Collective photon-atom states by Raman coupling inside a cavity: A dynamic field-mode approach

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We examine the collective quantum dynamics of photons and atoms driven by Raman transitions inside a low-Q cavity. At short times, we show that most of the Stokes photons can be captured by a single time-varying field mode. We determine an approximate analytical form of the mode function and construct a single-mode effective Hamiltonian. The model allows us to address the structure of the photon-atom state explicitly. In particular, we indicate that photons and atoms exhibit a form of photon-atom squeezing during the initial stage of the evolution.

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I. INTRODUCTION

Investigations of cavity quantum electrodynamics (cavity QED) in the last few decades have explored many intriguing phenomena of strongly coupled photons and atoms inside electromagnetic cavities [1]. In addition, a variety of mechanisms of controlling quantum states in cavities have been discovered for applications in quantum computations and quantum communications. One of the most widely employed processes is Raman coupling in which Λ-type three-level atoms emit or absorb Stokes (or anti-Stokes) photons by interacting with an external pump pulse. In early studies of cavity QED models, the quantum dynamics of various Raman couplings in ideal cavities has been studied [2]. If cavity field leakage is included, a single-atom system can serve as a deterministic source of single photons, and it has been demonstrated experimentally [3–5]. Recently, applications of cavity QED with an ensemble of Λ-type atoms have attracted considerable interest. With suitable combinations of Raman (or double Raman) interactions inside optical resonators, it is possible to manipulate squeezed light operators [6], entangled states of atoms [7], and optical Fock states [8] and to achieve interfacing between collective atomic excitations and single photons [9].

In this paper we investigate the quantum-state structure of photons generated by $N>1$ atoms via Raman scattering inside a leaky cavity. Unlike the single-photon single-atom problem [10], a full description of (pure) photon-atom state vectors becomes difficult as the number of atoms increases. The main challenge is the involvement of infinitely many frequency field modes when the cavity is not ideal. Even for the case with a few photons emitted, it could be quite tedious to express the multiphoton state vector in a frequency mode basis. However, knowledge of state vectors is important to determine the full quantum statistics, and the amplitudes in frequency modes would determine the spatial-temporal features of the field propagating out of the cavity.

Indeed, the same difficulty also occurs in collective spontaneous emission problems in free space. We have recently analyzed the superradiance problem in the Dicke limit [11] and found that the radiation from $N>1$ atoms can be optimally captured by several dynamical pulse modes [12]. The key feature of these dynamical mode functions is that their spectral amplitudes could change with time as the system evolves. Such a flexibility enables us to represent the state vector of photons in a much simplified manner, while keeping the same information as in the frequency-mode basis. In addition, if the time is not too long and the initial state is a vacuum, then the field is sufficiently described by a single dynamical mode. This interesting effect is general for linearized systems at short times, and it is independent of the frequency dependence of the atom-field coupling.

The main purpose of this paper is to formulate a dynamical mode model to describe the collective Raman scattering in the bad-cavity regime. Consideration of low-Q cavities is known to have some interesting applications in generating nonclassical states of atoms [13]. In our study, the cavity provides a well-defined output field channel that allows us to treat the field effectively in one dimension. In addition, since a bad cavity having a relatively large cavity linewidth can support a broad frequency range of vacuum noise, atomic transitions can roughly follow the exponential decay rule as in free space [14]. We can then make a comparison with results found in the free-space superradiance problem [12]. As an application, we will apply our model to study the buildup of a strong correlation between photons and atoms at short times. Such a correlation manifests as a form of squeezing, and hence it may serve as a resource for applications such as quantum teleportation.

II. MODEL AND HAMILTONIANS

We consider a system of N three-level Λ-type atoms interacting with a classical pump field and quantized Stokes cavity fields in Raman configuration (Fig. 1). The three atomic levels $|q\rangle$ $(q=1,2,3)$ have the corresponding atomic energies ε_q, and the pump field frequency is ω_p. The cavity has a nonzero leakage rate, and we will assume that different quasimodes of the cavity are widely separated such that there is only one quasimode with frequencies near the Raman resonance $\hbar\omega_c \approx \hbar\omega_p - \varepsilon_2 + \varepsilon_1$. However, the field associated with the resonant cavity quasimode still consists of a continuous range of frequencies limited by the cavity linewidth. Assuming that the atomic motion can be neglected and the size of atoms cloud is small compared with the resonant

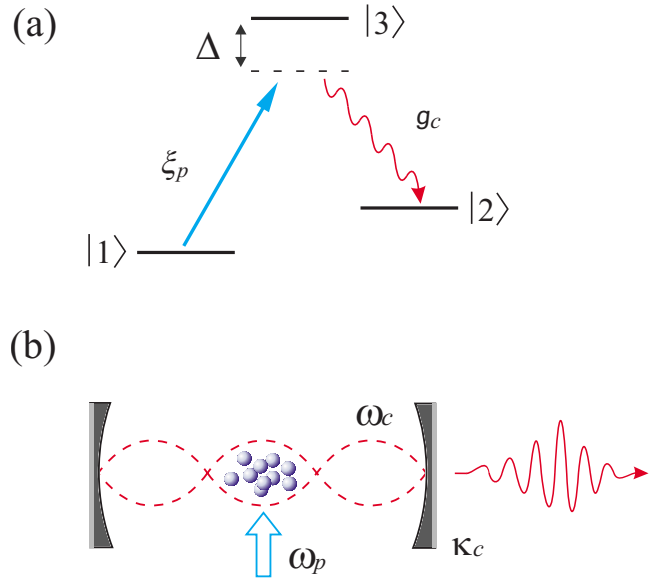

FIG. 1. (Color online) A schematic diagram of atomic levels and the Raman coupling configuration.

wavelength, the Hamiltonian of the system is given by ($\hbar=1$),

$$H=\int dk\omega_k a_k^\dagger a_k+\sum_{q=1}^{3}\sum_{j=1}^{N}\varepsilon_q|q\rangle_j\langle q|+\sum_{j=1}^{N}(\xi_p e^{-i\omega_p t}|3\rangle_j\langle 1|+\text{H.c.})+\sum_{j=1}^{N}\int dk(\eta_k a_k|3\rangle_j\langle 2|+\text{H.c.}), \quad (1)$$

where $a_k^\dagger$ and a_k are creation and annihilation operators for the quantized field mode with frequency ω_k, and the integrals are taken over all positive ω_k's. The ξ_p and η_k are coupling strengths proportional to their corresponding dipole transition matrix elements. Note that we have suppressed the polarization label for the field operators, and it is understood that $a_k^\dagger$ and a_k refer to the polarization selected by the $|2\rangle\leftrightarrow|3\rangle$ dipole transition.

For later purposes, it is convenient to introduce the coupling strength between the atom and the quasicavity mode, g_c, which is related to η_k (assumed Lorentzian) by [15]

$$\eta_k=\frac{g_c\sqrt{\kappa_c/\pi}}{\omega_k-\omega_c+i\kappa_c}, \quad (2)$$

where κ_c is the cavity field leakage rate. In the ideal cavity limit g_c corresponds to the vacuum Rabi frequency between the levels $|3\rangle$ and $|2\rangle$.

The loss due to spontaneous decay in the upper level $|3\rangle$ can be suppressed by choosing the detuning $\Delta=\varepsilon_3-\varepsilon_1-\omega_p$ such that it is much larger than the Rabi frequencies associated with $|1\rangle\leftrightarrow|3\rangle$ and $|2\rangle\leftrightarrow|3\rangle$ transitions—i.e., $\Delta\gg\xi_p$, g_c. In this way, the upper atomic level $|3\rangle$ is hardly populated and it can be eliminated adiabatically. This leads to the Hamiltonian

$$H'=\int dk\omega_k' a_k^\dagger a_k+\sum_{j=1}^{N}\left(\varepsilon_1-\frac{|\xi_p|^2}{\Delta}\right)|1\rangle_j\langle 1|+\sum_{j=1}^{N}(\varepsilon_2-h_s)|2\rangle_j\langle 2|+\sum_{j=1}^{N}\int dk\left(\frac{\xi_p\eta_k^*}{\Delta}a_k^\dagger|2\rangle_j\langle 1|+\text{H.c.}\right), \quad (3)$$

where $\omega_k'=\omega_k-\omega_p$, and hence the lower limit of the integral is now $-\omega_p$. For optical transitions, it is safe to extend the limit to $-\infty$. The term h_s is defined by $h_s=\frac{g_c^2}{\Delta}a_c^\dagger a_c$ with $a_c\equiv g_c^{-1}\int\eta_k a_k dk$ being a cavity-mode operator. Such a cavity-mode operator is an average of field operators weighted by the line shape of the cavity mode, and it is usually introduced in leaky cavity systems [15] in order to describe the quantized cavity field. Here the term h_s corresponds to an ac Stark shift due to the absorption and reabsorption of cavity photons by atoms in the state $|2\rangle$. Such processes in terms of bare field-mode operators a_k's would mean the scattering of photons between different k's. However, for bad-cavity systems considered in this paper, the influence of h_s can be ignored. This is seen by the small average number of cavity photon number $\langle a_c^\dagger a_c\rangle$ inside the cavity, and hence we can ignore h_s as an approximation [16]. In fact, as long as the last term of H' is dominant—i.e., when the Rabi frequency ξ_p controlled by the pump field is stronger than g_c—then the slight detuning due to h_s can be neglected. Consequently, the Hamiltonian (3) in the interaction picture reads

$$H''=\int dk(\mu_k J_+ a_k e^{-i\omega_k'' t}+\mu_k^* a_k^\dagger e^{i\omega_k'' t}J_-). \quad (4)$$

Here $\omega_k''=\omega_k'+|\xi_p|^2/\Delta+\varepsilon_2-\varepsilon_1$ and $\mu_k=\xi_p^*\eta_k/\Delta$ are defined. The J_i are collective spin operators, $J_z=\frac{1}{2}\Sigma_{n=1}^{N}(|2\rangle_n\langle 2|-|1\rangle_n\langle 1|)$, $J_-=\Sigma_{n=1}^{N}|2\rangle_n\langle 1|$, and $J_+=\Sigma_{n=1}^{N}|1\rangle_n\langle 2|$, which obey the usual angular momentum commutation relations. The effective Hamiltonian (4) is equivalent to the Dicke superradiance model but the coupling here is controlled by the external classical pump field as well as the cavity line shape.

Specifically, the bad-cavity regime that we are interested in is defined by the condition $\kappa_c\gg\sqrt{N}\frac{\xi_p g_c}{\Delta}\equiv\bar{g}$, which means that the collective Raman Rabi frequency is small compared with the cavity-field leakage rate. Under this condition, the transition rate between $|1\rangle$ and $|2\rangle$ can be estimated to be $\bar{g}^2/\kappa_c$. Such a transition rate should be greater than the spontaneous Raman transition rate in free space, $N\gamma_f\xi_p^2/\Delta^2$ (where γ_f is the spontaneous decay rate of $|3\rangle$), in order to have the most photons emitted in the cavity. This requires the atom-cavity coupling to be sufficiently strong so that $g_c^2\gg\kappa_c\gamma_f$. Combining these conditions together, we will study the system operating in the following regime:

$$\frac{g_c^2}{\gamma_f} \gg \kappa_c \gg \frac{\sqrt{N}\xi_p g_c}{\Delta}, \tag{5}$$

which is a (Raman) generalization of the one-dimensional atom regime previously discussed in the literature [17].

III. DOMINANT DYNAMIC FIELD MODE

In this section we determine an expression of a dominant field mode existing in a short-time domain. First, we employ the Schwinger's representation of angular momentum operators, $J_-=c^\dagger d$ and $J_+=d^\dagger c$, where c and d are bosonic annihilation operators associated with atoms in $|2\rangle$ and $|1\rangle$, respectively. The standard procedure of bosonizing $J_\pm$ at short times is to replace d by $\sqrt{N}$ so that $J_+ \to \sqrt{N}c$ and $J_- \to \sqrt{N}c^\dagger$, since the number of atoms in the state $|1\rangle$ changes little compared with N. Such an approximation is the same as keeping the leading term of the Holstein-Primakoff expansion of the angular momentum operators [18]. The main problem of the approximation, however, is that the depletion of atoms in the level $|1\rangle$ is completely ignored. To account for the depletion effect dynamically, we treat d as a real time-dependent amplitude $r(t)$, so that $J_+ \to r(t)c$ and $J_- \to r(t)c^\dagger$. In this way, the effective Hamiltonian (4) is quadratic:

$$H''' = r(t)\int dk[\mu_k c a_k e^{-i\omega_k'' t} + \mu_k^* c^\dagger a_k^\dagger e^{i\omega_k'' t}]. \tag{6}$$

Since $r^2(t)$ is interpreted as the average number of atoms in the state $|1\rangle$ at time t, conservation of particle number requires

$$r^2(t) + \langle c^\dagger c\rangle = N, \tag{7}$$

with $r(0)=\sqrt{N}$. This generalizes the standard bosonization scheme with the depletion effect included.

Noting that the Heisenberg equations of motion of c and a_k are linear, the solution of $a_k(t)$ formally takes the form

$$a_k(t) = \Gamma(k,t)c^\dagger(0) + \int \Lambda(k,k',t)a_{k'}(0)dk', \tag{8}$$

where $\Gamma(k,t)$ and $\Lambda(k,k',t)$ are some time-dependent coefficients. From this relation, we define the normalized dynamical mode function $u(k,t) \equiv \chi^{-1}\Gamma^*(k,t)$, where $\chi^2 = \int dk|\Gamma(k,t)|^2$, and the corresponding annihilation operator

$$b(t) \equiv \chi^{-1}\int dk\Gamma^*(k,t)a_k(t). \tag{9}$$

Assuming that the initial field is in the vacuum state and all atoms are in the state $|1\rangle$ [i.e., $\langle c^\dagger(0)c(0)\rangle=0$], then all the emitted photons at time t are contained by the dynamical mode. This can be seen by

$$\int dk\langle a_k^\dagger(t)a_k(t)\rangle = \int dk|\Gamma(k,t)|^2 = \langle b^\dagger(t)b(t)\rangle. \tag{10}$$

In other words, all other field modes orthogonal to $u(k,t)$ remain in the vacuum state.

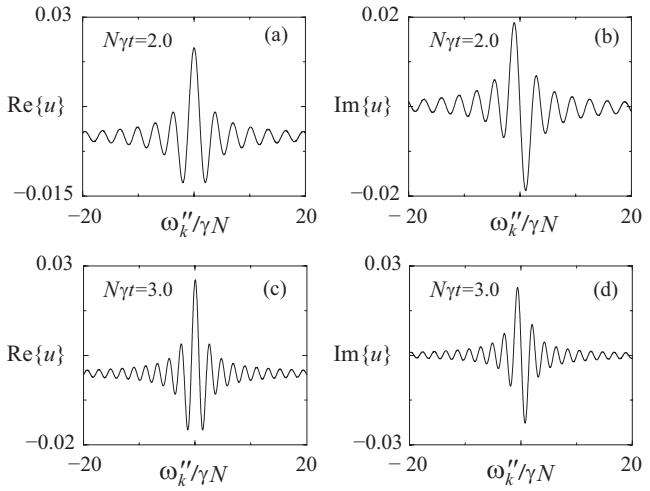

FIG. 2. The real and imaginary parts of the mode function $u(k,t)$ (in units of $1/\gamma$) as a function of ω_k'' at dimensionless times $N\gamma t=2$ [(a) and (b)] and $N\gamma t=3$ [(c) and (d)] for $N=500$ atoms.

By solving the Heisenberg's equation of motion for a_k and c in the bad-cavity regime, we obtain an analytical solution of $\Gamma(k,t)$ and $r(t)$ consistent with the condition of particle number conservation, Eq. (7),

$$u(k,t) = \chi^{-1}\int_0^t dt' r(t')e^{-i\omega_k'' t' + \theta(t')}, \tag{11}$$

$$r(t) = e^{-\gamma(N+1)t}\sqrt{\frac{N(1+N)}{1+Ne^{-2\gamma(N+1)t}}}, \tag{12}$$

where $\gamma=|\xi_p g_c|^2/\Delta^2\kappa_c$ and

$$\theta(t) = \gamma\int_0^t r^2(t')dt' = \ln\sqrt{(1+N)/(1+Ne^{-2\gamma(N+1)t})}, \tag{13}$$

$$\chi^{-2} = \frac{\gamma(1+Ne^{-2\gamma(N+1)t})}{N\pi(1-e^{-2\gamma(N+1)t})} \tag{14}$$

are obtained. In Fig. 2, we illustrate the shapes of $u(k,t)$ at various times. The resonance peak at $\omega_k''=0$ is well developed after one or two $N\gamma t$. At nonresonance frequencies, both the real and imaginary parts of $u(k,t)$ are increasing oscillatory as time increases.

The time-varying field mode $u(k,t)$ obtained above is based on the bosonization of the $J_\pm$ operators. An interesting question is whether $u(k,t)$ can also be a good approximation of the dominant mode for the full Hamiltonian (4), particularly when the time is beyond the linearization regime. To answer this question, we compare $u(k,t)$ with the leading mode obtained by the atomic-dipole correlation function method [12]. According to the formalism in [12], the radiation from N excited atoms in vacuum can be optimally projected onto a set of discrete field modes $\{f_n(k,t)\}$ defined by the eigenequation

$$\int W(k,k',t)f_n(k',t)dk' = \lambda_n f_n(k,t), \tag{15}$$

where the kernel W is related to the correlation function

$$W(k,k',t) \equiv \frac{\eta_k^* \eta_{k'} |\xi_p|^2}{\Delta^2} \int_0^t \int_0^t dt_2 dt_1 e^{-i\omega_k(t_2-t)} \times e^{i\omega_{k'}(t_1-t)} \langle \Psi(0)|J_+(t_2)J_-(t_1)|\Psi(0)\rangle, \tag{16}$$

and λ_n is the average photon number in the mode f_n. In the low-Q cavity limit where atoms experience a sufficiently broad band of vacuum noise, the atomic dynamics would follow the same form of the master equation [see Eq. (24)] appearing in free-space problems. We can then determine the correlation function $\langle J_+(t_2)J_-(t_1)\rangle$ via the quantum regression theorem [12]. However, we should remark that this is justified only in the bad-cavity regime, since a sufficiently large κ_c is required in order to provide a short memory time for the Markovian approximation.

By performing numerical diagonalization of $W(k,k',t)$ [12], we determine $f_1(k,t)$ corresponding to the largest eigenvalue λ_1. Our numerical investigations indicate that λ_1 is significantly higher than other eigenvalues at short times, which is consistent with the single-dominant-mode picture. In addition, $u(k,t)$ agrees very well with $f_1(k,t)$. To quantify the similarity, we examine the overlap integral $|\int f_1^*(k,t)u(k,t)dk|^2$. For example, at the earlier time $N\gamma t=2$ the overlapping is approximately 0.999 for $N=500$ atoms and at a longer time $N\gamma t=6$, the overlapping is about 0.981. The good agreement between $u(k,t)$ and $f_1(k,t)$ beyond several $N\gamma t$ suggests that the validity regime of linearization can be extended for the lowest-mode calculation. The nonlinear nature of $J_\pm$ is mainly responsible for the generation of photons in higher modes.

IV. SINGLE-MODE INTERACTION BY THE TIME-DEPENDENT VARIATIONAL PRINCIPLE

Based on knowledge of the dynamical mode $u(k,t)$ in Eq. (11), we can now construct a single-mode model that generates the dynamics of the system state vector approximately. Let us consider a trial time-dependent state vector defined by

$$|\Psi(t)\rangle = \sum_{n=0}^{N} q_n(t)|n\rangle_F |N-n,n\rangle_A \otimes |\text{vac}\rangle_X, \tag{17}$$

where $|N-n,n\rangle_A$ denotes the collective atomic state with $N-n$ atoms in the atomic level $|1\rangle$ and n atoms in the atomic level $|2\rangle$, $|n\rangle_F$ is the n-photon state in the mode $u(k,t)$, i.e.,

$$|n\rangle_F \equiv \frac{b^{\dagger n}}{\sqrt{n!}}|0\rangle, \tag{18}$$

and $|\text{vac}\rangle_X$ refers to the vacuum state of all field modes orthogonal to $u(k,t)$. It should be understood that Eq. (17) is an approximation which ignores the contribution of a small number of photons occupied in modes different from $u(k,t)$. This is justified by the analysis in the previous section.

By restricting Eq. (17) as the class of wave functions approximating the state of the system, we apply the time-dependent variation principle to determine the equation of motion of $q_n(t)$. This is achieved by minimizing the action

$$S = \int dt \langle \Psi(t)|(i\partial_t - H'')|\Psi(t)\rangle, \tag{19}$$

where $q_n(t)$ are variation parameters. The functional derivative leads to a set of linearly coupled first-order differential equations

$$i\dot{q}_n(t) = \sum_{m=0}^{N} M_{nm}(t)q_m(t), \tag{20}$$

in which the coefficients M_{nm} are recognized as the matrix elements of the *single-mode* effective Hamiltonian H_{eff}; i.e., Eq. (20) is equivalent to the Schrödinger equation

$$H_{eff}|\Psi(t)\rangle = i\partial_t|\Psi(t)\rangle. \tag{21}$$

After some calculations, we obtain

$$H_{eff} = \Omega(t)J_+ b + \Omega^*(t)J_- b^\dagger, \tag{22}$$

where

$$\Omega(t) = \int dk \mu_k u^*(k,t) e^{-i\omega_k'' t} \tag{23}$$

is the time-varying coupling strengths due to the evolution of the mode. It should be noted that $J_\pm$ are retained in the effective Hamiltonian without bosonization; H_{eff} can somehow capture the nonlinear behavior collective atomic operators.

In order to test the validity of the single-mode model (22), we compare the results with those obtained by the master equation of the atomic density matrix ρ_A. In the bad-cavity limit, where the heavily damped cavity-mode amplitude follows adiabatically with the atomic dipoles, one can eliminate the cavity field and obtain the master equation [19]

$$\dot{\rho}_A = \gamma(2J_-\rho_A J_+ - J_+ J_- \rho_A - \rho_A J_+ J_-), \tag{24}$$

which takes the same form as that which appears in the free-space Dicke superradiance problem [20]. The only modification is that the decay rate $\gamma = |\xi_p g_c|^2/\kappa_c \Delta^2$ now depends on the cavity properties. It should be noted that the derivation of Eq. (24) has included all the field modes in the Hamiltonian (4) and there is no single-mode approximation involved.

Let the atomic density matrix governed by the single-mode Hamiltonian (22) be $\rho_A^{(1)}$, and we compare it with the solution of Eq. (24). The fidelity between $\rho_A^{(1)}$ and ρ_A is defined by [21]

$$\mathcal{F} = \text{Tr}\sqrt{\sqrt{\rho_A}\rho_A^{(1)}\sqrt{\rho_A}}, \tag{25}$$

which measures the similarity between the two density matrices. Therefore $\mathcal{F}$ provides an indicator of the validity of the single-mode dynamics. We performed numerical calculations of $\rho_A^{(1)}$ and ρ_A for various parameters, and some of the typical results are shown in Fig. 3. We see that $\mathcal{F}$ is close to 1 at early times (up to a certain break time t_c), indicating that the single-mode theory can successfully recover the atomic

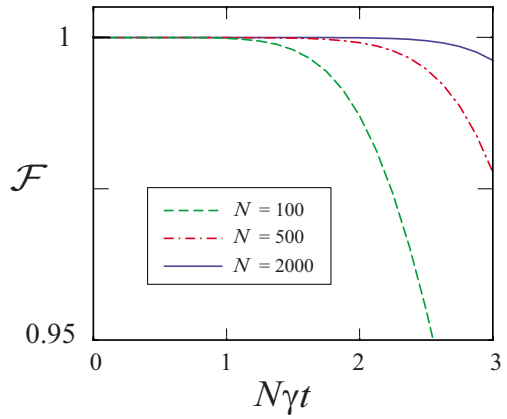

FIG. 3. (Color online) Fidelity of atomic density matrices obtained from the single-mode model as compared with the numerical solution of master equation (24).

dynamics. The break time t_c depends on the number of atoms, N. In the case of $100 \leqslant N \leqslant 2000$ atoms we have tested, we observed that $t_c \approx 2/N\gamma$ when $\mathcal{F} > 0.95$. At longer times $t > t_c$, the appreciable decrease of $\mathcal{F}$ is a result of the presence of the second and higher modes, although the dominant mode may still be approximated by $u(k,t)$.

V. PHOTON-ATOM SQUEEZING

The single-mode Hamiltonian (22) provides us with a convenient way to address the structure of the system state vector explicitly. As an application, we examine the time development of photon-atom correlation. Since all atoms are initially prepared in the state $|1\rangle$ in vacuum, the emitted number of photons and the number of atoms in the state $|2\rangle$ are always equal. Such a strong number correlation is seen in the state vector given in Eq. (17). We remark that Eq. (17) is already in the form of Schmidt decomposition, and hence the presence of more than one nonzero q_n in the summation indicates the existence of photon-atom entanglement. One could calculate, for example, the entropy $-\Sigma_n |q_n|^2 \ln |q_n|^2$, to quantify the degree of entanglement. Here we will address the quantum correlation from the squeezing properties of the system. To this end, let us define the following quadrature operators for atoms and photons:

$$X_1 = \frac{b^\dagger + b}{\sqrt{2}}, \quad X_2 = \frac{J_+ + J_-}{\sqrt{2N}}, \tag{26}$$

$$P_1 = \frac{i(b^\dagger - b)}{\sqrt{2}}, \quad P_2 = \frac{i(J_- - J_+)}{\sqrt{2N}}. \tag{27}$$

By the commutation relation

$$[X_1 + P_2, P_1 - X_2] = i\left(1 + \frac{2J_z}{N}\right), \tag{28}$$

the corresponding uncertainty relation reads

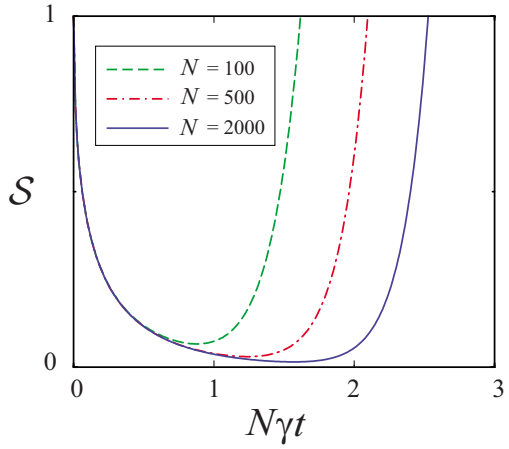

FIG. 4. (Color online) An illustration of the time variation of the ratio $\mathcal{S} = V(X_1 + P_2)/\sqrt{D}$ for $N = 100$ (green dashed line), $N = 500$ (red dash-dotted line), and $N = 2000$ (blue solid line). The $\mathcal{S} < 1$ corresponds to squeezing.

$$V(X_1 + P_2)V(P_1 - X_2) \geqslant \frac{1}{4}\left(1 + \frac{2\langle J_z\rangle}{N}\right)^2 \equiv D, \tag{29}$$

where $V(\hat{O}) = \langle \hat{O}^2 \rangle - \langle \hat{O} \rangle^2$ is the variance of an operator $\hat{O}$. We define a system exhibiting photon-atom squeezing if either $V(X_1 + P_2)$ or $V(P_1 - X_2)$ is smaller than $\sqrt{D}$. If $J_\pm$'s can be treated as harmonic oscillator operators as in the linearized regime, the photon-atom squeezing condition is reduced to the familiar two-mode squeezing condition. Note that knowledge of the dynamical field mode is important here in order to define the field quadratures associated with squeezing.

In Fig. 4, we illustrate the squeezing behavior by plotting $V(X_1 + P_2)$ as a function of time. The curves are obtained from the numerical solution of the Schrödinger equation governed by the single-mode Hamiltonian (22). At early times, $V(X_1 + P_2)$ decreases rapidly and it is smaller than $\sqrt{D}$; i.e., the squeezing effect occurs. Such a result is understood because $J_\pm$ are nearly bosonic, and hence H_{eff} is approximately the Hamiltonian of parametric down-conversion, giving the familiar result of two-mode squeezing,

$$V(X_1 + P_2) \approx \exp\left[-2\int_0^t \Omega(t')dt'\right]. \tag{30}$$

However, at longer times, nonlinear effects due to $J_\pm$ will become significant and the squeezing can be destroyed eventually. Our numerical calculations for $100 \leqslant N \leqslant 2000$ atoms indicate that there exists a characteristic time after which $V(X_1 + P_2)$ stops decreasing and becomes an increasing function of time (see Fig. 4). It remains an open problem to determine an analytical expression of the characteristic time. Here our numerical investigation suggests that Eq. (30) holds in the regime $N\gamma t \leqslant 1$.

VI. CONCLUSION

To summarize, we have formulated a single-mode model to study the quantum dynamics of Raman scattering from N

atoms inside a bad cavity. The key part of our formalism is employment of a dynamic field mode capturing the emitted photons. Under the condition (5), we determined the explicit form of the mode function and constructed a single-mode Hamiltonian by the time-dependent variation principle. By comparing the atomic dynamics predicted from the master equation (24), we find that the single-mode model is valid at times up to two to three $N\gamma t$. We should remark that although the master equation (24) provides information on the atomic density matrix, it does not determine the full atom-field state vector. With our single-mode approach, we can now access the state vector of the system and study the quantum correlation between atoms and photons within and beyond the linear regime. An example of an application is provided regarding the squeezing properties of photon-atom states. We have shown that a strong squeezing can be established in the joint photon-atom quadrature fluctuations, but such a squeezing behavior will diminish at longer times due to the nonlinear behavior of atomic-dipole operators.

As noted in the previous section, the state vector (17) indicates that photons and atoms are entangled as long as there is more than one nonzero term in the summation. Such an entanglement could be a useful resource for quantum teleportation, apart from the usual protocol based on two-mode squeezed light [22]. This is because the squeezed photon-atom states could in principle allow teleportation between light and matter—for example, teleporting an unknown quantum state of light to atoms for storage. In addition, we also note that the generation of photon-atom squeezed states is an intermediate step of generating temporally separated entangled light pulses [23] if the atomic state can be mapped to another light pulse by suitable excitation schemes. We hope to address these applications with the idea of dynamical modes in the future.

ACKNOWLEDGMENT

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