Fluctuations of the Casimir-Polder force between an atom and a conducting wall

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We consider quantum fluctuations of the Casimir-Polder force between a neutral atom and a perfectly conducting wall in the ground state of the system. In order to obtain the atom-wall force fluctuation we first define an operator directly associated with the force experienced by the atom considered as a polarizable body in an electromagnetic field and we use a time-averaged force operator in order to avoid ultraviolet divergences appearing in the fluctuation of the force. This time-averaged force operator takes into account that any measurement involves a finite time. We also calculate the Casimir-Polder force fluctuation for an atom between two conducting walls. Experimental observability of these Casimir-Polder force fluctuations is also discussed, as well as the dependence of the relative force fluctuation on the duration of the measurement.

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I. INTRODUCTION

A striking consequence of quantum electrodynamics is that the radiation field, even in its ground state, has fluctuations of the electric and magnetic fields around the zero value [1,2]. This theoretical prediction has many remarkable observable consequences. One of them is the prediction of the existence of electromagnetic forces between two or more uncharged objects in the vacuum. The existence of these forces was first predicted in two papers by Casimir [3] and by Casimir and Polder [4] in 1948, and, from then onwards, interest in this subject has grown exponentially. Many experiments have definitively proved these effects with remarkable precision, measuring the Casimir force between a lens and a wall [5,6], between a neutral atom and a wall [7–9], between a surface and a Bose-Einstein condensate [10,11], and between two metallic neutral parallel plates [12,13].

One aspect of Casimir-Polder forces has not received, in our opinion, enough attention: the value calculated for the force is actually an average value, and it may in principle exhibit quantum fluctuations. The study of fluctuations of Casimir-Polder forces could be relevant for the stability of micro- and nanoelectromechanical systems (MEMS and NEMS), which are devices based on controlling the movement of metallic objects separated by distances of the order of micrometers or nanometers, where Casimir forces may be relevant [14,15].

Casimir and Casimir-Polder force fluctuations have been studied, with different approaches, by Barton [16–18], Eberlein [19,20], Jaekel and Reynaud [21], and Wu *et al.* [22,23]. Our approach follows that of Barton, with the difference that, whereas Barton studied only entirely macroscopical systems, we apply his method of time-averaged operators to the study of systems with also one atom present.

In this paper we calculate the fluctuations of the Casimir-Polder force between a neutral atom and a perfectly conducting wall in the ground state of the system. We first introduce an operator directly associated with the force experienced by a polarizable body in an electromagnetic field. Since the quadratic mean value of the force proves to be divergent, we make use of the method of time-averaged operators introduced and widely used by Barton in his papers about fluctuations of Casimir forces for macroscopic bodies [16–18]. The analytical techniques used are introduced in Sec. II, whereas a detailed calculation is given in Sec. III. In Sec. IV the Casimir-Polder force fluctuation is obtained in the case of an atom between two parallel walls: this permits us to specialize our results to a system for which the atom-wall Casimir-Polder force has been measured with precision [7,8]. In the Conclusions, we make further remarks on our results and outline possible future developments.

II. FORCE OPERATOR AND THE METHOD OF TIME-AVERAGED OPERATORS

Let us first briefly review the method often used to calculate the average Casimir-Polder force between an atom and a neutral conducting wall. The calculation is carried out by considering the interaction energy of the atom with the radiation field in the vacuum state. A convenient choice is to use the effective interaction Hamiltonian given by [24]

$$W=-\frac{1}{2}\sum_{\mathbf{k}\mathbf{k}'jj'}\alpha(k)\mathbf{E}_{\mathbf{k}j}(\mathbf{r}_A)\cdot\mathbf{E}_{\mathbf{k}'j'}(\mathbf{r}_A), \quad (1)$$

where

$$\mathbf{E}(\mathbf{r})=\sum_{\mathbf{k}j}\mathbf{E}_{\mathbf{k}j}(\mathbf{r})=i\sum_{\mathbf{k}j}\sqrt{\frac{2\pi\hbar\omega_k}{V}}(a_{\mathbf{k}j}-a^{\dagger}_{\mathbf{k}j})\mathbf{f}(\mathbf{k}j,\mathbf{r}), \quad (2)$$

$\alpha(k)$ being the dynamical polarizability of the atom and $\mathbf{f}(\mathbf{k}j,\mathbf{r})$ the mode functions used for the quantization of the electromagnetic field in the presence of the wall. This Hamiltonian is correct up to order $\alpha\sim e^2$, e being the electron charge. This effective Hamiltonian allows considerable simplification in the calculation of Casimir-Polder potentials, in both stationary and dynamical cases [25–27]. The presence of the wall is taken into account by considering a conducting cubic cavity defined by

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$$-\frac{L}{2}<x<\frac{L}{2}, \quad -\frac{L}{2}<y<\frac{L}{2}, \quad 0<z<L, \tag{3}$$

where L is the side of the cavity and $V=L^3$ its volume. The mode functions for this box have components [1,28]

$$f_x(\mathbf{k}j,\mathbf{r}) = \sqrt{8}(\mathbf{e}_{\mathbf{k}j})_x \cos\left[k_x\left(x+\frac{L}{2}\right)\right]\sin\left[k_y\left(y+\frac{L}{2}\right)\right]\sin(k_z z),$$

$$f_y(\mathbf{k}j,\mathbf{r}) = \sqrt{8}(\mathbf{e}_{\mathbf{k}j})_y \sin\left[k_x\left(x+\frac{L}{2}\right)\right]\cos\left[k_y\left(y+\frac{L}{2}\right)\right]\sin(k_z z),$$

$$f_z(\mathbf{k}j,\mathbf{r}) = \sqrt{8}(\mathbf{e}_{\mathbf{k}j})_z \sin\left[k_x\left(x+\frac{L}{2}\right)\right]\sin\left[k_y\left(y+\frac{L}{2}\right)\right]\cos(k_z z), \tag{4}$$

where $\mathbf{e}_{\mathbf{k}j}$ are polarization unit vectors and the allowed values of $\mathbf{k}$ have components

$$k_x=\frac{l\pi}{L}, \quad k_y=\frac{m\pi}{L}, \quad k_z=\frac{n\pi}{L}, \quad l,m,n=0,1,2,\ldots\ . \tag{5}$$

We obtain a correct description of a conducting wall located in $z=0$ by taking the limit $L\to+\infty$.

We now calculate the quantum average of the operator (1) on the ground state $|0\rangle$ of the electromagnetic field. If we consider the atom located in $\mathbf{r}_A=(0,0,d)$, with $d>0$, we obtain

$$E(d) = \langle 0|W|0\rangle = -\frac{\pi\hbar c}{V}\sum_{\mathbf{k}j} k\alpha(k)[\mathbf{f}(\mathbf{k}j,\mathbf{r}_A)\cdot\mathbf{f}(\mathbf{k}'j',\mathbf{r}_A)].$$

Because this interaction energy depends on the z coordinate of the atom, in a quasistationary approach the atom experiences a force given by

$$F_A(d) = -\frac{\partial}{\partial d}E(d). \tag{6}$$

Using the explicit expression of the mode functions $\mathbf{f}(\mathbf{k}j,\mathbf{r})$ it is easy to get the result

$$F_A(d) = -\frac{3\hbar c\alpha}{2\pi d^5}, \tag{7}$$

where d is the atom-wall distance, α is the static polarizability of the atom, and the minus sign indicates that the force is attractive. The expression (7) is valid in the so-called *far zone* defined by the condition $d \gg c/\omega_0$, ω_0 being a typical atomic transition frequency. This result coincides with that obtained by Casimir and Polder [4]. Effects related to a possible motion of the atom have been recently considered in the literature by inclusion of the atomic translational degrees of freedom [29,30].

This method provides a physically transparent way for calculating the average force on the atom but it does not enable one to easily obtain the quadratic average value of the force, necessary for the fluctuation. Thus we introduce a new operator associated with the force on the atom. In order to define such an operator we formally take minus the derivative of the operator (1) with respect to the z coordinate of the atom d, treated as a parameter. So we take the following quantity as the force operator:

$$F_A = -\frac{\partial}{\partial d}W = -\frac{\pi\hbar c}{V}\sum_{\mathbf{k}\mathbf{k}'jj'}\sqrt{kk'}\alpha(k)(a_{\mathbf{k}j}-a^\dagger_{\mathbf{k}j})(a_{\mathbf{k}'j'}-a^\dagger_{\mathbf{k}'j'}) \times F_A(\mathbf{k}j,\mathbf{k}'j',d), \tag{8}$$

where

$$F_A(\mathbf{k}j,\mathbf{k}'j',d) = \frac{\partial}{\partial d}[\mathbf{f}(\mathbf{k}j,\mathbf{r}_A)\cdot\mathbf{f}(\mathbf{k}'j',\mathbf{r}_A)]. \tag{9}$$

It is easy to show that the quantum average of the operator (8) on the vacuum state $|0\rangle$ gives back the expression (7) of the force, since the derivation with respect to d commutes with the quantum average. This force operator is correct up to order α. We can now consider the operator corresponding to the square of the force—that is,

$$F_A^2 = \left(\frac{\pi\hbar c}{V}\right)^2 \sum_{\substack{\mathbf{k}\mathbf{k}'jj'\\ \mathbf{p}\mathbf{p}'ll'}}\sqrt{kk'pp'}\alpha(k)\alpha(p)(a_{\mathbf{k}j}-a^\dagger_{\mathbf{k}j})(a_{\mathbf{k}'j'}-a^\dagger_{\mathbf{k}'j'}) \times (a_{\mathbf{p}l}-a^\dagger_{\mathbf{p}l})(a_{\mathbf{p}'l'}-a^\dagger_{\mathbf{p}'l'})F_A(\mathbf{k}j,\mathbf{k}'j',d)F_A(\mathbf{p}l,\mathbf{p}'l',d). \tag{10}$$

Using this operator, however, we find that the average squared value of the force has an ultraviolet divergence that cannot be regularized by a cutoff function. An analogous problem was encountered by Barton in his works on force fluctuations for macroscopic bodies [16–18]. In order to solve this problem, he proposed to consider explicitly that every real measurement must involve a finite time T and thus considered a temporal average of the force. The basic idea is to integrate on time the quantum average value with a response function $f(t)$ describing the measurement process. Then, F_A being the force operator in the Schrödinger representation and H the Hamiltonian of the system, the time-averaged force with an instrument characterized by a normalized response function $f(t)$ is given by

$$\overline{F_A}(d) = \int_{-\infty}^{+\infty} dt f(t)\langle 0|F(t)|0\rangle = \int_{-\infty}^{+\infty} dt f(t)\langle 0|e^{(i/\hbar)Ht}Fe^{-(i/\hbar)Ht}|0\rangle. \tag{11}$$

Expression (11) can be thought of as the quantum average on the state $|0\rangle$ of the *time-averaged operator*

$$\overline{F_A} = \int_{-\infty}^{+\infty} dt f(t) e^{(i/\hbar)Ht}F_A e^{-(i/\hbar)Ht}, \tag{12}$$

which is a time-independent operator whose definition depends on the properties of the instrument used for the measurement.

The choice of using the operator $\overline{F_A}$ does not change our results for the average force, whereas it introduces, as we will show in the next section, a natural frequency cutoff in the average squared value of the force, such as $e^{-\omega T}$. This is indeed reasonable since an instrument with an integration time T does not resolve processes with frequencies larger than T^{-1}.

It is worth saying a few words on the idealized assumption of a perfectly conducting wall and considering if this assumption could be somehow related to the ultraviolet divergence of the force fluctuation. In order to address this point, we can compare the average of the squared force (10) with the analogous expression that is obtained in free space—i.e., in the absence of a wall. For an atom in free space the average force acting on it is zero, of course, but there are force fluctuations around the zero value, due to vacuum field fluctuations. Also in this case the force fluctuation is ultraviolet divergent, and it is given by an expression similar to Eq. (10), with the free-space field modes. Thus it seems evident that the divergence of the force fluctuation cannot be directly related to the assumption of a perfect mirror, because similar divergent fluctuations occur also in the absence of a wall. These divergences seem rather related to the well-known ultraviolet divergences of vacuum field fluctuations. However, it is likely that when the average value of Eq. (10) is evaluated, the actual result may depend on the properties of the mirror at high frequencies, in particular on its conductivity. Thus it can be relevant to consider the role of the finite conductivity of a real mirror, quite apart from the problem of divergences. This point will be discussed in the next section, in particular by comparing the finite duration of the force measurement with the inverse of the mirror's plasma frequency.

III. FLUCTUATION OF THE CASIMIR-POLDER FORCE BETWEEN AN ATOM AND A WALL

We now use Eq. (12) to define the time-averaged operator associated to the square of the force, which is

$$(\overline{F_A})^2 = \int_{-\infty}^{+\infty} dt f(t) \int_{-\infty}^{+\infty} dt' f(t') e^{(i/\hbar)Ht} F_A e^{-(i/\hbar)H(t-t')} \times F_A e^{-(i/\hbar)Ht'} . \qquad (13)$$

In this equation, H is the total Hamiltonian of the system—i.e., $H=H_F+H_A+W$, where H_F and H_A are, respectively, the Hamiltonian of the free electromagnetic field and of the atom, and W is the interaction term introduced in the previous section. We have obtained, for the mean force, a result correct to the first order in the polarizability α of the atom. As a consequence, a coherent result for the average value of the square of the force should contain the second power of α. Since F_A is an operator of order α, it is clear from Eq. (13) that we must retain only H_F+H_A instead of H in the exponentials, in order to have a mean quadratic value of $\overline{F_A}$ proportional to α^2. Besides, as the state $|0\rangle$ does not contain atomic variables, it is sufficient to put $H=H_F$ in Eq. (13).

Thus, taking the response function $f(t)$ as a Lorentzian of width T given by

$$f(t) = \frac{1}{\pi}\frac{T}{t^2+T^2}, \qquad (14)$$

we obtain the following expression for the fluctuation $\Delta\overline{F_A} = (\langle F_A^2\rangle - \langle F_A\rangle^2)^{1/2}$ of the Casimir-Polder force on the atom:

$$(\Delta\overline{F_A})^2 = \frac{1}{2}\sum_{\mathbf{k}\mathbf{k}'jj'} |\langle 0|F_A|1_{\mathbf{k}j}1_{\mathbf{k}'j'}\rangle|^2 |g(\omega+\omega')|^2, \qquad (15)$$

where

$$g(\omega) = \int_{-\infty}^{+\infty} dt f(t) e^{-i\omega t} = e^{-|\omega|T} \qquad (16)$$

is the Fourier transform of the response function $f(t)$. Expression (15) is valid both in the near and far zone. From this expression we obtain the final result for the force fluctuation, approximated to the far zone,

$$\Delta\overline{F_A} = \frac{\hbar c\alpha}{4\pi}\frac{1}{c^5T^5\left[1+\left(\frac{cT}{d}\right)^2\right]^4}\Bigg[5+40\left(\frac{cT}{d}\right)^2 + 145\left(\frac{cT}{d}\right)^4 + 317\left(\frac{cT}{d}\right)^6 + 400\left(\frac{cT}{d}\right)^8 + 285\left(\frac{cT}{d}\right)^{10} + 10\left(\frac{cT}{d}\right)^{12} + 86\left(\frac{cT}{d}\right)^{14}\Bigg]^{1/2}. \qquad (17)$$

It is evident from Eq. (15) that the use of the time-averaged operator introduces a frequency cutoff $e^{-c(k+k')T}$, where T is the duration of the measurement, which makes finite the force fluctuation.

When the high-frequency finite conductivity of a real mirror is taken into account, the mirror becomes transparent for frequencies larger than its plasma frequency:

$$\omega_P = \sqrt{\frac{4\pi Ne^2}{m}}, \qquad (18)$$

N being the density of free electrons, and e and m the electron charge and mass, respectively. The importance of this on the atom-wall force fluctuation depends on the ratio between T^{-1} (the cutoff due to the finite measurement time) and ω_P. If the plasma frequency is larger than T^{-1}, the finite conductivity of the mirror should not give relevant effects; on the contrary, it can yield relevant effects if $\omega_P \ll T^{-1}$. A typical value of the plasma frequency is $\sim 10^{16}$ Hz, and thus we can neglect finite-conductivity corrections for force measurements lasting more than 10^{-16} s. In the cases we shall consider in the next section, in particular when parameters from the experiment in Ref. [8] are considered, measurement times are much larger than this value and thus the corrections due to the finite conductivity of the mirror can be neglected.

We can easily study the behavior of the relative fluctuation, which is the standard deviation of the force divided by the absolute value of the average force, in two different limiting cases. When $d \ll cT$ we get

$$\frac{\Delta \overline{F_A}}{|\langle 0|F_A|0\rangle|} = \frac{1}{3}\sqrt{\frac{43}{2}}\left(\frac{d}{cT}\right)^6, \tag{19}$$

whereas in the case $d \gg cT$ we have

$$\frac{\Delta \overline{F_A}}{|\langle 0|F_A|0\rangle|} = \frac{\sqrt{5}}{6}\left(\frac{d}{cT}\right)^5. \tag{20}$$

In the first case, the force fluctuation seems to be negligible compared to the average force, while in the second case it would result much larger than the average force, thus experimentally observable. Similar conclusions concerning the experimental observability of the fluctuations of the atom-wall Casimir-Polder force have been obtained in Ref. [22] (only in the far zone), using a quite different approach based on the Langevin equation for the Brownian motion of a test particle due to field fluctuations in the presence of the wall. They also find that force fluctuations are negligible in experiments that measure the force averaged on a large time (compared to d/c), while they can be significant for shorter time scales.

When the atom-wall distance is of the order of $d \sim 1\ \mu$m (typical distance in actual experimental setups [5]) the time scale which separates the two regimes is $T \sim 10^{-14}$ s, which is a very short time scale but probably no longer impossible nowadays. According to our previous discussion on corrections due the properties of a real mirror at frequencies larger than its plasma frequency ω_P, the high-frequency finite conductivity of a typical mirror should not play a significant role even for such a short time scale (which is larger than the inverse of a typical plasma frequency). Hence, to evaluate the experimental observability of the fluctuations we need a reasonable value of the measurement time T. In order to compare our theoretical predictions for the force fluctuations with actual precision measurements of the atom-wall Casimir-Polder force in the far zone, we have extended our calculations to the system of one atom between two parallel metallic walls. In fact, for this system well-established precision measuements exist [8].

IV. FLUCTUATIONS OF THE CASIMIR-POLDER FORCE ON AN ATOM BETWEEN TWO CONDUCTING WALLS

In order to take into account the presence of the two parallel walls separated by a distance L, we make use of the mode functions associated to a conducting parallelepiped cavity defined by

$$-\frac{L_1}{2} < x < \frac{L_1}{2}, \quad -\frac{L_1}{2} < y < \frac{L_1}{2}, \quad -\frac{L}{2} < z < \frac{L}{2}, \tag{21}$$

which are easily found to be

$$f_x(\mathbf{k}j,\mathbf{r}) = \sqrt{8}(\mathbf{e}_{\mathbf{k}j})_x \cos\left[k_x\left(x+\frac{L_1}{2}\right)\right] \times \sin\left[k_y\left(y+\frac{L_1}{2}\right)\right]\sin\left[k_z\left(z+\frac{L}{2}\right)\right],$$

$$f_y(\mathbf{k}j,\mathbf{r}) = \sqrt{8}(\mathbf{e}_{\mathbf{k}j})_y \sin\left[k_x\left(x+\frac{L_1}{2}\right)\right] \times \cos\left[k_y\left(y+\frac{L_1}{2}\right)\right]\sin\left[k_z\left(z+\frac{L}{2}\right)\right],$$

$$f_z(\mathbf{k}j,\mathbf{r}) = \sqrt{8}(\mathbf{e}_{\mathbf{k}j})_z \sin\left[k_x\left(x+\frac{L_1}{2}\right)\right] \times \sin\left[k_y\left(y+\frac{L_1}{2}\right)\right]\cos\left[k_z\left(z+\frac{L}{2}\right)\right]. \tag{22}$$

In the limit $L_1 \to +\infty$ we obtain two infinite conducting walls located in $z = \pm L/2$. Following the same steps of Secs. II and III we obtain the following expression for the average force on the atom:

$$F_A(d) = -\frac{\pi^4\hbar c\alpha}{8L^5}\frac{\sin\left(\frac{3\pi d}{L}\right) - 11\sin\left(\frac{\pi d}{L}\right)}{\cos^5\left(\frac{\pi d}{L}\right)}, \tag{23}$$

where L is the distance between the two walls and $-L/2 < d < L/2$ is the distance of the atom from the plane in the middle of the plates. This force vanishes for $d=0$ for symmetry reasons. This result coincides with a result already obtained by Barton [31]. We have then calculated, using the time-averaged operator method described in Sec. II, the value of the relative fluctuation of the force. We find that also in this case the relative fluctuation depends on the measurement time. Since the experiment in [8] consists in the passage of a beam of atoms between the two walls, an estimate of the integration time T can be obtained from the length of the cavity (8 mm in the mentioned experiment) and the average speed of the atoms. This average velocity can be easily obtained from the Maxwell-Boltzmann distribution of the atoms, and thus we get $T \sim 10^{-5}$ s. In this case the expression of the relative fluctuation can be simplified, yielding

$$\frac{\Delta F_A}{|\langle 0|F_A|0\rangle|} \simeq \frac{e^{-\pi cT/L}}{\left(\frac{2\pi cT}{L}\right)^{5/2}} \times \frac{\cos^6[10^6\pi d\ (\mathrm{m}^{-1})]}{|\sin[3\times 10^6\pi d\ (\mathrm{m}^{-1})] - 11\sin[10^6\pi d\ (\mathrm{m}^{-1})]|}, \tag{24}$$

where we have used the fact that $L=1\ \mu$m and units for the distance d have been excplicitly specified. This function diverges for $d \to 0$ (since the average force vanishes for $d=0$), but is already negligible for $d \sim 10^{-10}$ m—that is, at a distance of the order of the Bohr radius. Consequently, we can conclude that in this experimental setup the fluctuation of the force is so small to be hardly observable. This does not exclude observability of the fluctuation of the Casimir-Polder force in future experimental setups characterized by shorter measurement times, of course.

V. CONCLUSIONS

In this paper we have considered the fluctuation of the Casimir-Polder force experienced by a neutral atom in front of an uncharged conducting wall or between two parallel uncharged walls. We have first introduced a quantum operator directly associated with the force on the atom, considered as a microscopic polarizable body, due to the electromagnetic field. This operator has been obtained by taking minus the derivative of the operator corresponding to the atom-field effective interaction energy with respect to the coordinate of the atom normal to the plate(s). This operator has been used to calculate the mean force in both configurations. As for the quadratic mean value, in order to go beyond the nonregularizable ultraviolet divergences encountered, we have used the method of time-averaged operators, previously used by Barton for the Casimir force fluctuation between macroscopic bodies. We have obtained the relative fluctuation both in the cases of one and two walls. In the case of one wall, the value of the relative force fluctuation strongly depends on the ratio between the atom-wall distance d and the distance cT traveled by the light during the measurement time T. Fluctuations are larger the smaller is the duration of the force measurement. In the case of two walls, we have been also able to estimate the experimental observability of this fluctuation in a recent precision experiment on the atom-wall Casimir-Polder force in the far zone [8], concluding that in this experiment the fluctuations are very small and hardly observable. Our results show that force fluctuations should, however, be observable in experiments in which the force is measured in much shorter time scales. Future extensions of this work involve the calculation of the Casimir-Polder force between two atoms (retarded van der Waals force), where one may expect that the relative fluctuation of the force could be significantly larger because only microscopic objects are involved.

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