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Photon-blockade-induced Mott transitions and *XY* spin models in coupled cavity arrays

Dimitris G. Angelakis,[1,*] Marcelo Franca Santos,[2] and Sougato Bose[3]
[1]*Centre for Quantum Computation, Department of Applied Mathematics and Theoretical Physics, University of Cambridge, Wilberforce Road,* CB3 0WA, *United Kingdom*
[2]*Departamento de Física, Universidade Federal de Minas Gerais, Belo Horizonte, 30161-970, Minas Gerais, Brazil*
[3]*Department of Physics and Astronomy, University College London, Gower Street, London,* WC1E 6BT, *United Kingdom*
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We propose a physical system where photons could exhibit strongly correlated effects. We demonstrate how a Mott-insulator phase of atom-photon excitations (polaritons) can arise in an array of individually addressable coupled electromagnetic cavities when each of these cavities is coupled resonantly to a single two-level system (atom, quantum dot, or Cooper pair). This Mott phase is characterized by the same integral number of net polaritonic excitations with photon blockade providing the required repulsion between the excitations in each site. Detuning the atomic and photonic frequencies suppresses this effect and induces a transition to a photonic superfluid. Finally, on resonance the system can straightforwardly simulate the dynamics of many-body spin systems.

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Strongly coupled many-body systems described by the Bose-Hubbard models [1] exhibit Mott-insulating phases whose realization in optical lattices [2] has opened varied possibilities for the simulation of many-body physics [3]. Can there be another *engineered* quantum many-body system which displays such phases? This will be especially interesting if the strengths of this system are "complementary" to that of optical lattices—for example, if it allowed the coexistence of accessibility to individual constituents of a many-body system and a strong interaction between them, or if it allowed the simulation of arbitrary networks rather than those derivable from superposing lattices. It will be particularly arresting to find such phases by modifying a system of photons which, by being noninteracting, are unlikely candidates for the studies of many-body phenomena. Here we propose such a system consisting of coupled electromagnetic cavities [4–7], doped with *single* two-level systems [8–15]. Using the nonlinearity generated from the corresponding photon blockade effect [16], we show the possibility of observing an insulator phase of total (atomic+photonic or simply *polaritonic*) excitations and its transition to a superfluid (SF) of photons. Compared to the optical lattice case, the different Hubbard-like model that describes the system involves neither purely bosonic nor purely fermionic entities; the transition from insulator to SF is also accompanied by a transition of the excitations from polaritonic to photonic; and excitations, rather than physical particles such as atoms, are involved. In addition, the possibility of simulating the dynamics of an *XY* spin chain with individual spin manipulation, using a mechanism different from that used in optical lattices [3], has been suggested.

Assume a chain of N coupled cavities. A realization of this has been studied in structures known as coupled resonator optical waveguides or couple-cavity waveguides (CCWs) in photonic crystals [6,17], in tapered fiber-coupled toroidal microcavities [10], and in coupled superconducting microwave resonators [8,13]. This has already stimulated proposals to use such systems to implement quantum computation [18] using coupled CCWs [19] and in a two-dimensional cluster state setup [20]. The production of entangled photons [21] and the interaction of polaritons [22] have been studied as well. The Hamiltonian corresponds to a series of quantum harmonic oscillators coupled through hopping photons and is given by $H=\sum_{k=1}^{N}\omega_d a_k^\dagger a_k+\sum_{k=1}^{N}A(a_k^\dagger a_{k+1}+\text{H.c.})$, where $a_k^\dagger$ (a_k) are the localized eigenmodes (Wannier functions). The photon frequency and hopping rate are ω_d and A, respectively, and no nonlinearity is present yet. Assume now that the cavities are doped with two-level systems (atoms, quantum dots, or superconducting qubits), and $|g\rangle_k$ and $|e\rangle_k$ are their ground and excited states at site k. The Hamiltonian describing the system is the sum of three terms, H^{free}, the Hamiltonian for the free light and dopant parts, H^{int}, the Hamiltonian describing the internal coupling of the photon and dopant in a specific cavity, and H^{hop}, for the light hopping between cavities:

$$H^{free}=\omega_d\sum_{k=1}^{N}a_k^\dagger a_k+\omega_0\sum_k |e\rangle_k\langle e|_k, \quad (1)$$

$$H^{int}=g\sum_{k=1}^{N}(a_k^\dagger|g\rangle_k\langle e|_k+\text{H.c.}), \quad (2)$$

$$H^{hop}=A\sum_{k=1}^{N}(a_k^\dagger a_{k+1}+\text{H.c.}), \quad (3)$$

where g is the light-atom coupling strength. The $H^{free}+H^{int}$ part of the Hamiltonian can be diagonalized in a basis of mixed photonic and atomic excitations, called polaritons. These polaritons, also known as dressed states, involve a mixture of photonic and atomic excitations and are defined by the operators $P_k^{(\pm,n)}=|g,0\rangle_k\langle n\pm|_k$, where $|n\rangle_k$ denotes the n-photon Fock state in the kth cavity. The polaritons of the kth atom-cavity system denoted by $|n\pm\rangle_k$ are given by $|n+\rangle_k=(\sin\theta_n|g,n\rangle_k+\cos\theta_n|e,n-1\rangle_k)/\sqrt{2}$ and

*dimitris.angelakis@qubit.org

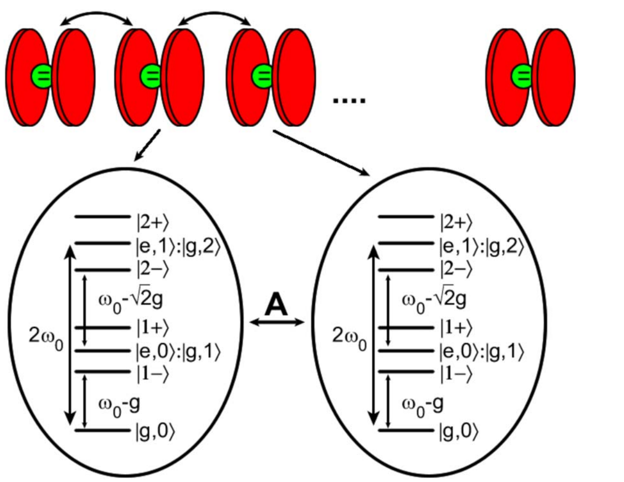

FIG. 1. (Color online) A series of cavities coupled through light and the polaritonic energy levels for two neighboring cavities.

$|n-\rangle_k=(\cos\theta_n|g,n\rangle_k-\sin\theta_n|e,n-1\rangle_k)/\sqrt{2}$ with energies $E_n^{\pm}=n\omega_d\pm g\sqrt{n+\Delta^2/g^2}$, $\tan 2\theta_n=-g\sqrt{n}/\Delta$, and atom-light detuning $\Delta=\omega_0-\omega_d$. They are also eigenstates of the sum of the photonic and atomic excitation operator $\mathcal{N}_k=a_k^{\dagger}a_k+|e\rangle\langle e|_k$ with eigenvalue n (Fig. 1).

We will now justify the statement that the lowest-energy state of the system consistent with a given number of net excitations per site (or filling factor) becomes a Mott state of the net (polaritonic) excitations for integer values of the filling factor. To understand this, we rewrite the Hamiltonian (for $\Delta=0$) in terms of the polaritonic operators as

$$H=\sum_{k=1}^{N}\left(\sum_{n=1}^{\infty}n(\omega_d-g)P_k^{(-,n)\dagger}P_k^{(-,n)}+\sum_{n=1}^{\infty}n(\omega_d+g)P_k^{(+,n)\dagger}P_k^{(+,n)}+\sum_{n=1}^{\infty}g(n-\sqrt{n})P_k^{(-,n)\dagger}P_k^{(-,n)}+\sum_{n=1}^{\infty}g(\sqrt{n}-n)P_k^{(+,n)\dagger}P_k^{(+,n)}\right)+A\sum_{k=1}^{N}(a_k^{\dagger}a_{k+1}+\text{H.c.}). \quad (4)$$

The above implies (assuming the regime $An\ll g\sqrt{n}\ll\omega_d$) that the lowest-energy state for a given number, say η, of net excitations at the kth site would be the state $|\eta-\rangle_k$ (this is because $|\eta+\rangle$ has a higher energy, but the same net excitation η). Thus one need consider only the first, third, and last terms of the above Hamiltonian H to determine the lowest-energy states. The first term corresponds to a linear spectrum, equivalent to that of a harmonic oscillator of frequency ω_d-g. If only that part were present in the Hamiltonian, then it would not cost any extra energy to add an excitation (of frequency ω_d-g) to a site already filled with one or more excitations, as opposed to an empty site. However, the term $g(n-\sqrt{n})P_k^{(-,n)\dagger}P_k^{(-,n)}$ raises the energy of an uneven excitation distribution such as $|(n+1)-\rangle_k|(n-1)-\rangle_l$ among any two sites k and l relative to the uniform excitation distribution $|n-\rangle_k|n-\rangle_l$ among these sites. Thus the third term of the above Hamiltonian can be regarded as an effective, nonlinear "on-site" photonic repulsion, and leads to the Mott state of the net excitations per site being the ground state for commensurate filling. Reducing the strength of the effective nonlinearity, i.e., the blockade effect, through detuning for example, should drive the system to the SF regime. This could be done by Stark-shifting the atomic transitions from the cavity by an external field. The new detuned polaritons are not as well separated as before, and their energies are merely shifts of the bare atomic and photonic ones by $\pm g^2n/\Delta$, respectively. In this case it costs no extra energy to add excitations (excite transitions to higher polaritons) in a single site, and the system moves to the SF regime. Note here that the mixed nature of the polaritons could in principle allow for mostly photonic excitations and a photonic Mott state. The required values of g and Δ for the corresponding nonlinearity, though, seem to be unrealistic within current technology [8–15].

To quantify the transition of the system from a Mott phase to a SF phase as the detuning $\Delta=\omega_0-\omega_d$ is increased, we have performed a numerical simulation using 3–7 sites with the Hamiltonian of Eqs. (1)–(3) (numerical diagonalization of the complete Hamiltonian without any approximations).[1] In the Mott phase the particle number per site is fixed and its variance is zero (every site is in a Fock state). In such a phase, the expectation value of the destruction operator for the relevant particles, the order parameter, is zero. In the traditional mean-field (and thus necessarily approximate) picture, this expectation value becomes finite on transition to a SF, as a coherent superposition of different particle numbers is allowed to exist per site. However, our entire system is a "closed" system and there is no particle exchange with the outside. SF states are characterized by a fixed "total" number of particles in the finite-site system and the expectation of a destruction operator at any given site is zero even in the SF phase. Thus this expectation value cannot be used as an order parameter for a quantum phase transition. Instead, we use the variance of the total number of excitations per site, the operator $\mathcal{N}_k$, at a given site (we choose the middle cavity, but any of the other cavities would do) to characterize the Mott-to-SF phase transition. This variance, var($\mathcal{N}_k$), has been plotted in Fig. 2 as a function of $\log_{10}\Delta$ for a filling factor of one net excitation per site. For this plot, we have taken the parameter ratio $g/A=10^2$ ($g/A=10^1$ gives very similar results), with Δ varying from $\sim10^{-3}g$ to $\sim g$ and $\omega_d,\omega_0\sim10^4g$. We have plotted both ideal graphs (if neither the atoms nor the cavity fields underwent any decay or decoherence) and also performed simulations explicitly using decay of the atomic states and photonic states in the range of $g/\max(\kappa,\gamma)\sim10^3$, where κ and γ are the cavity and atomic decay rates.

These decay rates are expected to be feasible soon in toroidal microcavity systems with atoms [10] and arrays of coupled stripline microwave resonators, each interacting

[1]We use finite number of sites for the simulation as our system compared to the optical lattice case is computationally more exhaustive. In addition to the bosonic occupation numbers per site, there is also the extra atomic degree of freedom at each site that cannot be eliminated [2,25].

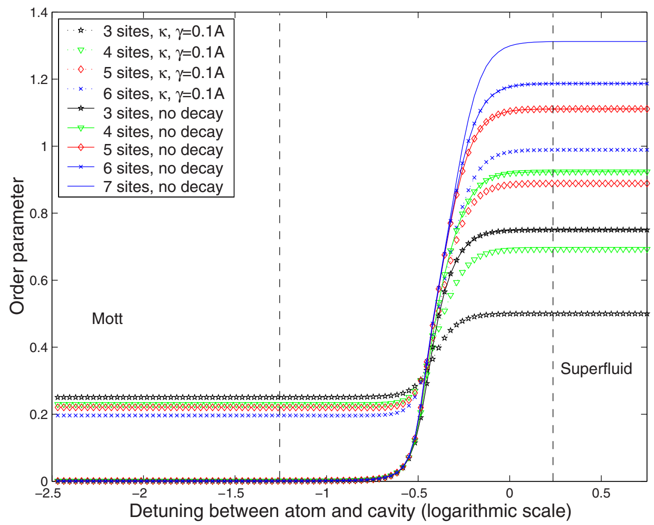

FIG. 2. (Color online) Order parameter as a function of the detuning between the hopping photon and the doped two-level system (in logarithimic units of the matter-light coupling g). Simulations include results for 3–7 sites, with and without dissipation due to spontaneous emission and cavity leakage. Close to resonance ($0 \leq \Delta/g \leq 10^{-1}$), where the photon-blockade-induced nonlinearity is maximum (and much larger than the hopping rate), the system is forced into a polaritonic Fock state with the same integral number of excitations per site (order parameter zero-Mott-insulator state). Detuning the system by applying external fields and inducing Stark shifts ($\Delta \geq g$) weakens the blockade and leads to the appearance of different coherent superpositions of excitations per site (a photonic SF). The increase in the number of sites leads to a sharper transition, as expected.

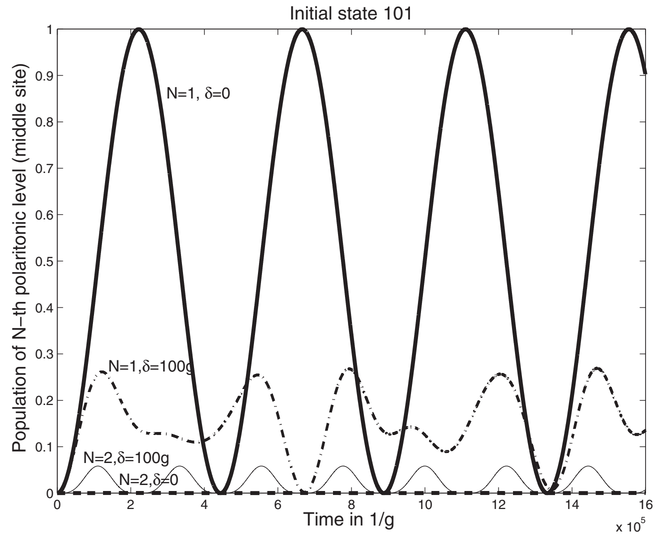

FIG. 3. Probability of exciting the polaritons corresponding to one (thick solid line) or two (thick dashed line) excitations for the middle cavity, taken initially to be empty (in the resonant regime). In this case, as we started with fewer polaritons than the number of cavities, oscillations occur for the single-excitation polariton. The double one is never occupied due to the blockade effect in the Mott-insulator phase (thick dashed line). For the detuned case, however, hopping of more than one polariton is allowed (SF regime), which is evident as the higher polaritonic manifolds are being populated now (dashed and thin solid lines).

with a superconducting qubit [13]. For these simulations, we have assumed that the experiment (of going from the Mott state to the SF state and back) takes place in a time scale of $1/A$ so that the evolution of one ground state to the other and back is adiabatic. The simulations of the state with decay have been done using quantum jumps, and it is seen that there is still a large difference of var($\mathcal{N}_k$) between the Mott and SF phases despite the decays. As expected, the effect of dissipation reduces the final value of the order parameter in the SF regime (population has been lost through decay), whereas in the Mott regime it leads to the introduction of fluctations, again due to population loss from the $|1-\rangle$ state. The Mott-[var($\mathcal{N}_k$)=0] to-SF [var($\mathcal{N}_k$)>0] transition takes place over a finite variation of Δ (because of the finiteness of our lattice) around $10g$, and as expected becomes sharper as the number of sites is increased.

In an experiment, one would start in the resonant (Mott) regime with all atom-cavity systems initially in their absolute ground ($|g,0\rangle^{\otimes k}$) states and prepare the atom-cavity systems in the joint state $|1-\rangle^{\otimes k}$ by applying a global external laser tuned to this transition. This is the Mott state with the total (atomic+photonic) excitation operator $\mathcal{N}_k$ having the value unity at each site. One would then Stark-shift and detune (globally again) the atomic transitions from the cavity by an external field, and observe the probability of finding $|1-\rangle$ and the predicted decrease of this probability (equivalent to the increase in our order parameter, the variance of $\mathcal{N}_k$) as the detuning is increased. To infer the fluctuations in $\mathcal{N}_k$ of our system, it basically suffices to check the population of the $|1-\rangle$ state as this is the only way a single excitation can be present in the kth site. For this, a laser is applied which is of the right frequency to accomplish a cycling transition between $|1-\rangle$ and another level. Through monitoring its fluorescence, accurate state measurements can be made [24].

We have also calculated the probabilities of populating the lowest polaritonic state of the first and second manifolds $|1-\rangle_k$, $|2-\rangle_k$ of a middle cavity out of an array of three cavities. The initial state is the polaritonic state $|1-\rangle_k$ excited at the right and the left cavities, with the middle cavity being in the ground state $|g,0\rangle_k$. Figure 3 shows, as expected from our discussion above, that on resonance the photon blockade prevents any excitation to any state higher than the first (Mott-insulator phase). However, by simply varying the atomic frequency and inducing some detuning (of the order of $100g$ in our simulation), the weakening of the blockade effect results in the probability of exciting to the second manifold becoming increasingly strong. This means that the polaritonic excitations (or better, the photonlike polaritonic particles now, as we are in the dispersive regime) can hop together in bunches of two or more from cavity to cavity (SF regime).

We will now show that in the Mott regime the system simulates an XY spin model with the presence and absence of polaritons corresponding to spin up and down. Let us assume that we initially populate the lattice only with polaritons of energy $\omega_0 - g$, in the limit $\omega_d \approx \omega_0$, Eq. (4) becomes

$$H_k^{free} = \omega_d \sum_{k=1}^{N} P_k^{(+)\dagger} P_k^{(+)} + P_k^{(-)\dagger} P_k^{(-)}, \tag{5}$$

$$H_k^{int} = g\sum_{k=1}^{N} P_k^{(+)\dagger}P_k^{(+)} - P_k^{(-)\dagger}P_k^{(-)}, \quad (6)$$

$$H_k^{hop.} = A\sum_{k=1}^{N} P_k^{(+)\dagger}P_{k+1}^{(+)} + P_k^{(-)\dagger}P_{k+1}^{(+)} + P_k^{(+)\dagger}P_{k+1}^{(-)} + P_k^{(-)\dagger}P_{k+1}^{(-)} + \text{H.c.}, \quad (7)$$

where $P_k^{(\pm)\dagger}=P_k^{(\pm,1)\dagger}$ is the polaritonic operator creating excitations to the first polaritonic manifold (Fig. 1). In the rotating-wave approximation, Eq. (7) reads (in the interaction picture) $H_I=A\Sigma_{k=1}^{N}P_k^{(-)\dagger}P_{k+1}^{(-)}+\text{H.c.}$ In deriving the above, the logic has two steps. First, note that terms of the type $P_k^{(-)\dagger}P_{k+1}^{(+)}$, which interconvert between polaritons, are fast rotating and they vanish. Second, if we create only the polaritons $P_k^{(-)\dagger}$ in the lattice initially with energy ω_0-g, then the polaritons corresponding to $P_k^{(+)\dagger}$ will never even be created, as the interconverting terms vanish. Thus the term $P_k^{(+)\dagger}P_k^{(+)}$ can also be omitted. Note that, because double occupancy of the sites is prohibited, one can identify $P_k^{(-)\dagger}$ with $\sigma_k^{+}=\sigma_k^{x}+i\sigma_k^{y}$, where σ_k^{x} and σ_k^{y} are standard Pauli operators. Then the Hamiltonian becomes $H_I=\sigma_k^{x}\sigma_{k+1}^{x}+\sigma_k^{y}\sigma_{k+1}^{y}$, which is the standard *XY* model of interacting spins with spin up (down) corresponding to the presence (absence) of a polariton. Note that, although this is different from optical lattice realizations of spin models, where, instead, the internal levels of a two-level atom are used for the two qubit states [3], the measurement could be done using very similar atomic state measurement techniques (utilizing the advantage of larger distances between sites here). Some simple applications of *XY* spin, chains in quantum-information processing, such as quantum-information transfer [23,26], can thus be readily implemented in our system. Very recently, a novel idea for efficient cluster state quantum computation was proposed in this system, where the database search and factoring quantum algorithms could be implemented using just two rows of cavities [20].

In conclusion, we showed that a range of many-body system effects, such as Mott transitions for polaritonic particles obeying mixed statistics, could be observed in optical systems of individual addressable coupled cavity arrays interacting with two-level systems. We also discussed possible implementations using photonic crystals, toroidal microcavities, and superconducting systems. Finally, we showed the capability and advantages of simulating *XY* spin models using our scheme and noted the ability of these arrays to simulate arbitrary quantum networks.

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