Resonance states of three self-gravitating bosons and fermions

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The resonance states of the systems consisting of three identical bosons and of three fermions interacting with an attractive $1/r$ potential are investigated using highly correlated exponential basis functions. The method of complex-coordinate rotation is used to extract energies and widths for the resonances lying below the $n=2$ excited state threshold of the respective two-body subsystem. For the system consisting of three identical bosons, we have obtained a total of 12 S-wave resonances. It is concluded that nine of them belong to one series of Rydberg states. For the system consisting of three fermions, we have obtained a total of 23 S-wave resonances in total spin three-halfs (the ^4S state) and 17 S-wave resonances total spin one-half (the ^2S states). The resonances for the ^4S states belong to three different series, while those for the ^2S states belong to two Rydberg series. The resonance energies and widths of these states for the three-body self-gravitating bosons and fermions are reported.

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I. INTRODUCTION

Recently a scheme for electromagnetically generating Bose-Einstein condensates (BECs) was realized that it is self-bound for sufficiently strong self-gravitation [1]. For a particular configuration scheme, it was demonstrated that a stable BEC with attractive $1/r$ interatomic interactions is achievable by irradiating the atoms with the intense, extremely off-resonant, electromagnetic fields [1,2]. Such an attractive potential can simulate the gravity between quantum systems. In a particular regime, the $1/r$ attraction balances the outward pressure due to the zero-point energy and the short range S-wave scattering [2]. With a proper choice of laser beams, the dipole-dipole interaction of order r^{-3} can be averaged out in the usual strong anisotropy regime. In such a regime, the interatomic potential takes a form of $-u/r$, where u is the strength of the potential that can be adjusted by changing the laser intensity. The factor u can be understood with the analog of GMm, where G is the Newton's constant and M, m are the masses. The investigation [1] with the gravitationlike interaction also suggests the possibility of realizing self-gravitating Bose stars in the laboratories [3]. It was further speculated that clusters consisting of BECs might be constructed in the laboratories [5]. Details of the self-gravitating three-boson systems have been highlighted in earlier works ([4–10], and references therein). Besides such self-gravitating identical boson systems, it is of great interest to study the system of three self-gravitating fermions. With the recent progresses in self-gravitating fermions ([11], and references therein) and with the recent developments on the degenerated Fermi gases [12], here we report investigations of resonances in these three-body systems.

From the theoretical aspect, several investigations have been performed for N identical, spinless (or spin stretched system), bosons interacting gravitationally [9,10,13]. The system for $N=3$ bosons was studied very recently in an adiabatic hyperspherical calculation [4]. The system of self-gravitating fermions has also been studied with attractive $1/r$ potentials ([14] and references therein). Recently, we have calculated the bound state energy levels of the self-gravitating three-identical-boson system and the self-gravitating three-fermion systems using a variational method with correlated wave functions [15]. While most of the above-mentioned references are for the ground state or for other bound states of the three-body self-gravitating bosons (also for the self-gravitating fermions [15]), there have been no investigations on the quantitative aspect about the three-body resonances for such systems, to the best of our knowledge. In a recent work using the hyperspherical treatment, resonances for a three-boson system were briefly discussed qualitatively, but no resonance energies and widths were reported [4]. In the present work, we have investigated the resonance states of a system consisting of three self-gravitating bosons and fermions lying below the $n=2$ threshold of the respective two-body subsystem. We employ the complex-coordinate rotation method together with using explicitly correlated exponential basis functions. In a first attempt to calculate S-wave resonances in such three boson systems, we have determined the resonance positions and widths for twelve states. Among such resonances, nine of them seem to belong to the members of a typical Rydberg series. Particles having totally symmetrized wave functions are called bosons and the totally antisymmetrized wave functions are called fermions. For the treatment of the system consisting of three fermions, we have employed the antisymmetrized wave functions as discussed on a study of Li isoelectronic sequence [16], and have considered the $1/r$ interaction potentials [14]. For a system consisting of three spin-1/2 particles, there are two possibilities to form their total spin, leading to total spin three-halfs (the ^4S state) and total spin one-half (the ^2S states), in a manner similar to the three electrons in the lithium atom (see, for example, Ref. [16]). For the system of three identical fermions, we have obtained a total twenty three ^4S resonances of which eight belongs to the first series, six belongs to the second series, and the other six belong to the third series. The ^2S resonances are similar to the ^4S states except for the third series that they do not appear in the total-spin-1/2 case. We believe investigations on fermions with antisymmetrized wave functions in the framework of attractive $1/r$ interaction potentials have not been reported in the literature until our preset work. The convergence of our calculations has been examined with increasing number of terms in the basis functions. In this paper, we have presented the theoretical details on Sec. II. The bound states of the systems of identical bosons and the systems of three fermions were reported in our earlier work [15], and selected few are highlighted in Sec. III. Results for resonance states of bosons and fermions are presented in Secs. IV and V, respectively.

II. CALCULATIONS

The Hamiltonian for the system of three identical particles having mass m interacting with gravitational potentials can be written as

$$H=T+V, \tag{1}$$

with

$$T=-\frac{\hbar^2}{2m}\sum_{i=1}^{3}\nabla_i^2 \tag{2}$$

and

$$V=-\frac{u}{r_{31}}-\frac{u}{r_{32}}-\frac{u}{r_{21}}, \tag{3}$$

where 1, 2, and 3 denote the three identical particles and $r_{ij}=|\vec{r}_i-\vec{r}_j|=r_{ji}$. For the system in Ref. [1], the coupling constant u is given by $u=(11/4\pi)(Iq^2\alpha_p^2/c\varepsilon_0^2)$, where I is the laser intensity, q the photon wave number, and α_p the atomic dynamic polarizability. This suggests that the strength of the interaction can be controlled by the laser parameters under certain conditions [1]. In this work, we have used an effective atomic unit (e.a.u.) with the analogy of the atomic units in which the two-body energy spectrum is represented as $-1/n^2$. The units of length and energy are expressed as $\hbar^2/2mu$ and $mu^2/2\hbar^2$, respectively. For S states of the proposed system with three identical bosons, we employ the explicitly correlated exponential wave functions

$$\Psi=\hat{S}\sum_{i=1}^{N_b}D_i\exp[(-a_ir_{31}-b_ir_{32}-c_ir_{21})\omega], \tag{4}$$

where the totally symmetrized operator $\hat{S}$ [16] is denoted by

$$\hat{S}=1+(12)+(13)+(23)+(123)+(132). \tag{5}$$

The wave functions for three fermions with total spin one-half (the ^2S states), we follow the treatment for the lithium, a three-electron atom [16], as

$$\Psi=\hat{A}\sum_{i=1}^{N_b}D_i\exp[(-a_ir_{31}-b_ir_{32}-c_ir_{21})\omega][(\alpha_1\beta_2\alpha_3-\beta_1\alpha_2\alpha_3)/\sqrt{2}], \tag{6}$$

with $\hat{A}$ the antisymmetrizer operator

$$\hat{A}=1-(12)-(13)-(23)+(123)+(132), \tag{7}$$

and α and β representing the spin-up and spin-down wave functions, respectively. As for the wave functions with total spin three-halfs (the ^4S states), we use the following:

$$\Psi=\hat{A}\sum_{i=1}^{N_b}D_i\exp[(-a_ir_{31}-b_ir_{32}-c_ir_{21})\omega](\alpha_1\alpha_2\alpha_3). \tag{8}$$

Here a_i, b_i, c_i are the nonlinear variation parameters, D_i ($i=1,\ldots,N_b$) are the linear expansion coefficients, and N_b is the number basis terms. The scaling factor $\omega=1$ for bound state calculations and it is varied for resonance calculations for the optimized values of a_i, b_i, c_i. The nonlinear variational parameters a_i, b_i, and c_i are chosen from a quasirandom process as used in the earlier works ([15,17–22], reference therein). The nonlinear parameters a_i, b_i, and c_i are chosen from the three positive interval $[A_1,A_2]$, $[B_1,B_2]$ and $[C_1,C_2]$;

$$a_i=\langle\langle i(i+1)\sqrt{2}/2\rangle\rangle(A_2-A_1)+A_1,$$

$$b_i=\langle\langle i(i+1)\sqrt{3}/2\rangle\rangle(B_2-B_1)+B_1,$$

$$c_i=\langle\langle i(i+1)\sqrt{5}/2\rangle\rangle(C_2-C_1)+C_1, \tag{9}$$

where the symbol $\langle\langle\cdots\rangle\rangle$ designates the fractional part of a real number. Using the similar basis functions (4) and the quasirandom process (6), we have calculated the resonance states of H^- [20], He [21], and Ps^- [22] using the stabilization method. The basis function (4) was used in an earlier study [23] for an investigation of the domain of coupling constants of three-boson systems.

In this work, we have calculated resonance parameters using the complex rotation method [24–26]. In this method, the interparticle radial coordinate r_{ij} are transformed into

$$r_{ij}=r_{ij}\exp(i\vartheta), \tag{10}$$

where ϑ is real and positive. The Hamiltonian (1) takes the form

$$H=T\exp(-2i\vartheta)+V\exp(-i\vartheta). \tag{11}$$

Under such a transformation one needs to calculate the matrix element for the kinetic energy term (2) and the potential energy term (3) separately, and then scale them according to Eq. (10). Resonances can be examined once the complex eigenvalue problem is diagonalized with the exponential basis functions mentioned above. A complex resonance eigenvalue $W=E_r$-$i\Gamma/2$, where E_r and Γ denote the resonance energy and width, respectively, that exhibits stationary behavior for small change of ϑ and the minimum of $\partial W/\partial\vartheta\approx$minimum will yield the resonance position and width. For the behavior of the spectrum of the transformed Hamiltonian and for other aspects of the complex rotation method, readers are referred to several reviews in the literature [24–26].

TABLE I. Ground state and excited-states energies in e.a.u. for bosons.

N	Present work [15]	Ref. [4]	Ref. [10]	Ref. [9]
1	−2.143 558 75	−2.136 527	−2.142 86	−2.134
2	−1.148 984 43	−1.145 881		
∞	−0.5			

III. BOUND STATES OF THE SYSTEMS OF THREE BOSONS AND FERMIONS

Very recently, we have carried out an investigation on the bound states for the three-boson and three-fermion systems bound to each other respectively by the gravitationlike forces [15]. Here we show some selected results in Table I. Our ground state energy, obtained by using a Ritz variational method, has a property of rigorous upper bound to the exact energy, and its lowest value as compared to others in the literature hence represents the best result for such a three-boson system to this date. Bound state energies for three fermions were also reported. Details can be found in our earlier work [15]. We therefore believe that using the wave functions (4), (6), and (8) to investigate resonances in these three-boson and three-fermion systems, together with employing the complex-coordinate rotation method for studies of resonances, will also lead to accurate results for such systems.

IV. RESONANCE STATES FOR THE SYSTEM OF THREE IDENTICAL BOSONS

As discussed above, to obtain resonance parameter (E_r, Γ) in the framework of complex coordinate rotation method, one needs to calculate the complex energy level $E(\vartheta,\omega)$ by diagonalizing the Hamiltonian (11) with the wave function (4). Here we vary the values of ω from 0.1 to 1.0, and ϑ from 0.10 to 0.70 radians in a mesh size of 0.05 radians. In our calculations we have found that it is sufficient to vary ϑ from 0.10–0.65 radians, and ω from 0.2 to 0.7. Once the resonance poles are identified by observing the energies in the complex energy plane, then near a particular resonance pole, the plots of $E(\vartheta,\omega)$ show that the rotational paths (see Figs. 1–3) are slowed down and all the energies for different values of ω in the complex plane will approximately meet at a point, and from which the resonance parameters can be determined. In Fig. 1, we present the $E(\vartheta,\omega)$ in the range of ϑ from 0.15 to 0.40 radians, and in Figs. 2 and 3, we use ϑ in the range of 0.10 to 0.35 radian, in a spacing of 0.05 radians. We next compute the minimum of changes in complex energy with respect to the change of ϑ, $|W(\vartheta+\Delta\vartheta)-W(\vartheta)|$ around the resonance pole for different ω values. Among all the minima for different ω values, the minimum value of the relative minima ($\partial W/\partial\vartheta\approx\min$) will yield the resonance position and width for that particular resonance. We present the feature in Tables II and III for a fixed ω and the increasing number of basis terms in Eq. (4). The above procedure can be repeated by using different expansion lengths to examine the convergence. Usually, accurate resonance parameters (E_r, Γ) can be obtained by using such a computational technique.

Following the above computational scheme, we have found twelve S-wave resonances for a system consisting of

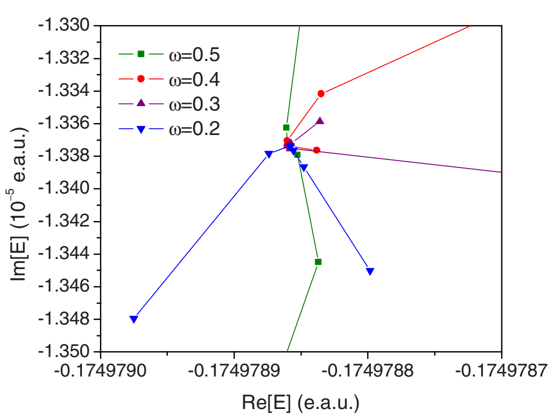

FIG. 1. (Color online) Rotational paths of the seventh resonance state for the system of three identical bosons in the complex energy plane for four different values of the scaling factor ω.

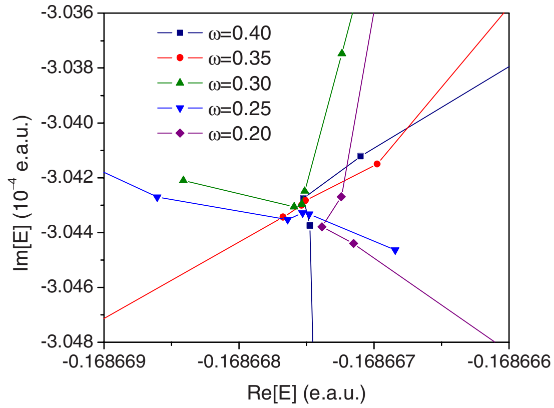

FIG. 2. (Color online) Rotational paths of the eighth resonance state for the system of three identical bosons in the complex energy plane for five different values of the scaling factor ω.

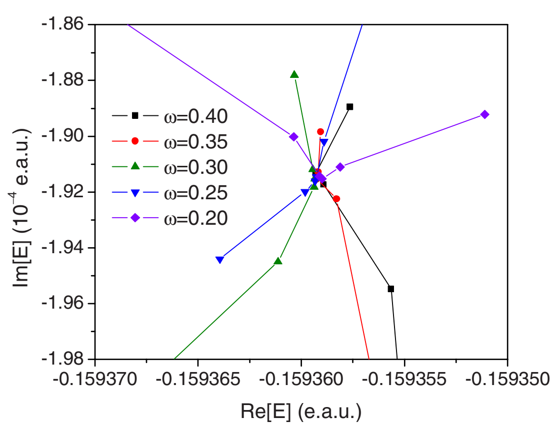

FIG. 3. (Color online) Rotational paths of the tenth resonance state for the system of three identical bosons in the complex energy plane for five different values of the scaling factor ω.

TABLE II. Convergence and stationary behavior of the lowest resonance of the system consisting of three identical bosons (in e.a.u). The numbers in square brackets denote powers of ten. The asterisk represents minimum value among all the minima.

ϑ	Re[E]	Im[E]	$\lvert W(\vartheta+\Delta\vartheta)-W(\vartheta)\rvert$	$\frac{\partial W}{\partial\vartheta}=\min$
$\omega=0.6$, $N_b=500$				
0.25	−0.410 235 586 41	−0.000 541 444 10		
0.30	−0.410 235 576 60	−0.000 541 465 00	0.231[−07]	
0.35	−0.410 235 576 01	−0.000 541 467 62	0.269[−08]	
0.40	−0.410 235 576 04	−0.000 541 467 92	0.301[−09]	
0.45	−0.410 235 576 07	−0.000 541 467 96	0.500[−10]	0.500[−10]
0.50	−0.410 235 575 85	−0.000 541 467 88	0.234[−09]	
0.55	−0.410 235 572 70	−0.000 541 457 60	0.108[−07]	
0.60	−0.410 235 536 53	−0.000 541 376 79	0.885[−07]	
$\omega=0.6$, $N_b=600$				
0.25	−0.410 235 572 35	−0.000 541 470 69		
0.30	−0.410 235 575 86	−0.000 541 468 24	0.428[−08]	
0.35	−0.410 235 576 02	−0.000 541 468 00	0.288[−09]	
0.40	−0.410 235 576 04	−0.000 541 467 99	0.224[−10]	
0.45	−0.410 235 576 04	−0.000 541 468 01	0.200[−10]	0.200[−10]*
0.50	−0.410 235 576 01	−0.000 541 468 11	0.104[−09]	
0.55	−0.410 235 576 00	−0.000 541 469 15	0.104[−08]	
0.60	−0.410 235 580 85	−0.000 541 479 02	0.110[−07]	

three identical bosons bound by gravitational forces. The convergence and the stationary behavior of the lowest and the fourth resonances are presented in Tables II and III, respectively. The rotational paths for the seventh, eighth, and tenth resonances are presented in Figs. 1–3. We present our calculated resonance energies in Fig. 4 and in Table IV. The results presented in Figs. 1–5 and Table IV are obtained by using 600-term of wave functions (4). Furthermore, it is observed that the results presented in Tables I and II are fairly converged with 400, 500-term basis functions and also with different sets of nonlinear variational parameters a_i, b_i, c_i. We estimate the uncertainties for the resonance energies and widths are within a few parts of the last digits quoted. From Fig. 4, it is evident that the resonances 1, 2, 3, 5, 8, 10, 11, and 12, according to the order of appearance, belong to a Rydberg series (like $2sns$ atomic Rydberg series in He) converging to the $n=2$ excited state threshold of the two-body subsystem. Next, we fit the binding energies of the first series measured from the $n=2$ excited state threshold (−0.125 e.a.u) of the two-body subsystem using the quantum defect formula

$$E_r^{/} = P/[2(n-\mu)^2], \tag{12}$$

for $n>5$. In Fig. 5, the circles and the dashed line denote the actual calculations using $E_r^{/}=-0.125-E_r$, and the triangles and the solid line denote the fitting using the quantum defect formula (12) for $n>5$. In this figure, we use the log-log scale and the fit is extended down to $n=3$. From the fit in Fig. 5, we have obtained $P=5.35227$ and $\mu=0.17347$. The fitting in Fig. 1 is quite good as the χ^2 is very small (in the order of 10^{-9}), and the square of correlation coefficient (ρ^2) is much closer to 1 that represents an almost perfect fit. We have not tried to fit the other resonances (4, 7, and 9) to the quantum defect formula as one (or more) of these resonances might be associated with the higher lying $n=3$ threshold of the two-

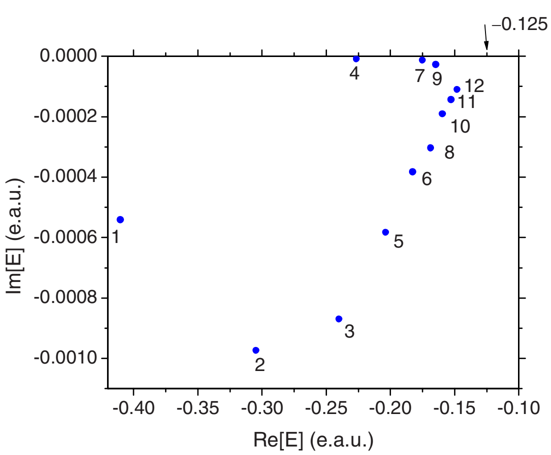

FIG. 4. (Color online) Calculated resonances for the system of three identical bosons in the complex energy plane below the $n=2$ threshold of the two-body subsystem.

TABLE III. Convergence and stationary behavior of the fourth resonance of the system consisting of three identical bosons (in e.a.u). The numbers in square brackets denote powers of ten. The asterisk represents minimum value among all the minima.

ϑ	Re[E]	Im[E]	$\lvert W(\vartheta+\Delta\vartheta)-W(\vartheta)\rvert$	$\frac{\partial W}{\partial\vartheta}$=min
ω=0.4, N_b=500				
0.20	−0.226 443 095 52	−0.952 888 66[−05]		
0.25	−0.226 443 093 67	−0.952 795 07[−05]	0.185[−08]	
0.30	−0.226 443 093 50	−0.952 782 67[−05]	0.210[−09]	
0.35	−0.226 443 093 49	−0.952 781 04[−05]	0.191[−10]	
0.40	−0.226 443 093 48	−0.952 780 85[−05]	0.102[−10]	0.102[−10]
0.45	−0.226 443 093 38	−0.952 781 79[−05]	0.100[−09]	
0.50	−0.226 443 092 42	−0.952 810 50[−05]	0.100[−08]	
ω=0.4, N_b=600				
0.20	−0.226 443 094 55	−0.952 737 75[−05]		
0.25	−0.226 443 093 62	−0.952 780 07[−05]	0.102[−08]	
0.30	−0.226 443 093 51	−0.952 780 80[−05]	0.110[−09]	
0.35	−0.226 443 093 50	−0.952 780 40[−05]	0.108[−10]	
0.40	−0.226 443 093 49	−0.952 780 41[−05]	0.100[−10]	0.100[−10]*
0.45	−0.226 443 093 57	−0.952 782 72[−05]	0.833[−10]	
0.50	−0.226 443 094 25	−0.952 805 99[−05]	0.719[−09]	

TABLE IV. Resonance energies and widths (in e.a.u) for the system consisting of three identical bosons. The notation $a[b]$ stands for $a\times 10^b$. The numbers in square brackets denote powers of ten.

First series Order of appearance	Quantum number (n)	E_r	$\Gamma/2$
1	3	−0.410 235 576	0.000 541 47
2	4	−0.304 561 023	0.000 973 98
3	5	−0.239 945 56	0.000 869 74
5	6	−0.203 661 35	0.000 583 3
6	7	−0.182 482 8	0.000 383 3
8	8	−0.168 667 5	0.000 304
10	9	−0.159 36	0.000 191
11	10	−0.152 76	0.000 144
12	11	−0.147 9	0.000 11
	∞	−0.125	
Other resonances			
Order of appearance		E_r	$\Gamma/2$
4		−0.226 443 09	0.952 78[−5]
7		−0.174 978 86	0.133 7[−4]
9		−0.164 530 3	0.277[−4]

body subsystem. Further investigations are needed to shed light on the nature of these three-body resonances.

V. RESULTS FOR THE SYSTEM OF THREE FERMIONS

Following the similar computational scheme as discussed above, we have obtained resonance parameters (E_r,Γ) for the system of three fermions with a total spin of three-halfs and of total spin one-half. We calculate the complex energy

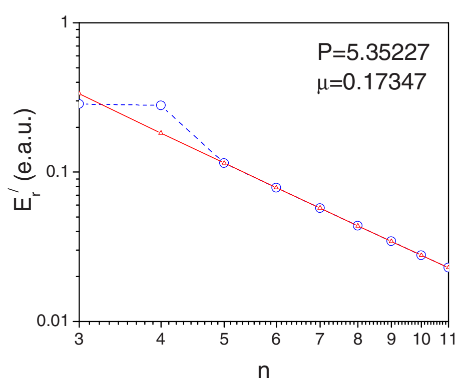

FIG. 5. (Color online) Calculated resonance energies $E_r^{/}$ (e.a.u.) (circles + dashed line) measured from the $n=2$ excited state threshold of the two-body subsystem and the fittings (triangles + solid line) using the quantum defect formula (12).

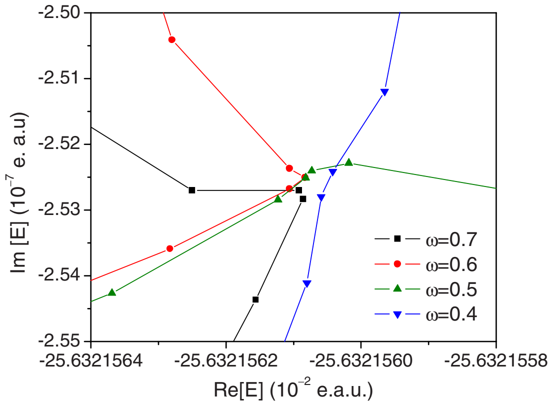

FIG. 6. (Color online) Rotational paths of the third resonance state for the system of three fermions (total spin-3/2) in the complex energy plane for four different values of the scaling factor ω.

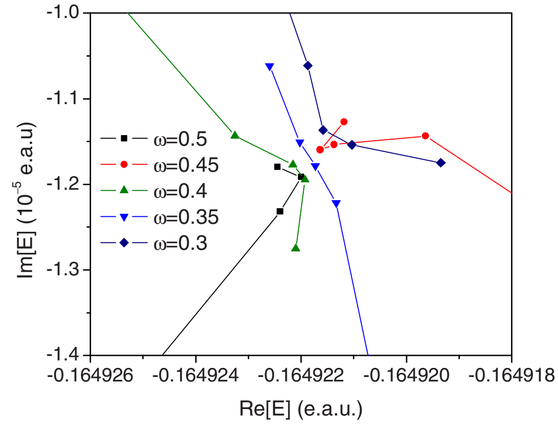

FIG. 7. (Color online) Rotational paths of the fifteenth resonance state for the system of three fermions (total spin-3/2) in the complex energy plane for five different values of the scaling factor ω.

eigenvalues $E(\vartheta,\omega)$ by diagonalizing the Hamiltonian (11) with the wave functions (8) and (6), respectively. We vary the values of ω from 0.1 to 1.0, and ϑ from 0.10 to 0.70 radians in a mesh size of 0.05 radians. In Figs. 6 and 7, we present the rotational paths for the third and fifteenth resonances, respectively. In Fig. 6, we present $E(\vartheta,\omega)$ in the range of ϑ from 0.10 to 0.40 radians. In Fig. 7, we use ϑ in the range of 0.10 to 0.35 radian in a spacing of 0.05 radians. In Tables V and VI, we present the convergence and the stationary behavior for the lowest and 5th resonances, respectively. We have found twenty three S-wave resonances for states with total spin three-halfs for a system consisting of three fermions bound by gravitational forces. We have also found seventeen S-wave resonances for total spin one-half and these resonances seem to have the same complex eigenvalues as those of the total spin three-halfs. We present our calculated resonance energies for states with total spin three-halfs in Fig. 8 and in Table VII. We present the resonances for the states with total spin one-half

TABLE V. Convergence and stationary behavior of the lowest resonance of the system consisting of three fermions using total spin three-half wave functions (in e.a.u). The numbers in square brackets denote powers of ten. The asterisk represents minimum value among all the minima.

ϑ	Re[E]	Im[E]	$\lvert W(\vartheta+\Delta\vartheta)-W(\vartheta)\rvert$	$\frac{\partial W}{\partial\vartheta}=\min$
ω=0.7, N=400				
0.25	−0.398 251 292 68	−0.000 338 749 17		
0.30	−0.398 251 210 39	−0.000 339 074 84	0.336[−6]	
0.35	−0.398 251 184 67	−0.000 339 136 87	0.672[−7]	
0.40	−0.398 251 178 13	−0.000 339 149 87	0.146[−7]	
0.45	−0.398 251 181 25	−0.000 339 162 76	0.133[−7]	0.133[−7]
0.50	−0.398 251 221 34	−0.000 339 224 61	0.737[−7]	
0.55	−0.398 251 473 84	−0.000 339 532 94	0.399[−6]	
ω=0.7, N=500				
0.25	−0.398 250 919 69	−0.000 339 135 58		
0.30	−0.398 251 132 56	−0.000 339 142 38	0.213[−6]	
0.35	−0.398 251 170 82	−0.000 339 148 71	0.388[−7]	
0.40	−0.398 251 177 33	−0.000 339 150 60	0.678[−8]	
0.45	−0.398 251 177 86	−0.000 339 151 46	0.101[−8]	0.101[−8]*
0.50	−0.398 251 174 41	−0.000 339 156 93	0.518[−7]	
0.55	−0.398 251 153 14	−0.000 339 208 63	0.559[−7]	

TABLE VI. Convergence and stationary behavior of the fifth resonance of the system consisting of three fermions with total spin-3/2 (in e.a.u.). The numbers in square brackets denote powers of ten. The asterisk represents minimum value among all the minima.

ϑ	Re[E]	Im[E]	$\lvert W(\vartheta+\Delta\vartheta)-W(\vartheta)\rvert$	$\frac{\partial W}{\partial\vartheta}=\min$
$\omega=0.5$, $N=400$				
0.20	−0.235 198 686 49	−0.000 230 073 97		
0.25	−0.235 198 624 48	−0.000 229 327 07	0.749[−6]	
0.30	−0.235 198 628 23	−0.000 229 178 10	0.149[−6]	
0.35	−0.235 198 500 94	−0.000 229 200 82	0.129[−6]	0.129[−6]
0.40	−0.235 197 551 28	−0.000 229 445 15	0.981[−6]	
0.45	−0.235 191 963 48	−0.000 230 058 56	0.562[−5]	
$\omega=0.5$, $N=500$				
0.20	−0.235 198 343 54	−0.000 229 640 64		
0.25	−0.235 198 563 79	−0.000 229 264 25	0.436[−6]	
0.30	−0.235 198 615 93	−0.000 229 208 40	0.764[−7]	
0.35	−0.235 198 641 45	−0.000 229 212 19	0.258[−7]	0.258[−7]*
0.40	−0.235 198 815 49	−0.000 229 289 37	0.190[−6]	
0.45	−0.235 200 626 40	−0.000 229 809 80	0.736[−6]	

in Table VII. The results presented in Figs. 6–11 and in Table VII are obtained using 500-term wave functions (8) and (6). It is found that the calculations with total spin one-half converge better than the calculations with total spin three-half. Furthermore, it is observed that the results presented in Table VII are fairly converged with 400, 500-term basis functions, and with different sets of nonlinear variational parameters a_i, b_i, c_i. We have examined the convergence of our calculations up to 800 terms with spin one-half wave functions. We estimate the uncertainties for the resonance energies and widths are within a few parts of the last digits quoted. From Fig. 8 and Table VII, it is seen that the resonances 1, 2, 5, 8, 11, 14, 17, and 21, according to the order of appearance, seem to belong to one Rydberg series converging to the $n=2$ threshold of the two-body subsystem. We denote this series as the first Rydberg series of resonances. We also analyze the results from Table VII and Fig. 8 that the resonances 3, 6, 9, 12, 15, and 18 belong to the second series and the resonances 4, 7, 10, 13, 16, and 19 belong to the third series. The resonances 20, 22, and 23 might belong to some lower-lying members of a Rydberg series converging to the $n=3$ threshold of the two-body subsystem. In Table VII, we have not presented the resonance widths for the resonances in the third series, and also for some other resonances (i.e., for 12 and 18 in the second series and 17 and 21 in the first series). While the real parts of these complex eigenvalues converge quite well, the imaginary parts do not converge to the desired accuracy, as such resonances have very narrow widths. Instead, we mark "very narrow" under the width column in Table VII. We have nevertheless been able to obtain the widths for the twelfth and seventeenth resonances for the spin-half case. In Fig. 8, for those resonances marked "very narrow," we simply put them down on the real axis for illustrative purpose. But in fact their widths (related to the imaginary part) are not "zero." Here we present the results for the three-fermion systems for both identical and nonidentical cases. For a spin-1/2 particle, there are two possibilities for its spin: Either spin-up or spin-down. So for the total spin one-half case, these fermions are not identical. But for the total spin three-halfs case, the three fermions are identical.

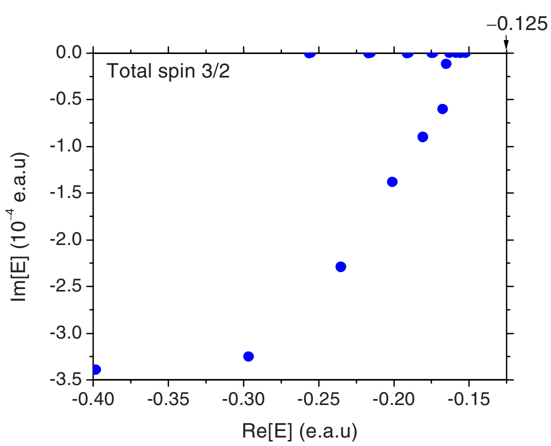

FIG. 8. (Color online) Calculated resonances for the system of three identical fermions with total spin-3/2 in the complex energy plane below the $n=2$ threshold of the two-body subsystem.

Next, we fit the binding energies of the first series measured from the $n=2$ excited state threshold (−0.125 e.a.u.) of the two-body subsystem using the quantum defect formula (12) for $n>4$. In Fig. 9, the circles and the dashed line denote the actual calculations using $E_r'=-0.125-E_r$, and the triangles and the solid line denote the fitting using the quan-

TABLE VII. Resonance energies and widths (in e.a.u.) for the system consisting of three fermions. The notation $a[b]$ stands for $a \times 10^b$. The numbers in square brackets denote powers of ten. The asterisk represents order of appearance for the spin-3/2 case.

		States with total spin of 3/2		States with total spin of 1/2	
First series					
Order*	Quantum number (n)	E_r	$\Gamma/2$	E_r	$\Gamma/2$
1	3	−0.398 251 18	−0.339 15[−3]	−0.398 251 18	−0.339 15[−3]
2	4	−0.296 598 2	−0.325 2[−3]	−0.296 598 2	−0.3252[−3]
5	5	−0.235 199	−0.229[−3]	−0.235 199	−0.229[−3]
8	6	−0.200 887	−0.138[−3]	−0.200 887	−0.138[−3]
11	7	−0.180 60	−0.9[−4]	−0.180 60	−0.9[−4]
14	8	−0.167 50	−0.6[−4]	−0.167 50	−0.6[−4]
17	9	−0.158 52	Very narrow	−0.158 53	−0.4[−4]
21	10	−0.152 1	Very narrow	−0.152 1	Very narrow
	∞	−0.125		−0.125	
Second series					
Order*	Quantum number (n)	E_r	$\Gamma/2$	E_r	$\Gamma/2$
3	5	−0.256 321 56	−0.25[−6]	−0.256 321 56	−0.25[−6]
6	6	−0.216 750 8	−0.33[−6]	−0.216 750 8	−0.33[−6]
9	7	−0.191 240 8	−0.24[−6]	−0.191 240 8	−0.24[−6]
12	8	−0.174 75	Very narrow	−0.174 749 5	−0.133[−6]
15	9	−0.164 92	−0.12[−4]	−0.164 92	−0.12[−4]
18	10	−0.155 8	Very narrow	−0.155 86	Very narrow
	∞	−0.125		−0.125	
Third series					
Order*	Quantum number (n)	E_r	$\Gamma/2$	E_r	$\Gamma/2$
4	5	−0.255 090 42	Very narrow		
7	6	−0.215 396 7	Very narrow		
10	7	−0.190 13	Very narrow		
13	8	−0.173 89	Very narrow		
16	9	−0.162 87	Very narrow		
19	10	−0.155 3	Very narrow		
	∞	−0.125		−0.125	
Other resonances					
Order of appearance		E_r	$\Gamma/2$	E_r	$\Gamma/2$
20		−0.153 54	−0.2[−4]	−0.153 54	−0.2[−4]
22		−0.136 28	−0.96[−4]	−0.136 28	−0.96[−4]
23		−0.127 513	−0.21[−4]	−0.127 513	−0.21[−4]

tum defect formula (12) for $n>4$. In this figure, we use the log-log scale and the fit is extended down to $n=3$. From the fit in Fig. 9, we have obtained $P=5.41464$ and $\mu=0.03039$. Similarly, we fit the binding energies of the second and third series with formula (12) for $n>6$. In Figs. 10 and 11, the circles denote the actual calculations using $E_r'=-0.125-E_r$ and the solid line denote the fitting using the quantum defect formula (12) for $n>6$. In these figures, we use the log-log scale and the fit is extended down to $n=5$. From the fit in Fig. 10, we obtain $P=5.65471$ and $\mu=0.45308$. Similarly, from the fit in Fig. 11, we obtain $P=5.51672$ and $\mu=0.47871$. The fittings in Figs. 9–11 are quite good as the χ^2 are very small (in the order of 10^{-8}), and the square of correlation coefficient (ρ^2) is much closer to 1, which represents

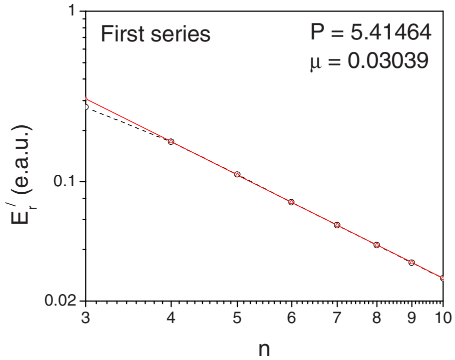

FIG. 9. (Color online) Calculated resonance energies E_r' (e.a.u.) (circles+dashed line) for first series in Table VII measured from the $n=2$ excited state threshold of the two-body subsystem and the fittings (triangles+solid line) using the quantum defect formula (12). Both total spin-3/2 and spin-1/2 have this series.

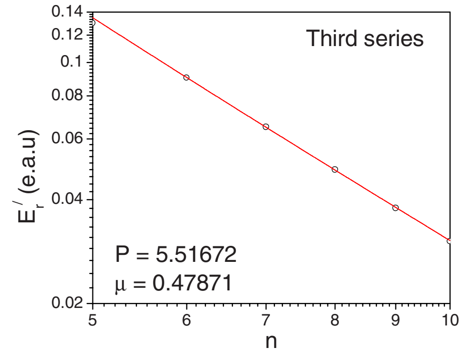

FIG. 11. (Color online) Calculated resonance energies E_r' (e.a.u.) (circles) for third series in Table VII measured from the $n=2$ excited state threshold of the two-body subsystem and the fittings (solid line) using the quantum defect formula (12). Only the total spin-3/2 case has this series. It is absent in the spin-1/2 case.

an almost perfect fit. We have not tried to fit the other resonances (20, 22, and 23) to the quantum defect formula as one (or more) of these resonances might be associated with the higher lying $n=3$ threshold of the two-body subsystem. Further investigations are needed to shed light on the nature of these three-body resonances.

VI. SUMMARY AND CONCLUSION

In the present work, we have made a first attempt to calculate the energies and widths for some S-wave resonances in self-gravitating three-identical-boson and three-fermion systems in the framework of complex-coordinate rotation method, which is a practical and powerful method to extract resonances of few-body systems. Highly correlated exponential wave functions supported by a widely used quasirandom process are employed to represent the correlation effects between the three identical particles. Below the $n=2$ threshold of the two-body subsystem, we have found one Rydberg series of resonances for bosons, and three series of resonances are obtained for the system with three fermions forming a total spin of 3/2. For the case when three fermions form a total spin of 1/2, we have found two series in this energy region. The first series of each system is found to have relatively broad widths, and the other series show relatively narrow widths. When the binding energies (relative to the $n=2$ threshold of the two-body subsystem) are fitted to a quantum defect formula, they seem to follow a very good fit, showing an interesting pattern for such resonances. Furthermore, we have also obtained three narrow resonances for bosons which might represent the other Rydberg series converging to the two-body excited $n=2$ threshold, or to the higher $n=3$ two-body threshold. For fermions, we have calculated the resonance states for both of total spin three-halfs and total spin one-half. Only one series of spin-3/2 containing six resonances does not belong to the spin-1/2, but the other resonances are very similar for both spin states of fermions. Finally, we should mention that with the recent advancement in the laboratory for self-gravitating Bose-Einstein condensates and degenerated Fermi gases, it would be of great interest in the future to study and compare the properties of the wave functions (i.e., the vibrational modes) between bosons and fermions. However, such an investigation is outside the scope of our present work. Here we present a first calculation of *S*-wave resonances in systems consisting of three identical bosons and of three fermions forming a total spin three-halfs and a total spin one-half. We hope our findings for the three-body systems with attractive $1/r$ potentials will provide useful information for further theoretical and experimental investigations for such intriguing three-body problems.

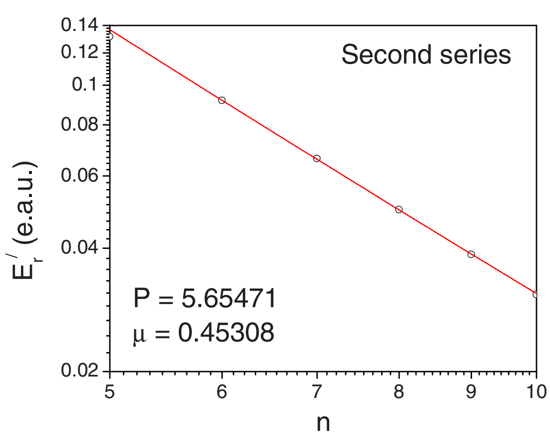

FIG. 10. (Color online) Calculated resonance energies E_r' (e.a.u.) (circles) for second series in Table VII measured from the $n=2$ excited state threshold of the two-body subsystem and the fittings (solid line) using the quantum defect formula (12). Both total spin-3/2 and spin-1/2 cases have this series.

ACKNOWLEDGMENT

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