Conditional squeezing of an atomic alignment

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We investigate the possibility to perform quantum nondemolition measurements of the collective alignment of an atomic ensemble in the case of a $F \geq 1$ spin. We compare the case of purely vectorial and purely tensorial Hamiltonians and show how to achieve conditional squeezing or entanglement of atomic alignment components.

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I. INTRODUCTION

The reduction in the quantum fluctuations of an atomic ensemble angular momentum has recently received much attention in connection with quantum information, high-sensitivity frequency measurements, and high-precision magnetometry. Such spin-squeezed atomic states may be obtained via nonlinear interaction processes between an ensemble of Λ atoms and cavity fields [1,2] or by direct mapping of a squeezed state of light onto the ground state atomic spin [3–6]. Another approach consists in probing the atomic angular momentum in a quantum nondemolition (QND) manner in order to reduce the quantum fluctuations of one of its components below the standard quantum noise [7,8]. The atomic squeezing is then conditioned on the QND measurement result and can be actively fed back to the atomic angular momentum using a magnetic field [9]. So far, these protocols have been implemented with cesium atoms [10,11]. Since the angular momentum is greater than 1/2 a full description of the atomic state not only requires to take into account the three components of the angular momentum, but also the higher order tensorial components. This means that one has to add to the simplified effective QND Hamiltonian $F_z S_z$ [10,11] three of the five components of the atomic alignment, which in general will perturb the measurement of the orientation F_z. It is, however, possible to choose the atomic detuning with the excited states such that their contribution is zero or negligible [12–14].

The goal of the present paper is to investigate high-angular momentum atom situations in which the Hamiltonian is not purely vectorial and show how it is actually possible to realize QND measurements of the atomic alignment components. Such measurements may then allow for squeezing not only the quantum fluctuations of the atomic orientation, but also those of the alignment, which are involved in several atom-light quantum interface protocols [2,15]. For instance, by achieving conditional squeezing of the alignment of an atomic ensemble combined by single atomic excitation retrievals using the Duan-Lukin-Cirac-Zoller (DLCZ) protocol [15,16], it is possible in principle to produce exotic atomic states with a non-Gaussian Wigner function, in a way similar to non-Gaussian optical states [17,18]. In addition to being a tool for atomic quantum noise studies, controlling the fluctuations of the atomic alignment may be of interest for improving the precision of magnetometers [10,19,20]. In order to draw simple conclusions, we shall limit ourselves to a first order linear atom-field interaction in the optical pumping regime, but we note that interesting possibilities may also be offered by orientation or alignment conversion [21] and nonlinear selective addressing of high-rank atomic polarization moments [22].

In Sec. II we give the effective Hamiltonian and derive the atom-field evolution equations. After reviewing in Sec. III the well-known vectorial Hamiltonian situation leading to QND squeezing of the orientation, we examine in Sec. IV the purely tensorial Hamiltonian situation. We highlight the differences with the vectorial situation and show how QND measurements of the alignment can be performed, leading to conditional squeezing or entanglement of the atomic components. The effect of spontaneous emission losses on the obtainable squeezing and the experimental feasibility are discussed in Sec. V in the case of rubidium atoms.

II. HAMILTONIAN AND EVOLUTION OF THE SYSTEM

We consider an optical field propagating along z which interacts with an N-atom ensemble in the low saturation regime and consider slow processes as compared to the evolution of the excited state populations and optical coherences, which can be adiabatically eliminated. In this case, the effective Hamiltonian describing the atom-light interaction can be written as [12,13,23]

$$H_{\rm int} = \sum_{F'} \hbar \frac{\sigma_{F'}}{2A} \frac{\Gamma/2}{\Delta_{F'} + i\Gamma/2} \int_0^L dz \Bigg\{ \frac{\alpha_V^{F'}}{2} F_z(z,t) S_z(z,t) - \frac{\alpha_{F'}^T}{2(F+1)} \left[F_z^2(z,t) - \frac{F(F+1)}{3} \right] S_0(z,t) + \frac{\alpha_T^{F'}}{2(F+1)} (F_x^2 - F_y^2)(z,t) S_x(z,t) + \frac{\alpha_T^{F'}}{2(F+1)} (F_x F_y + F_y F_x)(z,t) S_y(z,t) \Bigg\}. \tag{1}$$

F (F') is the total angular momentum of the ground state (of one of the excited states), and its Cartesian components are denoted by $F_{x,y,z}$. $\sigma_{F'}$ is the resonant cross-section of the

$F \rightarrow F'$ transition, and $\Delta_{F'}$ is the probe one-photon detuning with respect to this transition (>0 if blue detuned). A is the field cross-section and L the length of the N-atom medium. The vectorial and tensorial polarizabilities are denoted by $\alpha_V^{F'}$ and $\alpha_T^{F'}$ and their exact form, given in Refs. [13,23], is recalled in Appendix A. The definition for the Stokes operators used throughout the paper is

$$S_x = a_x^\dagger a_x - a_y^\dagger a_y, \tag{2}$$

$$S_y = a_x^\dagger a_y + a_y^\dagger a_x, \tag{3}$$

$$S_z = i(a_y^\dagger a_x - a_x^\dagger a_y), \tag{4}$$

$$S_0 = a_x^\dagger a_x + a_y^\dagger a_y, \tag{5}$$

where the field a with frequency ω is defined by $E=\mathcal{E}_0(a+a^\dagger)$ and $\mathcal{E}_0=\sqrt{\hbar\omega/2\epsilon_0 Ac}$.

To simplify the discussion and relate it to the experimental situation which will be considered in Sec. V, we assume in the following an $F=1$ total ground state spin, but the physical conclusions would actually remain the same for a higher angular momentum. The irreducible tensor operators T_q^k for $F=1$ are given by [24]

$$T_0^1 = F_z/\sqrt{2}, \tag{6}$$

$$T_\pm^1 = \pm F_\pm/2, \tag{7}$$

$$T_0^2 = (3F_z^2 - 2)/\sqrt{6}, \tag{8}$$

$$T_{\pm 1}^2 = \pm (F_z F_\pm + F_\pm F_z)/2, \tag{9}$$

$$T_{\pm 2}^2 = (F_\pm^2)/2, \tag{10}$$

with $F_\pm = F_x \pm iF_y$. In this case, the Hamiltonian reads

$$\begin{aligned} H_{\rm int} = \sum_{F'} \hbar \frac{\sigma_{F'}}{2A} \frac{\Gamma/2}{\Delta_{F'} + i\Gamma/2} \int_0^L dz \Bigg\{ & \frac{\alpha_V^{F'}}{\sqrt{2}} T_0^1(z,t) S_z(z,t) \\ & - \frac{\alpha_T^{F'}}{2\sqrt{6}} T_0^2(z,t) S_0(z,t) + \frac{\alpha_T^{F'}}{4} (T_2^2 + T_{-2}^2)(z,t) S_x(z,t) \\ & + \frac{\alpha_T^{F'}}{4i} (T_2^2 - T_{-2}^2)(z,t) S_y(z,t) \Bigg\}. \end{aligned} \tag{11}$$

The anti-Hermitian terms in the Hamiltonian of Eq. (1) are due to optical pumping. For an off-resonant interaction, these anti-Hermitian terms may be neglected, although their contribution should be considered carefully when it comes to optimizing the squeezing as it will be shown in Sec. V. If these terms are neglected the evolution of the atomic operators is simply given by $(d/dt)\hat{A} = (1/i\hbar)[\hat{A}, H_{\rm int}]$, which yields

$$\frac{d}{dt}\begin{bmatrix} T_0^1 \\ (T_2^2 + T_{-2}^2)/\sqrt{2} \\ (T_2^2 - T_{-2}^2)/(i\sqrt{2}) \end{bmatrix} = \sum_{F'} \frac{\sigma_{F'}\Gamma}{4A\Delta_{F'}} \begin{bmatrix} 0 & -\alpha_T^{F'} S_y/2 & \alpha_T^{F'} S_x/2 \\ \alpha_T^{F'} S_y/2 & 0 & -\alpha_V^{F'} S_z \\ -\alpha_T^{F'} S_x/2 & \alpha_V^{F'} S_z & 0 \end{bmatrix} \begin{bmatrix} T_0^1 \\ (T_2^2 + T_{-2}^2)/\sqrt{2} \\ (T_2^2 - T_{-2}^2)/(i\sqrt{2}) \end{bmatrix}. \tag{12}$$

We limited ourselves to this set of three operators, since it is a closed system under $\hat{H}_{\rm int}$ and allows for conditional squeezing of $T_2^2 + T_{-2}^2$ or $T_2^2 + T_{-2}^2$ as we will show later. The terms $\propto \alpha_V$ in Eq. (12) correspond to light-shifts, and the ones $\propto \alpha_T$ to Raman processes involving coherences between sublevels with $|\Delta m_F| = 2$. Under the slowly varying envelope and paraxial approximations [5], the field evolution equations read

$$\left(\frac{\partial}{\partial z} + \frac{1}{c}\frac{\partial}{\partial t}\right)\begin{bmatrix} S_x \\ S_y \\ S_z \end{bmatrix} = \sum_{F'} \frac{\sigma_{F'}\Gamma}{4A\Delta_{F'}} \begin{bmatrix} 0 & -\sqrt{2}\alpha_V^{F'} T_0^1 & \frac{\alpha_T^{F'}}{\sqrt{2}} \frac{T_2^2 - T_{-2}^2}{i\sqrt{2}} \\ \sqrt{2}\alpha_V^{F'} T_0^1 & 0 & -\frac{\alpha_T^{F'}}{\sqrt{2}} \frac{T_2^2 + T_{-2}^2}{\sqrt{2}} \\ -\frac{\alpha_T^{F'}}{\sqrt{2}} \frac{T_2^2 - T_{-2}^2}{i\sqrt{2}} & \frac{\alpha_T^{F'}}{\sqrt{2}} \frac{T_2^2 + T_{-2}^2}{\sqrt{2}} & 0 \end{bmatrix} \begin{bmatrix} S_x \\ S_y \\ S_z \end{bmatrix}. \tag{13}$$

The terms $\propto \alpha_V$ in Eq. (13) correspond to the well-known Faraday rotation. In the following, we will consider the Stokes operators *before (in)* and *after (out)* the interaction, integrated over the pulse duration T: $\mathbf{S}^{in/out} = \int_0^T dt\, s(0/L,t)$, and the collective atomic operators *before*/*after* the interaction $\mathbf{A}^{in/out} = \int_0^L dz\, \hat{A}(z,0/T)$. s and $\hat{A}$ have been normalized so that $\mathbf{S}^{in/out}$ and $\mathbf{A}^{in/out}$ are dimensionless. We note that the evolution Eqs. (12) and (13) can alternatively be deduced following the methods of [25,26] for the atoms and [27,28] for the photons.

III. VECTORIAL HAMILTONIAN

For $\alpha_T=0$, the Hamiltonian (1) reduces to the well-known "QND" Hamiltonian $\hbar(\sigma\Gamma/4A\Delta)(\alpha_V/2)S_zF_z$, which allows for nondestructively measuring F_z via a measurement of the conjugate observable of S_z, as was shown in [7,8,10,11]. We briefly review the principle of this conditional squeezing of the orientation before generalizing it to an alignment in the next section.

Prior to the measurement of $\mathbf{F}_z$, the atoms are prepared in a coherent spin state oriented along x, i.e., the atoms are pumped into an eigenstate of $\mathbf{F}_x$. The values of the components orthogonal to the mean spin, $\mathbf{F}_y$ and $\mathbf{F}_z$, are unknown *a priori*, and because of the commutation relation $[\mathbf{F}_y, \mathbf{F}_z]=i\mathbf{F}_x=iN$, their standard deviations satisfy $\Delta\mathbf{F}_y\Delta\mathbf{F}_z\geq N/2$. When there exists no correlation between the transverse components, such as in a sample prepared by optical pumping, $\Delta\mathbf{F}_y=\Delta\mathbf{F}_z=\sqrt{N/2}$. The atoms are placed in zero-magnetic field. The probe is linearly polarized ($\langle\vec{\mathbf{S}}\rangle=n\vec{x}$). Integrating the evolution equations, one obtains the following input-output relations:

$$x^{out}=x^{in}+\kappa_V s_z^{in}, \tag{14}$$

$$p^{out}=p^{in}, \tag{15}$$

$$s_y^{out}=s_y^{in}+\kappa_V p^{in}, \tag{16}$$

$$s_z^{out}=s_z^{in}. \tag{17}$$

The operators have been normalized so as to have unity variance when they are in coherent states ($x,p=F_{y,z}/\sqrt{N/2}$ and $s_{y,z}=\mathbf{S}_{y,z}/\sqrt{n}$). It is clear that by measuring the fluctuations of s_y^{out} one acquires information about the fluctuations of p (F_z is measured nondestructively via the Faraday rotation of the probe polarization it induces). The measurement is all the more accurate that the *vectorial* coupling strength

$$\kappa_V=\alpha_V\frac{\sigma\Gamma}{4A\Delta}\sqrt{\frac{Nn}{2}} \tag{18}$$

is large. One therefore conditionally squeezes the atomic orientation. The variances of the transverse components after the measurement-induced projection of s_y^{out} can easily be shown to be those of a minimal spin-squeezed state [29–31]

$$V[x^{out}|s_y^{out}]=1+\kappa_V^2,\quad V[p^{out}|s_y^{out}]=\frac{1}{1+\kappa_V^2}. \tag{19}$$

IV. TENSORIAL HAMILTONIAN

A. Single-pass interaction

Another interesting situation is the opposite case of a purely tensorial Hamiltonian, in which $\alpha_V=0$. In practice, the interaction involves several hyperfine excited states F', so that it is possible to choose the detuning such that the various vectorial contributions vanish $\Sigma_{F'}\sigma_{F'}\alpha_V^{F'}(\Gamma/\Delta_{F'})\simeq 0$, while the total tensorial contribution $\Sigma_{F'}\sigma_{F'}\alpha_T^{F'}(\Gamma/\Delta_{F'})$

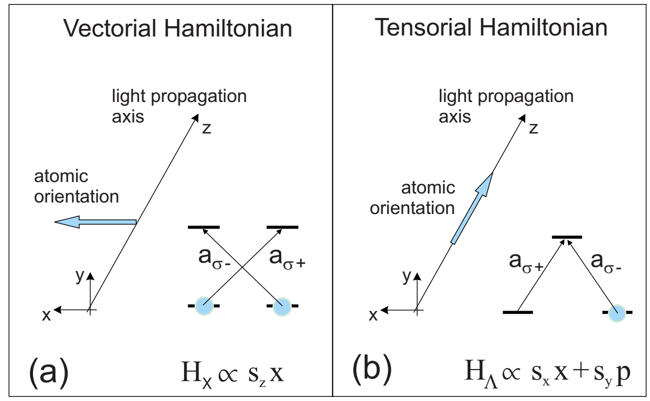

FIG. 1. (Color online) (a) Four-level scheme leading to a vectorial effective Hamiltonian $H_X\propto s_zx$. (b) Three-level Λ scheme leading to a tensorial effective Hamiltonian $H_\Lambda\propto s_xx+s_yp$.

does not. It is then possible to realize a conditional measurement of the alignment in this particular situation. Let us assume that the atoms are prepared in a coherent spin state along z. The conjugate transverse components in this case are $(\mathbf{T}_2^2+\mathbf{T}_{-2}^2)/\sqrt{2}$ and $(\mathbf{T}_2^2-\mathbf{T}_{-2}^2)/i\sqrt{2}$, since $[(\mathbf{T}_2^2+\mathbf{T}_{-2}^2)/\sqrt{2}, (\mathbf{T}_2^2-\mathbf{T}_{-2}^2)/i\sqrt{2}]=iN$. We normalize them as previously: $x=(\mathbf{T}_2^2+\mathbf{T}_{-2}^2)/\sqrt{N}$ and $p=(\mathbf{T}_2^2-\mathbf{T}_{-2}^2)/(i\sqrt{N})$ and assume a circularly polarized probe: $\langle\vec{\mathbf{S}}\rangle=n\vec{z}$.

The result of the integration of Eqs. (12) and (13) can be found in Ref. [6] and is recalled in Appendix B. It yields input-output relationships involving complex spatiotemporal modes for the fields and the atoms. For a thin medium ($\kappa_T\ll 1$), they lead to the following input-output relations:

$$x^{out}=x^{in}+\kappa_T s_y^{in}, \tag{20}$$

$$p^{out}=p^{in}-\kappa_T s_x^{in}, \tag{21}$$

$$s_x^{out}=s_x^{in}+\kappa_T p^{in}, \tag{22}$$

$$s_y^{out}=s_y^{in}-\kappa_T x^{in}, \tag{23}$$

with a *tensorial* coupling strength given by

$$\kappa_T=\alpha_T\frac{\sigma\Gamma}{8A\Delta}\sqrt{Nn} \tag{24}$$

[$\Sigma_{F'}\alpha_T^{F'}(\sigma_{F'}\Gamma/8A\Delta_{F'})\sqrt{Nn}$ if several excited states are involved].

The interaction is obviously not QND, since both components of the spin are now modified by the field, and conversely. This arises from the fact that the effective Hamiltonian in this case, $H_\Lambda\propto s_xx+s_yp$, is quite different from the previous vectorial situation $H_X\propto s_zx$, and now involves both quadratures (Fig. 1). As noted in [4], this tensorial Hamiltonian corresponds to a linear coupling between two harmonic oscillators which, when resonant, allows for efficient quantum state transfer between atomic and light variables and may be used in quantum memory protocols. As the coupling strength κ_T is increased, x^{out} and s_y^{out} (and p^{out} and s_x^{out}, respectively) coherently exchange their fluctuations, and it

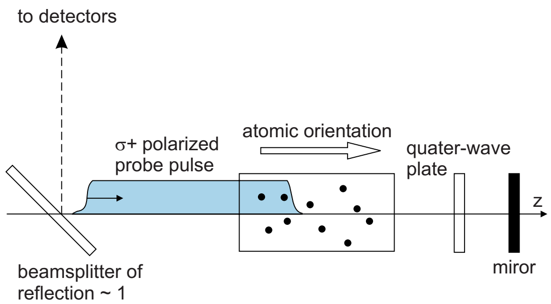

FIG. 2. (Color online) Schematic of the double pass configuration proposed to perform a QND measurement of a collective atomic alignment.

can indeed be shown that, when the collective coupling strength κ_T is large, the field fluctuations are efficiently mapped onto the atoms and vice versa [5,6].

However, since the atomic variables evolve during a single-pass "tensorial" interaction, it is *a priori* not well-suited for QND measurements. Nevertheless, it is still possible to perform a conditional measurement of the alignment by using two ensembles a and b (or by making two successive passes in one ensemble) with opposite mean orientations, such that $H_{\text{int}} \propto (x_a+x_b)s_x+(p_a+p_b)s_y$ and $[x_a+x_b,\ p_a+p_b]=0$. As will be detailed in the following sections, this restores the QND character of the interaction. The physical interpretation is that both field quadratures are written onto the atoms in each ensemble, but, because of the opposite orientations, their contributions cancel out, leaving the total alignment components unchanged, while the field still carries out information about both atomic alignment components.

B. Double-pass interaction

We first consider the double-pass geometry depicted on Fig. 2. A quarter-wave plate is inserted between the cell and the mirror, its neutral axis being aligned along x. We assume that the same pulse *successively* propagates back and forth in the atomic ensemble, with no temporal overlap. This is different from the situation of Refs. [32,33], where the pulse interacts with itself in the atomic medium, so that the nonlinear coupling allows for unconditional squeezing. We also note that similar ideas have been proposed for quantum memories and squeezing generation in Ref. [34] in the case of a purely vectorial Hamiltonian. In [34], the second pass is used to couple the second quadrature of light to the atoms. Again, the tensorial situation is quite different, since the Hamiltonian directly couples the two quadratures of light to the atoms. However, due to the $s_x x+s_y p$ form of the Hamiltonian, in order to perform a QND measurement of the alignment, one has to compensate for extra precession terms, which can be successfully done with a double-pass.

The double-pass interaction subsequently leads to

$$x^{out'} = x^{in}, \tag{25}$$

$$p^{out'} = -p^{in} + 2\kappa_T s_x^{in}, \tag{26}$$

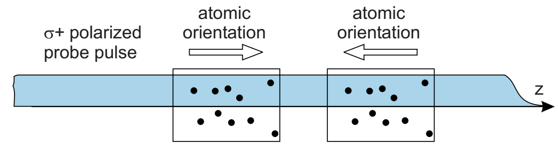

FIG. 3. (Color online) Schematic of the double ensemble configuration proposed to perform a QND measurement of a collective atomic alignment.

$$s_x^{out'} = s_x^{in}, \tag{27}$$

$$s_y^{out'} = s_y^{in} - 2\kappa_T x^{in}. \tag{28}$$

The measurement of $s_y^{out'}$ ($s_x^{out'}$) projects $x^{out'}$ ($p^{out'}$) in a state with reduced variance $V[x^{out'}|s_y^{out'}]=V[p^{out'}|s_x^{out'}]=1-4\kappa_T^2+o(\kappa_T^2)$. For a moderate value of $\kappa_T=0.35$, these variances are ~ 0.5, significantly smaller than the standard quantum limit. A rigorous derivation of the conditional variance, including the terms of order κ_T^2 in the input-output relations (20)–(23), can be obtained from the exact results of Eqs. (B1)–(B4) and leads to exactly the same conditional variance. The measurement of x performed this way is fully QND only for small values of κ_T, and will only result in a limited squeezing in principle. We now turn to a situation allowing for a QND measurement of the alignment for any value κ_T.

C. Double-cell interaction

Alternatively, a single-pass interaction can be performed with two atomic cells having opposite orientations—as in [11]—in order to entangle the alignment components of two atomic ensembles. As shown in Fig. 3(b) the light pulse propagates through two ensembles (a) and (b) prepared with opposite orientation $\langle \mathbf{F}_z^a \rangle = -\langle \mathbf{F}_z^b \rangle = N$, so that the input-output relationships now read

$$(x_a + x_b)^{out} = (x_a + x_b)^{in}, \tag{29}$$

$$(p_a + p_b)^{out} = (p_a + p_b)^{in}, \tag{30}$$

$$s_x^{out} = s_x^{in} + \kappa_T (p_a + p_b)^{in}, \tag{31}$$

$$s_y^{out} = s_y^{in} - \kappa_T (x_a + x_b)^{in}. \tag{32}$$

If the probe pulse duration is much longer than the time required to propagate through the two cells, the pulse interacts *simultaneously* with the two ensembles. In this experimentally accessible situation, the previous relations hold to any order in κ_T, and the measurement is perfectly QND. Similar to the vectorial situation, measuring s_y^{out} squeezes the variance of x_a+x_b to $2/(1+2\kappa_T^2)$. Note that one has $[x_a+x_b, p_a+p_b]=i(F_z^a+F_z^b)2/N=0$, since the two ensembles have opposite orientations. It is therefore possible to squeeze not only the fluctuations of x_a+x_b, but also those of p_a+p_b. As can be seen from Eq. (31), sending a second pulse and detecting s_x^{out} instead of s_y^{out} allows for squeezing p_a+p_b, leaving the alignment of the two ensembles entangled. The expected value of entanglement obtained is $\Delta_{\text{EPR}}=\Delta^2(x_a+x_b)+\Delta^2(p_a+p_b)=4/(1+2\kappa_T^2)<4$. Note that the result is the same as in the vectorial situation of [11], but the physical situation is rather different, since the tensorial situation requires a double pass for the alignment measurement to be completely QND.

V. ATOMIC NOISE AND EXPERIMENTAL VALUES FOR ^{87}Rb

A. General Hamiltonian and nonzero frequency noise measurements

For a single pass and in the case of a nonzero κ_V, the input-output relations read to first order in κ_V,κ_T

$$x^{out}=x^{in}+\kappa_T s_y^{in}-\kappa_V\sqrt{2n/N}p^{in}, \tag{33}$$

$$p^{out}=p^{in}-\kappa_T s_x^{in}+\kappa_V\sqrt{2n/N}x^{in}, \tag{34}$$

$$s_x^{out}=s_x^{in}+\kappa_T p^{in}-\kappa_V\sqrt{2N/n}s_y^{in}, \tag{35}$$

$$s_y^{out}=s_y^{in}-\kappa_T x^{in}+\kappa_V\sqrt{2N/n}s_x^{in}. \tag{36}$$

In the double-pass geometry described in Sec. IV B, the vectorial contributions cancel out and Eqs. (25)–(28) are left unchanged, so that the alignment can still be conditionally squeezed in this scheme. In the double-cell configuration, the vectorial contributions to the field evolution (Faraday rotation) naturally cancel out, but the vectorial contributions to the atom evolution (light shifts) do not. However, a z-aligned magnetic field with Larmor frequency $\Omega_L=-(\sigma\Gamma/8A\Delta)\alpha_V S_z$ can compensate for these light shifts.

Another experimentally relevant issue is the measurement of the Stokes parameters fluctuations. Technical noise is in general smaller than the quantum fluctuations of light only for higher-frequency components (typically above 0.1–1 MHz). It is therefore important to consider whether the schemes proposed in Sec. IV B and Sec. IV C can be extended to nonzero frequency noise measurements. In the double-cell configuration, it can easily be done by means of a z-aligned magnetic field. The Larmor precession couples x and p, but in the frame rotating at 2Ω (Ω is defined by $\hbar\Omega=\mu B_z$, where μ is the magnetic moment of the ground level and B_z the magnetic field value). The input-output relations (29)–(32) then remain unchanged when making the substitution $x\to(x)_{2\Omega}=x\cos(2\Omega t)+p\sin(2\Omega t)$, $p\to(p)_{2\Omega}=p\cos(2\Omega t)-x\sin(2\Omega t)$, etc. It is thus possible to measure in a QND manner the atomic operators $x_{2\Omega}$ and $p_{2\Omega}$ through their imprints on the sidebands components of S_x or S_y. Unfortunately, this technique cannot be used in the double-pass configuration, since the magnetic field would have to be reversed between the first and the second pass, which is not very realistic experimentally. However, if measurements of the Stokes operators at nonzero frequency are more easily shot-noise limited, it was shown in [10] that strong spin-squeezing could still be obtained experimentally in a zero-magnetic field (and so zero-frequency) situation.

B. Atomic noise considerations

We now discuss the intrinsic limitations brought by spontaneous emission noise in the tensorial Hamiltonian case. For the sake of simplicity we study the case of a $1\to 0,\ 1$ transitions, with atoms oriented along z. Using the Heisenberg-Langevin evolution equations and the quantum regression theorem, we obtain for a $1\to 0$ transition (for which $\alpha_V=-1/2$, $\alpha_T=-1$, and $\kappa_T=\kappa_V\sqrt{2}\equiv\kappa_0$)

$$x^{out}=x^{in}\sqrt{1-\varepsilon_a}+\sqrt{\varepsilon_a}f_x+\kappa_0 s_y^{in}-\kappa_0\sqrt{\frac{n}{N}}p^{in}-\frac{\gamma}{\Delta\sqrt{2}}\kappa_0 s_x^{in}, \tag{37}$$

$$p^{out}=p^{in}\sqrt{1-\varepsilon_a}+\sqrt{\varepsilon_a}f_p-\kappa_0 s_x^{in}+\kappa_0\sqrt{\frac{n}{N}}x^{in}-\frac{\gamma}{\Delta\sqrt{2}}\kappa_0 s_y^{in}, \tag{38}$$

$$s_x^{out}=s_x^{in}\sqrt{1-\varepsilon_p}+\sqrt{\varepsilon_p}f_{s_x}+\kappa_0 p^{in}-\kappa_0\sqrt{\frac{n}{N}}s_y^{in}-\frac{\gamma}{\Delta}\kappa_0 x^{in}, \tag{39}$$

$$s_y^{out}=s_y^{in}\sqrt{1-\varepsilon_p}+\sqrt{\varepsilon_p}f_{s_y}-\kappa_0 x^{in}+\kappa_0\sqrt{\frac{n}{N}}s_x^{in}-\frac{\gamma}{\Delta}\kappa_0 p^{in}, \tag{40}$$

with $\varepsilon_a=-(\kappa_0\Gamma/\Delta)\sqrt{n/N}$, $\varepsilon_p=-(\kappa_0\Gamma/\Delta)\sqrt{N/n}$ and $f_{x,p}$, f_{s_x,s_y} standard vacuum noise operators with variance unity.

For the $1\to 1$ transition, one has $\alpha_V=-3/4$ and $\alpha_T=3/2$, so that $\kappa_T=-\kappa_V\sqrt{2}\equiv\kappa_1$ and similar equations can be derived. Choosing the detunings such that $\kappa_0=\kappa_1\equiv\kappa/2$ cancels the vectorial terms finally yields the following input-output relationships

$$x^{out}=x^{in}\sqrt{1-\varepsilon_a}+\sqrt{\varepsilon_a}f_x+\kappa s_y^{in}+\frac{\varepsilon'}{\sqrt{2}}s_x^{in},$$

$$p^{out}=p^{in}\sqrt{1-\varepsilon_a}+\sqrt{\varepsilon_a}f_p-\kappa s_x^{in}+\frac{\varepsilon'}{\sqrt{2}}s_y^{in},$$

$$s_x^{out}=s_x^{in}\sqrt{1-\varepsilon_p}+\sqrt{\varepsilon_p}f_{s_x}+\kappa p^{in}+\varepsilon' x^{in},$$

$$s_y^{out}=s_y^{in}\sqrt{1-\varepsilon_p}+\sqrt{\varepsilon_p}f_{s_y}-\kappa x^{in}+\varepsilon' p^{in},$$

with $\varepsilon_a=(\kappa\Gamma/2)\sqrt{n/N}(1/\Delta_1-1/\Delta_0)$, $\varepsilon_p=(\kappa\Gamma/2)\sqrt{N/n}(1/\Delta_1-1/\Delta_0)$, and $\varepsilon'=-\kappa(\Gamma/4)(1/\Delta_1+1/\Delta_0)$. One retrieves beamsplitter-like relations for the losses, similar to those of [8]. ε_p simply describes absorption of the probe caused by spontaneous emission: The probe field is damped by a factor $\sqrt{1-\varepsilon_p}$, and some uncorrelated vacuum noise $\sqrt{\varepsilon_p}f_{s_x,s_y}$ is consequently added, as for the propagation through a beamsplitter with transmission $\sqrt{1-\varepsilon_p}$. ε_a describes the symmetrical process for the atoms: The probe, because of spontaneous emission, induces optical pumping toward a z-aligned coherent spin state (which is similar to mixing the probe with some vacuum). A difference with the vectorial situation is the presence of small contamination terms $\propto\varepsilon'$. They also correspond to optical pumping processes which tend to align x and p along s_x and s_y. To minimize the effect of spontaneous emission noise, one has to choose $n\sim N$ in order to have $\varepsilon_a\sim\varepsilon_p$, as in Ref. [8]. Finally, the total spontaneous emission contribution in the double-pass or double-cell configurations is finally obtained by doubling ε_a, ε_p, and ε' in the above equations.

C. Experimental values for ^{87}Rb

Based on these considerations, we discuss the values of squeezing or entanglement that can be expected in experiments with ^{87}Rb. We assume an interaction on the D_2 line with the atoms in the $F=1$ ground state. For room temperature vapor cells, taking into account the Doppler broadening, no detuning allows for completely canceling κ_V. On the contrary, for cold atoms with negligible Doppler broadening, κ_V can be canceled for a probe laser blue-detuned by $\Delta_0=38$ MHz from the $F'=0$ excited level (red-detuned by 34 MHz from $F'=1$ and 191 MHz from $F'=2$). For typical values for the density and volume (10^{11} cm^{-3} and 0.5 mm^3) of a cold atom cloud produced using a magneto-optical trap (the latter being switched off during the measurement), leading to $N=0.5\times10^8$ and taking a pulse of intensity 1 μW and duration 0.5 μs containing $n\sim0.5\times10^8$ photons (saturation parameter $\sim10^{-3}$, considering a cross section $A=1$ mm^2), the previous calculations predict $\kappa_T\sim-0.42$ and ~-5 dB of squeezing in the quantum fluctuations of $\mathbf{T}_2^2+\mathbf{T}_{-2}^2$ or $\mathbf{T}_2^2-\mathbf{T}_{-2}^2$. Higher values of κ_T (and hence higher squeezing values) can be reached for longer probe pulses, provided that the duration of the pulses remains smaller than the relaxation time of the Zeeman coherence, or by the use of a dipole trap to increase the optical depth [14]. For these parameters, in a double-pass or double-cell configuration, $\varepsilon_a=\varepsilon_p=0.14$ and $\varepsilon'\lesssim10^{-4}$ (the contribution of the $F'=2$ level is ~30 times smaller than those of the $F'=0,\ 1$ levels, and is not considered). As the fluctuations are predicted here to be reduced by a factor smaller than $\sim1/0.14$, the noise added by spontaneous emission can be neglected.

For the sake of comparison, we now discuss the relative strengths of tensorial and vectorial conditional measurements. In atomic vapors close to room temperatures, the detuning is usually chosen bigger than the Doppler broadening in order to avoid absorption [11]. It implies that the detuning has to be large as compared to the hyperfine structure and, since one has $\Sigma_{F'}\sigma_{F'}\alpha_T^{F'}=0$ for alkali atoms, it means that $\kappa_T\propto\Sigma_{F'}\sigma_{F'}\alpha_T^{F'}/\Delta'_F\sim0$, i.e., the effective Hamiltonian is then almost purely vectorial. This situation is obviously much more favorable for orientation than for alignment squeezing.

However, for a Doppler-free medium, it is possible to reduce the detuning while maintaining a small absorption. In this case, as can be seen from Fig. 4, both κ_T and κ_V (and hence alignment and orientation squeezing) may have similar values. To compare these values, we consider the case of a purely tensorial Hamiltonian (i.e., $\kappa_V=0$, obtained for $\Delta_0=38$ MHz and $\overline{\Delta}_0=13.2$), and the case of a purely vectorial one (i.e., $\kappa_T=0$, obtained for $\Delta_0=222$ MHz and $\overline{\Delta}_0=77$). In the first one, for the experimental parameters given above, $\kappa_T=-0.42$, $\kappa_V=0$, and $\varepsilon_p=\varepsilon_a=0.14$, whereas, in the second, $\kappa_T=0$, $\kappa_V=0.03$, and $\varepsilon_p=\varepsilon_a=0.01$. This shows that the common idea that the vectorial coupling strength is bigger than the tensorial one is not necessarily true for cold atom samples when the hyperfine structure is taken into account.

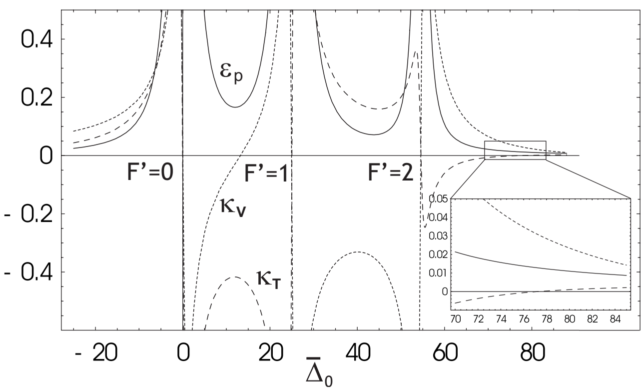

FIG. 4. Vectorial and tensorial coupling strength, κ_V (dotted) and κ_T (dashed), and amplitude of the noise added to the probe ε_p (plain) as functions of the normalized detuning $\overline{\Delta}_0=\Delta_0/(\Gamma/2)$ between the probe and the $F=1\rightarrow F'=0$ transition of ^{87}Rb D_2 line. The experimental parameters are detailed in Sec. V C. The inset zooms on the detuning area where the Hamiltonian is purely vectorial.

VI. CONCLUSION

We have shown how to perform a QND measurement of a collective atomic alignment. This extends the possibility to manipulate high-angular momentum components of a collective spin beyond the vectorial Hamiltonian interaction commonly used so far in experiments [7,10,11]. Noticeable physical differences are found between the purely vectorial Hamiltonian situation and the tensorial situation. In particular, if it had been noted in previous work [4,6] that the tensorial situation may lead to coherent atom-field quantum state transfer and storage, we have shown here that it also allows for performing a QND measurement of the atomic alignment, provided that two ensembles or two successive passes are used. Substantial conditional squeezing values are still predicted for realistic experimental situations with cold atomic samples. We also note that these measurements can be used to continuously control the atomic spin fluctuations via feedback [10]. The different feedback mechanisms that may be used to squeeze an atomic alignment will be presented elsewhere.

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APPENDIX A: POLARIZABILITY

The polarizabilities $\alpha_V^{F'}$ and $\alpha_T^{F'}$ are given by

$$\alpha_V^{F'}=\frac{3(2J'+1)}{2(2F'+1)(2J+1)}\left(-\frac{2F-1}{F}\delta_{F-1}^{F'}-\frac{2F+1}{F(F+1)}\delta_F^{F'}+\frac{2F+3}{F+1}\delta_{F+1}^{F'}\right),$$

$$\alpha_T^{F'} = -\frac{3(F+1)(2J'+1)}{2(2F'+1)(2J+1)}\left(\frac{1}{F}\delta_{F-1}^{F'} - \frac{2F+1}{F(F+1)}\delta_F^{F'} + \frac{1}{F+1}\delta_{F+1}^{F'}\right),$$

where $\delta_{F'}^{F}$ is Kronecker's symbol. The resonant cross section between two levels with an isotropically populated ground state is

$$\sigma_{F'} = \sigma_{2\ \mathrm{level}}\frac{2(2J+1)(2F'+1)}{3}\begin{Bmatrix} J' & 1 & J \\ F & I & F' \end{Bmatrix}^2,$$

with $\sigma_{2\ \mathrm{level}}=3\lambda^2/2\pi$. The commutators between the irreducible tensorial operators T_q^k are [24]

$$[T_{q_1}^{k_1}(F_g), T_{q_2}^{k_2}(F_g)] = \sum_{K,Q}(-1)^{K+2F_g}\sqrt{(2k_1+1)(2k_2+1)} \times \begin{Bmatrix} k_1 & k_2 & K \\ F_g & F_g & F_g \end{Bmatrix}\langle k_1k_2q_1q_2, KQ\rangle \times[1-(-1)^{k_1+k_2+K}]T_Q^K(F_g).$$

APPENDIX B: TENSORIAL SITUATION: SOLUTIONS OF THE EVOLUTION EQS. (12) and (13)

We assume a single-pass interaction with $\alpha_V=0$, as in Sec. IV A. After changing the spatiotemporal frame $(z,t)\to(z=z, t=t-z/c)$ and making the system dimensionless $(z,t)\to(\mathbf{z}=z/L, \mathbf{t}=t/T)$, the integration of Eqs. (12) and (13) yields [6]

$$x^{out} = x^{in} - \kappa_T\int_0^1 d\mathbf{z}\int_0^{\mathbf{z}} d\mathbf{z}' x^{in}(\mathbf{z}')\frac{J_1(2\kappa_T\sqrt{\mathbf{z}-\mathbf{z}'})}{\sqrt{\mathbf{z}-\mathbf{z}'}} + \kappa_T\int_0^1 d\mathbf{t}\, s_y^{in}(\mathbf{t})\left(\int_0^1 d\mathbf{z}\, J_0(2\kappa_T\sqrt{\mathbf{z}(1-\mathbf{t})})\right), \quad (B1)$$

$$p^{out} = p^{in} - \kappa_T\int_0^1 d\mathbf{z}\int_0^{\mathbf{z}} d\mathbf{z}' p^{in}(\mathbf{z}')\frac{J_1(2\kappa_T\sqrt{\mathbf{z}-\mathbf{z}'})}{\sqrt{\mathbf{z}-\mathbf{z}'}} - \kappa_T\int_0^1 d\mathbf{t}\, s_x^{in}(\mathbf{t})\left(\int_0^1 d\mathbf{z}\, J_0(2\kappa_T\sqrt{\mathbf{z}(1-\mathbf{t})})\right), \quad (B2)$$

and symmetrical equations for the fields

$$s_x^{out} = s_x^{in} - \kappa_T\int_0^1 d\mathbf{t}\int_0^{\mathbf{t}} d\mathbf{t}' s_x^{in}(\mathbf{t}')\frac{J_1(2\kappa_T\sqrt{\mathbf{t}-\mathbf{t}'})}{\sqrt{\mathbf{t}-\mathbf{t}'}} + \kappa_T\int_0^1 d\mathbf{z}\, p^{in}(\mathbf{z})\left(\int_0^1 d\mathbf{t}\, J_0(2\kappa_T\sqrt{\mathbf{t}(1-\mathbf{z})})\right), \quad (B3)$$

$$s_y^{out} = s_y^{in} - \kappa_T\int_0^1 d\mathbf{t}\int_0^{\mathbf{t}} d\mathbf{t}' s_y^{in}(\mathbf{t}')\frac{J_1(2\kappa_T\sqrt{\mathbf{t}-\mathbf{t}'})}{\sqrt{\mathbf{t}-\mathbf{t}'}} - \kappa_T\int_0^1 d\mathbf{z}\, x^{in}(\mathbf{z})\left(\int_0^1 d\mathbf{t}\, J_0(2\kappa_T\sqrt{\mathbf{t}(1-\mathbf{z})})\right), \quad (B4)$$

where J_0 and J_1 are the standard first order Bessel functions and the operators have been normalized so as to have unity variances when in coherent states. At first order in κ_T, one retrieves Eqs. (20)–(23).

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