Entanglement generation in a two-mode quantum beat laser

Manzoor Ikram,[1] Gao-xiang Li,[1,2] and M. Suhail Zubairy[1,3,4]
[1]*Centre for Quantum Physics, COMSATS Institute of Information Technology, Islamabad, Pakistan*
[2]*Department of Physics, Huazhong Normal University, Wuhan 430079, China*
[3]*Institute for Quantum Studies and Department of Physics, Texas A&M University, College Station, Texas 77843-4242, USA*
[4]*Texas A&M University at Qatar, Education City, P.O. Box 23874, Doha, Qatar*
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We analyze the quantum correlations between side modes of a quantum beat laser when a two-level atomic medium is driven strongly by a classical field. The squeezing and the entanglement generation of the cavity radiation are investigated. It turns out that there is neither squeezing nor entanglement when the strong driving field is resonant with the atomic transition but the generated light exhibits both two-mode squeezing and entanglement when the driving field is tuned away from the atomic transition.

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I. INTRODUCTION

The field of quantum information has recently attracted great interest due to its applications in information storage, communication, and computing [1]. Quantum entanglement is viewed as the key resource for these applications, especially for quantum teleportation [2], dense coding [3], and quantum computation [4]. A variety of physical systems presenting entanglement have been investigated both theoretically and experimentally. The vast majority of schemes concentrate on discrete variable systems, such as trapped ions [5], few photons electromagnetic field in the cavity QED [6], spontaneous parametric down conversion [7], and nuclear magnetic resonance [8].

Continuous quantum-variable systems are proposed as an alternative to discrete level systems for performing quantum information tasks. Gaussian states play a central role as they can be produced from reliable sources and can be controlled experimentally using accessible set of operations such as beam splitters, phase shifters, and different detection systems. Entanglement between two Gaussian modes can be generated in the laboratory, e.g., two output beams of a parametric down converter [9]. Schemes for the generation of macroscopic entanglement in atomic ensembles have been demonstrated via quantum state transfer from nonclassical light to atoms [10]. A number of schemes for the generation of such entangled states for photons or the bright light beams have also been proposed and experimentally verified [11–19]. Recently, we proposed the generation and evolution of entangled light in a correlated spontaneous emission laser [16] and two-mode laser from a three-level cascade atom [17]. In a two-level quantum beat laser [20], the gain medium is two-level atoms. When the two-level atomic medium is driven strongly by an intense light, the three-peak Mollow spectrum of the fluorescence light appears due to ac Stark splitting. In the present paper, we examine the squeezing and the entanglement between the side modes for cases where the driving field is either resonant or off resonant with the atomic transition. We find that the side modes are entangled only when the frequency of the driving field is not resonant with the atomic frequency when the system operates below and above threshold.

II. MODEL AND THE MASTER EQUATION

We consider a linear theory of two-level quantum beat laser which describes a system in which a strong field mode interacts with two-level atomic medium. These levels are being pumped and decayed to another level. For convenience we take the upper level to ground level decay and consider the situation when atoms are being excited to upper level only. Two side modes of frequencies ν_1 and ν_3 are buildup because of spontaneous emission. The coherence is produced by the pump mode of frequency ν_2, which is responsible for the coherent superposition of the lasing levels. The side modes are then correlated with each other in a particular direction. The side mode frequencies are locked in a doubly resonant cavity and the mode locking condition $\nu_2-\nu_1=\nu_3-\nu_2$ is satisfied, as shown in Fig. 1. The symmetric placement of the side band frequencies ν_1 and ν_3 about the pump frequency ν_2 is shown in Fig. 2. We take the pump field to be arbitrarily intense and treat it classically. Side modes of frequencies ν_1 and ν_3 are considered weak and treated quantum mechanically up to second order in coupling constant. Under these conditions the Hamiltonian for the system takes the form

$$H=\hbar(\omega-\nu_2)\sigma_z+\sum_{j=1}^{3}\hbar[(\nu_j-\nu_2)a_j^\dagger a_j+(ga_jU_j\sigma^\dagger+\text{H.c.})] \tag{1}$$

In this expression a_j is the annihilation operator for the jth field mode, $U_j=U_j(r)$ is the corresponding spatial mode

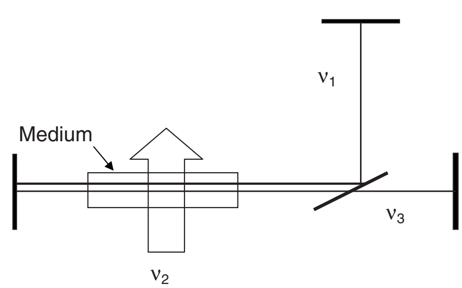

FIG. 1. Cavity configuration of a doubly resonant cavity.

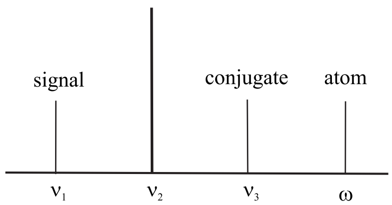

FIG. 2. Diagram showing symmetric placement of side band frequencies ν_1 and ν_3 about the pump frequency ν_2. For good squeezing and entanglement they are detuned from the atomic line center ω.

function, σ and σ_z are the atomic spin flip and probability difference operators, ω is the atomic frequency and ν_j is the frequency of jth mode, and g is the atom-field coupling constant. The rotating wave approximation has been made and the Hamiltonian is in the interaction picture rotating at the strong pump frequency ν_2.

The time dependence of an atom-field density operator $\rho_{a\text{-}f}$ can be obtained from the standard density operator equation of motion

$$\dot{\rho}_{a\text{-}f} = -i[H,\rho_{a\text{-}f}] + \cdots, \tag{2}$$

where the ellipsis represents the relaxation terms related to the cavity dissipation and the spontaneous emission of the atoms [20,21]. Under the adiabatic approximations the steady-state solution of the atomic equations of motion can be obtained by considering the slowly varying field modes as compared to atomic decay times [21]. By taking the trace of $\rho_{a\text{-}f}$ over the atomic states, the slowly varying density matrix equation of motion for the quantized field modes becomes

$$\begin{aligned}\dot{\rho} = &-A_1(\rho a_1 a_1^\dagger - a_1^\dagger \rho a_1) - \left(B_1 + \frac{\upsilon}{Q}\right)(a_1^\dagger a_1 \rho - a_1 \rho a_1^\dagger)\\ &+ C_1(a_1^\dagger a_3^\dagger \rho - a_3^\dagger \rho a_1^\dagger) + D_1(\rho a_3^\dagger a_1^\dagger - a_1^\dagger \rho a_3^\dagger) + [1 \leftrightarrow 3]\\ &+ \text{H.c.},\end{aligned} \tag{3}$$

where $1 \leftrightarrow 3$ represents the same terms with subscript 1 and 3 interchanged and ν/Q is the cavity loss rate for a given cavity configuration. The expressions for the coefficients A_1, B_1, C_1, D_1 are

$$A_1 = \frac{Ng^2 E_1}{1 + I_2 L_2}\left[\frac{I_2 L_2}{2} - \frac{(I_2 \gamma F/2)[(I_2 L_2 E_1)/2 - E_2^*(1 + \Gamma/i\Delta)/2]}{1 + I_2 F \gamma (E_1 + E_3^*)/2}\right], \tag{4}$$

$$B_1 = \frac{Ng^2 E_1}{1 + I_2 L_2}\left[1 + \frac{I_2 L_2}{2} - \frac{(I_2 \gamma F/2)\{[1 + (I_2 L_2/2)]E_1 + E_2^*(1 - \Gamma/i\Delta)/2\}}{1 + I_2 F \gamma (E_1 + E_3^*)/2}\right], \tag{5}$$

$$C_1 = \frac{-Ng^2 E_1}{1 + I_2 L_2}\left[\frac{(I_2 \gamma F/2)[(I_2 L_2 E_3^*)/2 - E_2(1 + \Gamma/i\Delta)/2]}{1 + I_2 F \gamma (E_1 + E_3^*)/2}\right], \tag{6}$$

$$D_1 = \frac{-Ng^2 E_1}{1 + I_2 L_2} \times \left[\frac{(I_2 \gamma F/2)\{[1 + (I_2 L_2/2)]E_3^* + E_2(1 - \Gamma/i\Delta)/2\}}{1 + I_2 F \gamma (E_1 + E_3^*)/2}\right], \tag{7}$$

where following the notations of Ref. [21], the complex Lorentzian denominator E_n is

$$E_n = \frac{1}{\gamma + i(\omega - \nu_n)}, \tag{8}$$

the dimensionless Lorentzian L_2 is

$$L_2 = \frac{\gamma^2}{\gamma^2 + (\omega - \nu_2)^2}, \tag{9}$$

the dimensionless Intensity I_2 is

$$I_2 = 4|V_2|^2 T_1 T_2, \tag{10}$$

and the dimensionless "population pulsation" term F is

$$F = \frac{\Gamma}{\Gamma + i\Delta}. \tag{11}$$

Here $V_2 = \mu \mathcal{E}_2 U_2/2\hbar$, where $\mathcal{E}_2$ is the field amplitude in mode 2 and U_2 is the corresponding spatial mode factor, μ is the electric dipole matrix element, N is the total number of interacting atoms, and $\Delta = \nu_2 - \nu_1$ is the beat frequency between modes 1 and 2. These coefficients assume that the only relaxation processes are upper-to-lower-level decay described by $\Gamma(=1/T_1)$, and the dipole decay described by γ $(=1/T_2)$. This is the usual experimental situation in laser spectroscopy. For pure spontaneous decay, γ is equal to $\Gamma/2$. The quantity $A_1 + A_1^*$ can be interpreted as the spectrum of resonance florescence, while $A_1 - B_1$ is the semiclassical complex gain and/or absorption coefficient and $C_1 - D_1$ is the semiclassical complex coupling coefficient. The expressions for the coefficients A_3, B_3, C_3, E_3 can be obtained by replacing 1 and 3 in the above expressions for A_1, B_1, C_1, E_1.

III. ENTANGLEMENT ANALYSIS

We now discuss the entanglement of the two side modes in the cavity based on the time-dependent solution of the master Eq. (3). If the two side modes are initially in the vacuum state, the cavity field is always in a two-mode Gaussian state since the master Eq. (3) only contains the quadratic terms of the bosonic operators a_j and $a_j^\dagger$ $(j=1,3)$. Under this initial condition, the time evolution of the nonzero expectation values for the quadratic operators obey

$$\frac{d}{dt}\langle a_1^\dagger a_1\rangle = \left(A_1 + A_1^* - B_1 - B_1^* - \frac{2\upsilon}{Q}\right)\langle a_1^\dagger a_1\rangle + (C_1 - D_1) \times \langle a_1^\dagger a_3^\dagger\rangle + (C_1^* - D_1^*)\langle a_1 a_3\rangle + A_1 + A_1^*, \tag{12}$$

$$\frac{d}{dt}\langle a_3^\dagger a_3\rangle = \left(A_3 + A_3^* - B_3 - B_3^* - \frac{2v}{Q}\right)\langle a_3^\dagger a_3\rangle + (C_3 - D_3) \times\langle a_1^\dagger a_3^\dagger\rangle + (C_3^* - D_3^*)\langle a_1 a_3\rangle + A_3 + A_3^*, \tag{13}$$

$$\frac{d}{dt}\langle a_1 a_3\rangle = \left(A_1 + A_3 - B_1 - B_3 - \frac{2v}{Q}\right)\langle a_1 a_3\rangle + (C_1 - D_1) \times\langle a_3^\dagger a_3\rangle + (C_3 - D_3)\langle a_1^\dagger a_1\rangle + C_1 + C_3. \tag{14}$$

In free space no buildup of photon number occurs, and $d\langle a_1^\dagger a_1\rangle/dt = A_1 + A_1^*$. Thus we interpret $A_1 + A_1^*$ as the spectrum of resonance fluorescence. Similarly the inhomogeneous term in the Eq. (14), $C_1 + C_3$, is the source contribution for the quantum combination tone $\langle a_1 a_3\rangle$. We will show that this quantity is responsible for entanglement and squeezing. Because the field in the cavity is in a two-mode Gaussian state, inseparability of the field in the cavity can be judged sufficiently and necessarily by use of the criteria proposed by Duan *et al.* [22] and Simon [23]. This criterion mentions that if a state of two-party quantum systems is inseparable, then the uncertainties in a pair of EPR-like operators $\hat{u}$ and $\hat{v}$ satisfy

$$M = (\Delta\hat{u})^2 + (\Delta\hat{v})^2 - m^2 - \frac{1}{m^2} < 0, \tag{15}$$

where

$$\hat{u} = |m|\hat{x}_1 + \frac{1}{m}\hat{x}_2,$$

$$\hat{v} = |m|\hat{p}_1 - \frac{1}{m}\hat{p}_2, \tag{16}$$

for any arbitrary nonzero real number m. Here, $\hat{x}_j = (a_j e^{i\theta} + e^{-i\theta} a_j^\dagger)/\sqrt{2}$ and $\hat{p}_j = (a_j e^{i\theta} - e^{-i\theta} a_j^\dagger)/\sqrt{2}i$ (with $j = 1, 2$) are the quadrature operators for the two modes 1 and 2 satisfying the relation $[\hat{x}_j, \hat{p}_k] = i\delta_{jk}$. Any state of two-party systems which obeys the inequality (15) must be an entangled state. The criterion depends upon the parameter m. By taking $m = \sqrt{\langle a_3^\dagger a_3\rangle / \langle a_1^\dagger a_1\rangle}$, the inequality becomes

$$M = \sqrt{\langle a_3^\dagger a_3\rangle\langle a_1^\dagger a_1\rangle} - |\langle a_1^\dagger a_3^\dagger\rangle| < 0. \tag{17}$$

That is to say, the entanglement between the two modes requires the nonclassical correlation between the two modes. For a two-mode Gaussian state with the standard covariance matrix [22,23], the condition becomes sufficient and necessary.

Another important quantity in the two-mode field is the two-mode squeezing. Defining the difference operator for the cavity modes as

$$d_1 = \frac{a_1 e^{i\theta} - a_3 e^{i\theta} + a_1^\dagger e^{-i\theta} - a_3^\dagger e^{-i\theta}}{2\sqrt{2}}, \tag{18}$$

for the field generated in the two-mode cavity in the present scheme, its variance is expressed as

$$\langle(\Delta d_1)^2\rangle = \frac{1}{4} + \frac{1}{4}[\langle a_1^\dagger a_1\rangle + \langle a_3^\dagger a_3\rangle - 2|\langle a_1 a_3\rangle|]. \tag{19}$$

Here the parameter θ is chosen so that $\langle\Delta d_1\rangle^2$ takes its minimum value [24]. The state is called a squeezed state if $\langle(\Delta d_1)^2\rangle < 1/4$. Evidently, the condition for the appearance of two-mode squeezing can be written as

$$|\langle a_1 a_3\rangle| > \sqrt{\langle a_1^\dagger a_1\rangle\langle a_3^\dagger a_3\rangle + \frac{1}{4}\langle a_1^\dagger a_1 - a_3^\dagger a_3\rangle^2}. \tag{20}$$

Comparing Eq. (17) with (20), it is easy to see that the condition for the appearance of the two-mode squeezing is more strict than that for the entanglement, that is, there may exist entanglement but no two-mode squeezing appears. Only when $\langle a_1^\dagger a_1\rangle = \langle a_3^\dagger a_3\rangle$, i.e., the cavity field reduces to a two-mode symmetric Gaussian state, do both the conditions for the appearance of the two-mode squeezing and the entanglement become the same. In the following discussion, according to the conditions (17) and (20) we will investigate the entanglement and squeezing property in detail.

A. Resonant case

First we consider that the atomic transition frequency ω is resonant to central mode frequency ν_2. In this case, the coefficients $A_1 = A_3^*$, $B_1 = B_3^*$, $C_1 = C_3^*$, and $D_1 = D_3^*$. Therefore, the two side-modes are in a symmetrical Gaussian state. The solution of Eqs. (12)–(14) is

$$\langle a_1^\dagger a_1\rangle = \langle a_3^\dagger a_3\rangle = \frac{(A+C)[1 - e^{2t(A-B-\nu/Q+C-D)}]}{2(\nu/Q - A + B - C + D)} + \frac{(A-C)[1 - e^{2t(A-B-\nu/Q-C+D)}]}{2(\nu/Q - A + B + C - D)},$$

$$\langle a_1 a_3\rangle = \frac{(A+C)[1 - e^{2t(A-B-\nu/Q+C-D)}]}{2(\nu/Q - A + B - C + D)} - \frac{(A-C)[1 - e^{2t(A-B-\nu/Q-C+D)}]}{2(\nu/Q - A + B + C - D)}, \tag{21}$$

where A, B, C, and D are the real parts of A_1, B_1, C_1, and D_1, respectively. The entanglement occurs only when $|\langle a_1 a_3\rangle| > \sqrt{\langle a_1^\dagger a_1\rangle\langle a_3^\dagger a_3\rangle}$ and the two mode squeezing occurs when $|\langle a_1 a_3\rangle| > \sqrt{\langle a_1^\dagger a_1\rangle\langle a_3^\dagger a_3\rangle + \frac{1}{4}\langle a_1^\dagger a_1 - a_3^\dagger a_3\rangle^2}$. In the above equations we always have $|\langle a_1 a_3\rangle| < \sqrt{\langle a_1^\dagger a_1\rangle\langle a_3^\dagger a_3\rangle}$. This means that in the resonance case there is no entanglement between the two side modes and there is no squeezing. Physically, it means that side band field are less correlated with each other than with themselves. This result is consistent with Refs. [24–26], where it is shown that the squeezing occurs only when pump field ν_2 is highly detuned from the atomic resonance ω.

B. Nonresonant case

Now we consider another case when the atomic frequency is nonresonant with the pump mode frequency ν_2 by an amount $\Delta_2 = \omega - \nu_2$. When the atoms are nonresonantly driven by the laser field, the nonclassical correlation may be established in the two-mode cavity field and entanglement may appear. In the nonresonance case, solving differential Eqs. (12)–(14) becomes complicated. By defining

$$f_i = A_i - B_i - \frac{\nu}{Q},$$

$$g_i = C_i - D_i,$$

$$h_i = A_i + A_i^*,$$

$$h_2 = C_1 + C_3$$

with $i=1,3$, we find the solution of Eq. (12)–(14) for the initial vacuum state

$$\langle a_1^\dagger a_1\rangle(t) = \frac{1}{\sqrt{RR^*}}\Bigg(\frac{h_1}{4}\Bigg[|\sqrt{R}+f_1-f_3^*|^2\frac{e^{s_1t}-1}{s_1} + |\sqrt{R}-f_1+f_3^*|^2\frac{e^{s_2t}-1}{s_2} + \Bigg((\sqrt{R}-f_1+f_3^*)(\sqrt{R^*}+f_1^*-f_3)\frac{e^{s_3t}-1}{s_3} + \text{c.c.}\Bigg)\Bigg]$$
$$+\Bigg\{\frac{h_2g_1^*}{2}\Bigg[(\sqrt{R}+f_1-f_3^*)\Bigg(\frac{e^{s_1t}-1}{s_1}-\frac{e^{s_4t}-1}{s_4}\Bigg) - (\sqrt{R}-f_1+f_3^*)\Bigg(\frac{e^{s_2t}-1}{s_2}-\frac{e^{s_3t}-1}{s_3}\Bigg)\Bigg] + \text{c.c.}\Bigg\}$$
$$+h_3|g_1|^2\Bigg(\frac{e^{s_1t}-1}{s_1}+\frac{e^{s_2t}-1}{s_2}-\frac{e^{s_3t}-1}{s_3}-\frac{e^{s_4t}-1}{s_4}\Bigg)\Bigg), \tag{22}$$

$$\langle a_3^\dagger a_3\rangle(t) = \frac{1}{\sqrt{RR^*}}\Bigg(\frac{h_3}{4}\Bigg[|\sqrt{R}-f_1+f_3^*|^2\frac{e^{s_1t}-1}{s_1} + |\sqrt{R}+f_1-f_3^*|^2\frac{e^{s_2t}-1}{s_2} + \Bigg((\sqrt{R}+f_1-f_3^*)(\sqrt{R^*}-f_1^*+f_3)\frac{e^{s_3t}-1}{s_3} + \text{c.c.}\Bigg)\Bigg]$$
$$+\Bigg\{\frac{h_2g_3^*}{2}\Bigg[(\sqrt{R^*}-f_1^*+f_3)\Bigg(\frac{e^{s_1t}-1}{s_1}-\frac{e^{s_3t}-1}{s_3}\Bigg) - (\sqrt{R^*}+f_1^*-f_3)\Bigg(\frac{e^{s_2t}-1}{s_2}-\frac{e^{s_4t}-1}{s_4}\Bigg)\Bigg] + \text{c.c.}\Bigg\}$$
$$+h_1|g_3|^2\Bigg(\frac{e^{s_1t}-1}{s_1}+\frac{e^{s_2t}-1}{s_2}-\frac{e^{s_3t}-1}{s_3}-\frac{e^{s_4t}-1}{s_4}\Bigg)\Bigg), \tag{23}$$

$$\langle a_1^\dagger a_3^\dagger\rangle(t) = \frac{1}{\sqrt{RR^*}}\Bigg(\frac{h_1g_3^*}{2}\Bigg[(\sqrt{R^*}+f_1^*-f_3)\Bigg(\frac{e^{s_1t}-1}{s_1}-\frac{e^{s_3t}-1}{s_3}\Bigg) - (\sqrt{R^*}-f_1^*+f_3)\Bigg(\frac{e^{s_2t}-1}{s_2}-\frac{e^{s_4t}-1}{s_4}\Bigg)\Bigg]$$
$$+h_2g_1^*g_3^*\Bigg(\frac{e^{s_1t}-1}{s_1}+\frac{e^{s_2t}-1}{s_2}-\frac{e^{s_3t}-1}{s_3}-\frac{e^{s_4t}-1}{s_4}\Bigg) + \frac{h_2^*}{2}\Bigg[(\sqrt{R}-f_1+f_3^*)(\sqrt{R^*}+f_1^*-f_3)\frac{e^{s_1t}-1}{s_1}$$
$$+(\sqrt{R}+f_1-f_3^*)(\sqrt{R^*}-f_1^*+f_3)\frac{e^{s_2t}-1}{s_2} + (\sqrt{R}+f_1-f_3^*)(\sqrt{R^*}+f_1^*-f_3)\frac{e^{s_3t}-1}{s_3} + (\sqrt{R}-f_1+f_3^*)(\sqrt{R^*}-f_1^*+f_3)\frac{e^{s_4t}-1}{s_4}\Bigg]$$
$$+\frac{h_3g_1^*}{2}\Bigg[(\sqrt{R}-f_1+f_3^*)\Bigg(\frac{e^{s_1t}-1}{s_1}-\frac{e^{s_4t}-1}{s_4}\Bigg) - (\sqrt{R}+f_1-f_3^*)\Bigg(\frac{e^{s_2t}-1}{s_2}-\frac{e^{s_3t}-1}{s_3}\Bigg)\Bigg]\Bigg), \tag{24}$$

where

$$s_1 = \frac{f_1+f_1^*+f_3+f_3^*}{2} + \frac{1}{2}(\sqrt{R}+\sqrt{R^*}),$$

$$s_2 = \frac{f_1+f_1^*+f_3+f_3^*}{2} - \frac{1}{2}(\sqrt{R}+\sqrt{R^*}),$$

$$s_3 = \frac{f_1+f_1^*+f_3+f_3^*}{2} + \frac{1}{2}(\sqrt{R^*}-\sqrt{R}),$$

$$s_4 = \frac{f_1+f_1^*+f_3+f_3^*}{2} - \frac{1}{2}(\sqrt{R^*}-\sqrt{R}), \tag{25}$$

with

$$R = (f_1-f_3^*)^2 + 4g_1g_3. \tag{26}$$

Figures 3–5 show the time evolution (κt) of the average photon number, the entanglement quantity M, and the variance $(\Delta d_1)^2$, where $\kappa=\nu/Q$. When the system operates below the threshold, the system evolves similar to a damped coupled oscillator. At higher intensities the atom saturates and Stark shifting of the atomic energy level occurs. The inelastic scattering process now involves two laser photons

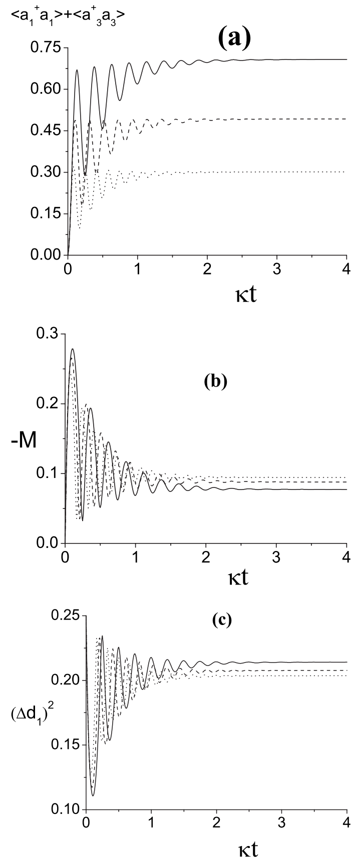

FIG. 3. Time evolution of the total photon number, the entanglement quantity M and the variance $(\Delta d_1)^2$. Here $C=2000$, $\Delta_2/\gamma=-100$, $\Delta/\gamma=50$, and $I_2=10\,000$ (solid), 8000 (dash), 6000 (dot).

at frequency ν_2 and scattered photons at frequencies ν_1 and ν_3. In the presence of detuning between the pump and the atomic frequency ω, the nonclassical correlation between side modes may dominate the self-correlation terms. Both the ac Stark shifts and the nonclassical correlation depend on the collective cooperativity $C=Ng^2/(\gamma\nu/Q)$, the detunings Δ_2 and Δ, and the laser intensity I_2. The ac Stark shifts restrict the increase in the photon number and the squeezing in time. So it could not get a larger photon number below threshold as shown in Fig. 3(a). Since the strength of the two-photon correlated emission process is proportional to $1/I_2$ when the driving field is strong, increasing I_2 may decrease the squeezing and the entanglement as shown in Figs. 3(b) and 3(c). As shown in Figs. 4, by increasing the collective cooperativity C (larger C has been achieved in a recent experiment [27]) the photon number can be increased but the squeezing and the entanglement can be enhanced in the very short time region. In the long time limit, the squeezing and the entanglement may be reduced with increasing C. Since in the nonresonance case the absorption and the gain for the modes 1 and 3 are asymmetric, the generated state is an asymmetric two-mode Gaussian state. This leads to the appearance of the entanglement but the absence of the two-mode squeezing as shown in Figs. 4(b) and 4(c).

In order to get bright field in the cavity, the system should operate above threshold. However, although the photon num-

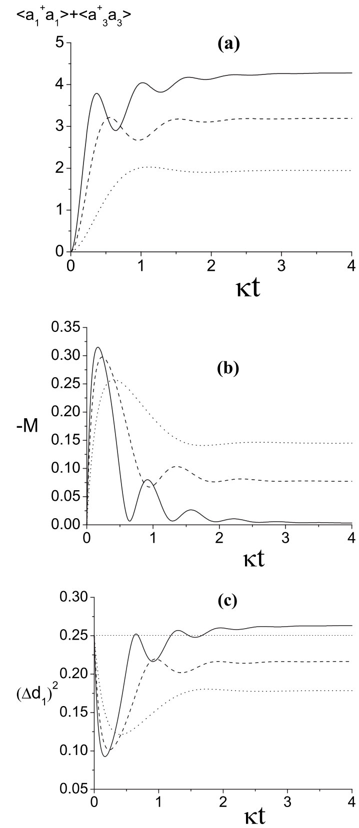

FIG. 4. Time evolution of the total photon number, the entanglement quantity M and the variance $(\Delta d_1)^2$. Here $I_2=30\,000$, $\Delta_2/\gamma=-85$, $\Delta/\gamma=50$, and $C=3000$ (solid), 2000 (dash), 1000 (dot).

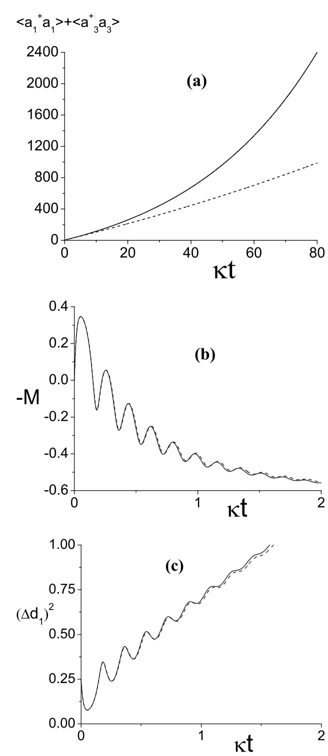

FIG. 5. Time evolution of the total photon number, the entanglement quantity M and the variance $(\Delta d_1)^2$. Here $I_2=30\,000$, $\Delta_2/\gamma=-85$, $\Delta/\gamma=70$, and $C=10\,900$ (solid), 10 800 (dash).

ber increases exponentially, the noises from the spontaneous emission and the photon dissipation also grow quickly. These noises destroy the entanglement very quickly so that the entanglement and squeezing can only appear in the very short time region as shown in Fig. 5. Therefore, the present system is not suitable for the generation of bright entangled field which may be maintained in a long-time regime.

IV. CONCLUSIONS

The squeezing and entanglement properties of two sideband modes of a two-mode quantum beat laser with strongly driven two-level atomic medium are investigated. When the pump mode is resonant to the atomic transition, we show that neither entanglement nor squeezing in the side modes appear because the side band field modes are less correlated with each other than with themselves. If the pump mode is tuned away from the atomic transition, under threshold, the entanglement and the squeezing can be generated and maintained in a longer period. We also notice that there is a region in which the entanglement exists but the squeezing is absent since the two side modes are in an asymmetric position for gain and absorption. However, above threshold, although the photon number in the two side modes increases exponentially, only the weak squeezing and entanglement can be generated and maintained in a short time region because the noise from atomic spontaneous emission and photon dissipation are amplified.

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