that time-bin qubits are used for carrying the information, each of them should be a superposition of well-separated wave packets of the optimal shape, which provides high efficiency and fidelity of quantum memory devices. The latter characterizes reproducibility of the relative phase and amplitude ratio of time-bin single-photon pulses during storage and retrieval.

In the simplest case, of an additive white noise, the maximization of the signal-to-noise ratio reduces to that of the amplitude of the output signal at some moment of time, given the energy of the input signal and transfer function of the medium. Although such a procedure is performed usually in the context of classical signals [19,30,31], it works exactly the same when a single photon should be detected at the output of a memory device. The only difference is that the amplitude and intensity of classical light are replaced by the photon probability amplitude density and photon probability density, respectively. A relevant situation may be the detection of single photons amid broadband background light in free-space communication setups [32]. The maximization of single-photon probability density at some moment of time means shortening of the single-photon wave packet to be stored and recalled using a given absorption line. This allows one to minimize the time windows which are necessary for the writing and read out of information and consequently to minimize the probability of detection of stray photons instead of information carriers. The same argument is true in the context of noise due to the dark counts of single-photon detectors.

A. Basic equations

Consider a system of $N \gg 1$ identical two-level atoms, with positions $\mathbf{r}_j$ $(j=1,\ldots,N)$ and resonance frequency ω_0, interacting among themselves and with the external world only through the electromagnetic field. We are interested in the interaction of the atomic system with a single-photon wave packet. In a one-dimensional light propagation model it is usually assumed that the excitation volume may be approximated by a cylinder with the cross section S and the length L, the Fresnel number of the excitation volume $F=S(L\lambda)^{-1} \geqslant 1$, a single-photon wave packet propagates in the z direction, and the wave front of the packet is planar inside the excitation volume. Let us divide the medium into n identical slices of mean position $z_p=pL/n$ $(p=1,2,\ldots,n)$. The length of each slice Δz is large compared to the wavelength $\lambda=2\pi c\omega_0^{-1}$, but small compared to L. We assume that each slice contains a large number $N_p=N/n$ of atoms, but has a small optical density $\alpha\Delta z \ll 1$, where α is a resonant absorption coefficient. Therefore, hereafter we assume that slowly time-varying envelopes of the field and atomic probability amplitudes are constant in each slice and consider "coarse-grained" functions on coordinate z. Besides, we assume that the time of propagation of photons through the system L/c is negligibly short compared to the evolution time of the slowly time-varying envelopes.

Let us denote the ground and excited states of the jth atom by $|0_j\rangle$ and $|1_j\rangle$. The Hamiltonian of the system, in the interaction picture and rotating-wave approximation, reads

$$H=\sum_{j,\mathbf{k},s}\hbar g_{\mathbf{k},s}^{*}b_j^{\dagger}a_{\mathbf{k},s}e^{i\mathbf{k}\cdot\mathbf{r}_j}e^{i(\omega_0-\omega)t}+\text{H.c.} \tag{1}$$

Here $g_{\mathbf{k},s}=\frac{i}{\hbar}\left(\frac{\hbar\omega}{2\varepsilon_0 V}\right)^{1/2}(\mathbf{d}\cdot\boldsymbol{\varepsilon}_{\mathbf{k},s})$ is the atom-field coupling constant, $b_j=|0_j\rangle\langle 1_j|$ is the atomic transition operator, $a_{\mathbf{k},s}$ is the photon annihilation operator in the radiation field mode with the frequency $\omega=kc$ and polarization unit vector $\boldsymbol{\varepsilon}_{\mathbf{k},s}$ $(s=1,2)$, V is the quantization volume of the radiation field (we take V much larger than the volume of the atomic system), and $\mathbf{d}$ is the dipole moment of the atomic transition. For the sake of simplicity we assume that the vectors $\boldsymbol{\varepsilon}_{\mathbf{k},s}$ and $\mathbf{d}$ are real.

First, consider the system of a slice of atoms and electromagnetic field. For each slice with coordinate z_p we can define the following collective atomic operators:

$$R_p=\sum_{j=1}^{N_p}b_j e^{-i\mathbf{k}_0\cdot\mathbf{r}_j}, \tag{2}$$

where $\mathbf{k}_0$ is directed along the z axis and $|\mathbf{k}_0|=\omega_0/c$, and the general form of the state of the system can be written as

$$|\psi(t,z_p)\rangle=\sum_{\mathbf{k},s}f_{\mathbf{k},s}(t,z_p)|0\rangle|1_{\mathbf{k},s}\rangle+c(t,z_p)|1\rangle|\text{vac}\rangle \tag{3}$$

with normalization condition $\Sigma_{\mathbf{k},s}|f_{\mathbf{k},s}(t,z_p)|^2+|c(t,z_p)|^2=1$, where $|0\rangle=|0_1,0_2,\ldots,0_{N_p}\rangle$ is the ground state of the slice's atomic system, $|\text{vac}\rangle$ is the vacuum state of the radiation field, $|1_{\mathbf{k},s}\rangle=a_{\mathbf{k},s}^{\dagger}|\text{vac}\rangle$, and $|1\rangle=N_p^{-1/2}R_p^{\dagger}|0\rangle$. It should be noted that the normalization condition right after Eq. (3) is written for the system consisting of only one slice and the emf. This normalization condition will not be used when considering the whole atomic system.

Substituting Eqs. (1) and (3) in the Schrödinger equation we obtain

$$\frac{\partial f_{\mathbf{k},s}(t,z_p)}{\partial t}=-ig_{\mathbf{k},s}\sqrt{N_p}\phi(\mathbf{k}_0-\mathbf{k})c(t,z_p)e^{-i(\omega_0-\omega)t}, \tag{4}$$

$$\frac{\partial c(t,z_p)}{\partial t} = -i\sqrt{N_p}\sum_{\mathbf{k},s} g^*_{\mathbf{k},s}\phi^*(\mathbf{k}_0-\mathbf{k})f_{\mathbf{k},s}(t,z_p)e^{i(\omega_0-\omega)t}, \quad (5)$$

where $\phi(\mathbf{x})=N_p^{-1}\Sigma_j \exp(i\mathbf{x}\cdot\mathbf{r}_j)$ is the diffraction function.

The photon density for the incoming wave packet at the slice reads

$$F_{\text{in}}(t,z_p) = \frac{1}{L^{3/2}}\sum_{\mathbf{k},s} f_{\mathbf{k},s}(-\infty,z_p)e^{i(\omega_0-\omega)t}, \quad (6)$$

and for the emitted radiation we have the analogous equation with $F_{\text{in}}(t,z_p)$ and $f_{\mathbf{k},s}(-\infty,z_p)$ replaced by $F(t,z_p)$ and $f_{\mathbf{k},s}(t,z_p)$, respectively. Then the solution of Eqs. (4) and (5) may be written as

$$c(t,z_p) = c(-\infty,z_p)e^{-(N_p\mu+1)t/2T_1} - \sqrt{\frac{N_p\mu}{T_1}}\int_0^\infty d\tau F_{\text{in}}(t-\tau,z_p)e^{-(N_p\mu+1)\tau/2T_1}, \quad (7)$$

$$F(t,z_p) = F_{\text{in}}(t,z_p) + \sqrt{\frac{N_p\mu}{T_1}}c(t,z_p). \quad (8)$$

Here $\mu=3\lambda^2(8\pi S)^{-1}$ is a geometrical factor [33], which describes the result of the integration

$$\int d\Omega_{\mathbf{k}}\sum_s (\mathbf{d}\cdot\boldsymbol{\varepsilon}_{\mathbf{k},s})^2\phi^2(\mathbf{k}_0-\mathbf{k}) = \frac{8\pi}{3}\left(\mu+\frac{1}{N_p}\right)d^2 \quad (9)$$

for identical dipole moments oriented perpendicular to the z axis, and

$$\frac{1}{T_1} = \frac{1}{4\pi\varepsilon_0}\frac{4d^2\omega_0^3}{3\hbar c^3}. \quad (10)$$

If we consider the case when $c(-\infty,z_p)=0$ and substitute Eq. (7) into Eq. (8), we obtain a solution for super-radiant resonant forward scattering of photons by an optically thin atomic medium [34,35],

$$F(t,z_p) = F_{\text{in}}(t,z_p) - b(\Delta z)\int_0^\infty d\tau F_{\text{in}}(t-\tau,z_p)e^{-\tau/T_2}, \quad (11)$$

where $b(x)=\alpha x/2T_2$, $T_2=2T_1$, $\alpha=4\mu N_p/\Delta z$ is the resonant absorption coefficient, and we have omitted $N_p\mu \ll 1$ from the exponential.

The solution (11) can be written in terms of the impulse-response function or transfer function of the slice. If we define

$$F_{\text{in}}(\omega) = \frac{1}{\sqrt{2\pi}}\int_{-\infty}^\infty dt\, F_{\text{in}}(t)e^{i\omega t}, \quad (12)$$

then

$$F(t,z_p) = \int_{-\infty}^\infty d\tau F_{\text{in}}(\tau,z_p)H(t-\tau) = \frac{1}{\sqrt{2\pi}}\int_{-\infty}^\infty d\omega\, F_{\text{in}}(\omega,z_p)H(\omega)e^{-i\omega t}, \quad (13)$$

where

$$H(t) = \delta(t) - b(\Delta z)\theta(t)e^{-t/T_2}, \quad (14)$$

$$H(\omega) = 1 - b(\Delta z)\frac{i}{\omega+i/T_2}. \quad (15)$$

Here $\theta(t)$ is equal to 0 for $t<0$, 1 for $t>0$, and 1/2 for $t=0$.

Now we return to the case of an optically dense medium considered as a sequence of optically thin slices, each of them characterized by the impulse-response (14) or transfer (15) function. In this case we can consider the quantity $c'(t,z)=\lim_{\Delta z\to 0} c(t,z_p)/\sqrt{\Delta z}$ as a probability amplitude density and assume that $z\in[0,L]$. For an optically thick medium the transfer function becomes

$$H(\omega,L) = \lim_{n\to\infty}\left[1 - b\left(\frac{L}{n}\right)\frac{i}{\omega+i/T_2}\right]^n = \exp\left(-b(L)\frac{i}{\omega+i/T_2}\right). \quad (16)$$

By expanding the exponential in Eq. (16) in power series and performing the Fourier transformation we obtain the following impulse-response function of a resonant medium with arbitrary optical density [35]:

$$H(t,L) = \delta(t) - \Phi(t),$$

$$\Phi(t) = b(L)\frac{J_1(2\sqrt{b(L)t})}{\sqrt{b(L)t}}\theta(t)e^{-t/T_2}. \quad (17)$$

Here $J_1(x)$ is the Bessel function of the first kind. Taking into account the coordinate dependence $F(t,z)$ for the optically thick sample, Eq. (13) should be written as

$$F(t,L) = \int_{-\infty}^\infty d\tau F_{\text{in}}(\tau,0)H(t-\tau,L) = \frac{1}{\sqrt{2\pi}}\int_{-\infty}^\infty d\omega\, F_{\text{in}}(\omega,0)H(\omega,L)e^{-i\omega t}. \quad (18)$$

B. Optimal pulse shape

First, find a shape of single-photon wave packets to be stored, which maximizes the signal-to-noise ratio (SNR) at the retrieval. In the theory of linear filters [19] it is well known that the maximum SNR is achieved for so-called matched filters, the impulse-response function of which is a time-reversed replica of an input (detected) signal, provided that the signal is read through a white noise. In that case the maximum peak value of the output signal, given energy of

the input signal, is achieved at some moment of time. In the present case of a homogeneously broadened absorption line, it is necessary to obtain the highest possible peak value of the photon density

$$|F(t,L)|^2 = \left[\int_{-\infty}^{\infty} d\tau F_{\text{in}}(\tau,0)H(t-\tau,L)\right]^2$$
$$= F_{\text{in}}^2(t,0) - 2F_{\text{in}}(t,0)\int_{-\infty}^{\infty} d\tau F_{\text{in}}(\tau,0)\Phi(t-\tau,L)$$
$$+ \left[\int_{-\infty}^{\infty} d\tau F_{\text{in}}(\tau,0)\Phi(t-\tau,L)\right]^2 \quad (19)$$

at some moment of time $t \geqslant 0$, assuming that the incoming pulse terminates at $t=0$ and $F_{\text{in}}(t,0)$ is a real function. The field generated after the moment $t=0$ is determined only by the last term in Eq. (19), the maximum peak value of which is achieved at the moment $t=0$ provided that $F_{\text{in}}(\tau,0) \propto \Phi(-\tau,L)$, which follows from the Cauchy-Bunyakowsky-Schwartz inequality. So, taking into account Eq. (17), we suggest the following single-photon wave packet:

$$F_{\text{in}}^L(t,0) = -A(L)\Phi(-t)$$
$$= \frac{1}{\sqrt{2\pi}}\int_{-\infty}^{\infty} d\omega \frac{A(L)}{\sqrt{2\pi}}[H(-\omega,L)-1]e^{-i\omega t} \quad (20)$$

as an optimal one from the standpoint of the SNR criterion. Here $A(L)=[b(L)(1-g(L))]^{-1/2}$ is the normalization constant, $g(x)=e^{-\alpha x/2}[I_0(\alpha x/2)+I_1(\alpha x/2)]$, $I_n(x)$ is the modified Bessel function of the first kind. The pulse begins at $t=-\infty$ and terminates at $t=0$, but in fact almost all energy is concentrated in several last oscillations of its amplitude. Substituting Eq. (20) in Eq. (18) and using Eq. (16) or Eq. (17) we obtain

$$F(t,L) = \gamma(t,L) - F_{\text{in}}^L(-t,0), \quad (21)$$

where

$$\gamma(t,L) = A(L)\sqrt{\frac{\pi|t|}{2T_2}}\sum_{m=1}^{\infty}\frac{(\alpha L/2T_2)^m}{m!(m-1)!}|t|^{m-1}$$
$$\times[I_{m-1/2}(|t|/T_2) - I_{-m+1/2}(|t|/T_2)] \quad (22)$$

is a near bell-shaped function, which in the case $\alpha L \gg 1$ is approximated by

$$\gamma(t,x) = -\sqrt{b(x)}\frac{g(x)}{\sqrt{1-g(x)}}\exp(-|t|\sqrt{\alpha x}/T_2), \quad (23)$$

and in the limit $\alpha L \to 0$ takes the form $\gamma(t)=-\sqrt{2/T_2}\times\exp(-|t|/T_2)$. In the last case the optimal shape of the incoming pulse becomes $F_{\text{in}}(t)=-\sqrt{2/T_2}\theta(-t)\exp(t/T_2)$, therefore from Eq. (21) it follows that the super-radiant forward scattering is negligible as expected. In the opposite case $\alpha L \to \infty$ the role of the function $\gamma(t,L)$ on the right-hand side of Eq. (21) tends to zero since $\int_{-\infty}^{\infty}|\gamma(t,L)|^2dt=2(\pi\sqrt{\alpha L})^{-1}$, so that no field goes through the medium until $t \geqslant 0$ (see Fig. 2). At the moment $t=0$ corresponding to the end of the incoming pulse the atomic system starts to emit the outcoming pulse

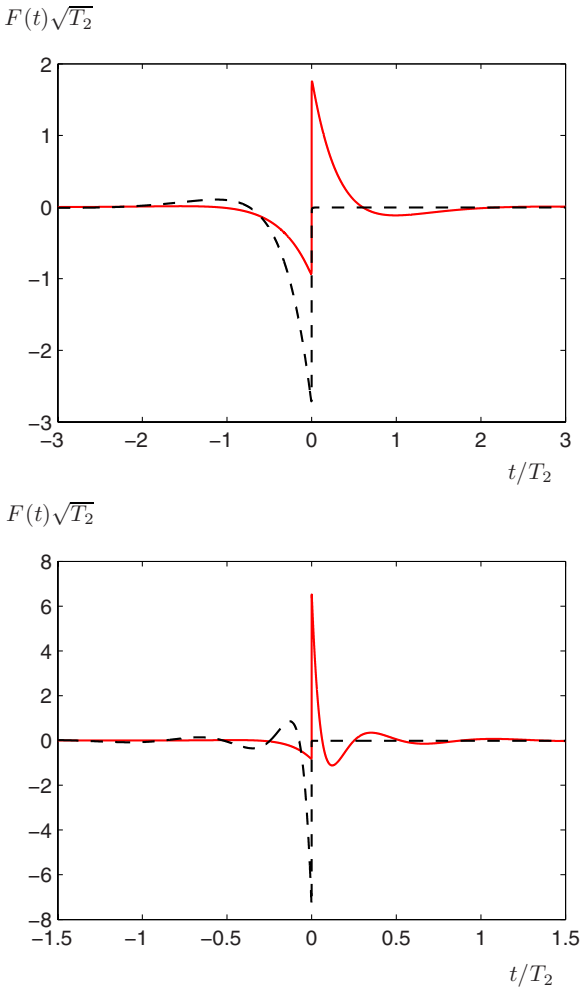

FIG. 2. (Color online) The amplitudes $F_{\text{in}}(t,0)$ (dashed line) and $F(t,L)$ (solid line) as functions of time t for $\alpha L=10$ (above) and $\alpha L=100$ (below).

$F(t,L)=-F_{\text{in}}^L(-t,0)$, which is the time-reversed (and opposite in phase) replica of the input pulse. Thus the moment $t=0$ is optimal for instantaneous creation of a subradiant state and mapping a single-photon state onto the collective atomic one.

Now, consider the efficiency of the proposed quantum memory scheme, defined by

$$\mathcal{E} \equiv \frac{\int_0^{\infty}|F_{\text{out}}(t)|^2dt - \int_{-\infty}^{0}|F_{\text{out}}(t)|^2dt}{\int_{-\infty}^{0}|F_{\text{in}}(t)|^2dt}. \quad (24)$$

The denominator is equal to unity by definition and the second term in the numerator corresponds to the probability of the photon loss due to its emission before the moment $t=0$, corresponding to the end of the incoming pulse and creation of a subradiant state. Substituting Eqs. (20), (21), and (23) we obtain

$$\mathcal{E} = 1 - \frac{4}{\sqrt{\pi\alpha L}}. \tag{25}$$

As expected, the efficiency tends to unity in the limit $\alpha L \to \infty$ according to a square-root law, which is typical of propagation effects in homogeneously broadened resonant systems. The main source of the photon loss is its leakage through the sample and incoherent emission in transverse modes during the whole process of writing and read out. It should be noted also that with increasing of αL the duration of the optimal single-photon wave packet decreases.

Finally, we consider the spatial distribution of probability amplitude density $c'(t,z)$ at $t=0$. From Eq. (18) it follows that the incoming pulse (20) after propagation of distance $z \leqslant L$ takes the form

$$F(t,z) = \frac{A(L)}{A(z)A(L-z)} \int_{-\infty}^{\infty} d\tau\, \gamma(t-\tau,z) F_{\text{in}}^{L-z}(\tau,0) + \frac{A(L)}{A(z)} \gamma(t,z) + \frac{A(L)}{A(L-z)} F_{\text{in}}^{L-z}(t,0) - \frac{A(L)}{A(z)} F_{\text{in}}^{z}(-t,0). \tag{26}$$

On the other hand from Eqs. (7) and (18) we can write

$$c'(0,z) = \sqrt{\frac{\alpha}{2T_2}} \int_{-\infty}^{\infty} d\omega\, F_{\text{in}}^{L}(\omega,0) H(\omega,z) \frac{i}{\sqrt{2\pi}(\omega + i/T_2)} = -\sqrt{\frac{2T_2}{\alpha}} \frac{\partial F(0,z)}{\partial z}. \tag{27}$$

Using the approximation (23) we obtain from Eq. (26)

$$F(0,z) = A(L) \left[-b(z) g(z) \exp\left(-\frac{b(L-z)T_2}{1+\sqrt{\alpha z}} \right) - \frac{b(L-z)}{2} + \frac{b(z)}{2} \right], \tag{28}$$

therefore

$$c'(0,z) = -\frac{A(L)\sqrt{b(L)}}{\sqrt{L}} \left[1 - \frac{1}{\sqrt{\pi}} \exp\left(-\frac{\alpha(L-z)}{2\sqrt{\alpha L}} \right) \right], \tag{29}$$

provided that $\alpha L \gg 1$.

We conclude that in the case of an optically thick sample the energy of the incoming wave packet, having optimal pulse shape, is distributed almost uniformly in the medium at the moment $t=0$ except for the far end of the sample ($z=L$) with the thickness $\Delta z/L \approx 2 \ln 2/\sqrt{\alpha L}$ (see Fig. 3). The probability of photon absorption at the moment $t=0$ is equal to

$$p_{\text{abs}}(t=0) = \int_0^L |c'(0,z)|^2 dz = 1 - \frac{2}{\sqrt{\pi\alpha L}} + \frac{1}{\pi\sqrt{\alpha L}}. \tag{30}$$

The second term on the right-hand side of Eq. (30) corresponds to the photon loss before the moment $t=0$, which is half of the total photon loss probability in Eq. (25).

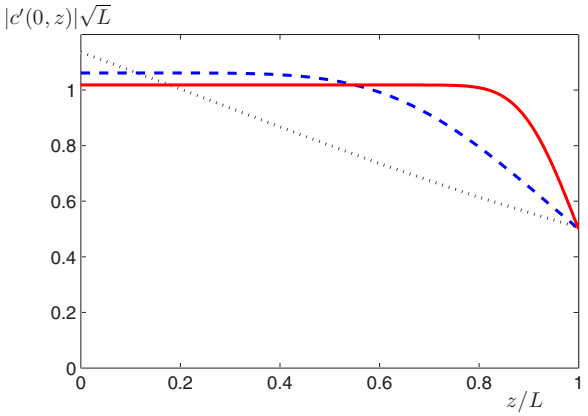

FIG. 3. (Color online) The amplitude density $|c'(0,z)|$ as a function of coordinate z for $\alpha L=10$ (dotted line), $\alpha L=100$ (dashed line), and $\alpha L=1000$ (solid line). The results of numerical calculation using Eq. (27).

When our results are compared with those of [17,18], it is apparent that the pulse shape (20) does not provide the maximum of efficiency (24). The latter needs the maximization of the total probability $\int_0^\infty dt[\int d\tau \Phi(t-\tau,L) F_{\text{in}}(\tau,0)]^2$ instead of the value $[\int d\tau \Phi(-\tau,L) F_{\text{in}}(\tau,0)]^2$. As a result the discontinuities of the amplitude of the atomic excitation at the edges of the sample arise and the error scales as one over square root of optical depth [see Eq. (25)]. On the other hand, the pulse shape considered here gives the maximum peak value of the probability density of the retrieved single-photon wave packet. Numerics show that in the case $\alpha L \gg 1$ the first burst of the output pulse contains about 90% of total photon probability, whereas that of the pulse optimized in respect to the efficiency is only about 70% having approximately the same duration [36].

IV. CONCLUSION

In summary, the quantum storage on a subradiant state in a macroscopic atomic ensemble is analyzed, provided that on a homogeneously broadened absorption line, no control field and forward retrieval are used. In possession of the impulse-response function of the sample it is possible to optimize the process of photon storage and retrieval from the standpoint of the signal-to-noise ratio and write down an explicit expression for the optimal pulse shape, which may be useful in the context of its experimental preparation. The light pulse to be stored should have a shape which is a time-reversed replica of the impulse-response function (to be more precise, of its regular part) of the atomic system. At the moment of time corresponding to the end of the absorbed pulse and the beginning of the emitted pulse the probability of excitation in the medium is distributed almost uniformly along the propagation direction. Therefore this is the optimal moment of time for coherent manipulation of the collective atomic state

aimed at the capture of the photon [16] or preparation of the subradiant state in an extended atomic ensemble. Obviously, the results obtained here can be easily generalized for absorption systems with another impulse-response function and for different kinds of noise, if the conditions for the linear response theory are fulfilled.

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