Unconventional geometric quantum computation in a two-mode cavity

Chunfeng Wu,[1,2] Zisheng Wang,[1] Xun-Li Feng,[1] Hsi-Sheng Goan,[3] L. C. Kwek,[1,4] C. H. Lai,[1] and C. H. Oh[1,*]
[1]*Department of Physics, National University of Singapore, 2 Science Drive 3, Singapore 117542*
[2]*School of Physics, Northeast Normal University, Changchun 130024, People's Republic of China*
[3]*Department of Physics and Center for Theoretical Sciences, National Taiwan University and National Center for Theoretical Sciences, Taipei 10617, Taiwan*
[4]*Nanyang Technological University, National Institute of Education, 1 Nanyang Walk, Singapore 637616*
(Received 15 May 2007; published 9 August 2007)

We propose a scheme for implementing unconventional geometric quantum computation by using the interaction of two atoms with a two-mode cavity field. The evolution of the system results in a nontrivial two-qubit phase gate. The operation of the proposed gate involves only metastable states of the atom and hence is not affected by spontaneous emission. The effect of cavity decay on the gate is investigated. It is shown that the evolution time of the gate in the two-mode case is less than that in the single-mode case proposed by Feng *et al.* [Phys. Rev. A **75**, 052312 (2007)]. Thus the gate can be more decay tolerant than the previous one. The scheme can also be generalized to a system consisting of two atoms interacting with an *N*-mode cavity field.

DOI: 10.1103/PhysRevA.76.024302 PACS number(s): 03.67.Lx, 03.65.Vf, 03.67.Pp

Decoherence plays a bothersome role in the realization of quantum gates for a quantum processor, since physical qubits inevitably interact with external systems. Thus, it is essential to consider fault-tolerant systems. One possible way to achieve fault-tolerant quantum gates is through the geometric phase. Geometric phases depend only on the solid angle enclosed by the parameter path [1] and generally not on the dynamics of the evolution. Thus geometric phases can be rendered very robust for computation. However, in practice, geometric phases are often accompanied by dynamic phases. These dynamic phases may destroy the potential robustness of the computation scheme. Hence, an important consideration of geometric quantum computation is the removal or avoidance of the dynamic phase.

One simple method to remove dynamic phases involves the choice of dark states as the qubit space. In this scheme, the dynamic phase is always zero [2]. In another method, one removes the dynamic phase by canceling the dynamic phases accumulated in different loops [3–5]. We refer to the latter method for canceling the dynamic phase as conventional geometric quantum computation.

Recently, there is another kind of scheme [6] in which one ensures that the dynamic phase γ^d is proportional to the geometric phase γ^g as $\gamma^d=\eta\gamma^g$ with $\eta\neq 0,-1$. Thus, the total phase is proportional to the geometric phase. This realization is sometimes called unconventional geometric quantum computation (UQC). To date, UQC has been proposed to be realized in many systems, such as cavity QED [6–9] and trapped ions [10].

The existing cavity QED schemes [8,9] involve only a single cavity mode; to the best of our knowledge, an UQC scheme utilizing two or more cavity modes has never been proposed. In this paper, we present a two-mode UQC scheme. In our scheme the atomic states act as the quantum computational basis, and the phases acquired by the atomic states to realize a certain phase gate are due to the cyclic evolution of the cavity modes. In comparison with the single-cavity-mode scheme [9], the scheme proposed in this paper is more efficient and more tolerant against decoherence. This is because there are two cavity modes contributing to the phase accumulation and the time consumed for realizing a certain phase gate is thus doubly reduced. We also show that the scheme can be generalized to a system consisting of two atoms interacting with an *N*-mode cavity field.

This paper is organized as follows. In Sec. I, we briefly describe the model and derive the effective interaction Hamiltonian. In Sec. II, we show how a two-qubit UQC gate can be realized in the case of cavity decay. We also show that the evolution time of the gate in the two-mode case is less than that of the gate in the one-mode case [9]. The fidelity of the gate is investigated. The generalization of the scheme to an *N*-mode cavity is also discussed. In Sec. III, we end with some conclusions.

We consider two three-level atoms inside a two-mode cavity with high quality value. The atomic energy levels and the interactions between atom and cavity modes (driving laser fields) are shown in Fig. 1. For each atom, there are three different states: $|e\rangle_i$, $|g\rangle_i$, and $|c\rangle_i$, with $i=1,2$, where $|e\rangle_i$, $|g\rangle_i$ are metastable states and $|c\rangle_i$ is an excited state. The dipole-

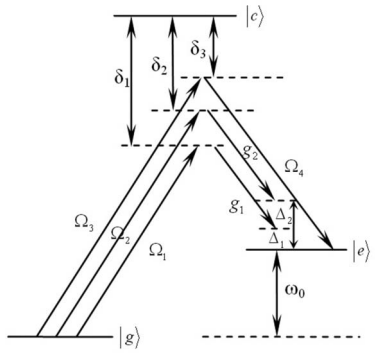

FIG. 1. Each atom in the Λ configuration with levels $|g\rangle$, $|e\rangle$, and $|c\rangle$ interacts with the excitation fields. Here δ_1, δ_2, δ_3, Δ_1, and Δ_2 are frequency detunings, and g_1, g_2, Ω_1, Ω_2, Ω_3, and Ω_4 are the respective coupling strengths.

*On leave at National Taiwan University, Taiwan. phyohch@nus.edu.sg

allowed transitions are $|c\rangle_i \leftrightarrow |g\rangle_i$ and $|c\rangle_i \leftrightarrow |e\rangle_i$ via three Raman channels due to the laser fields and the cavity modes. Two of the channels contain classical fields Ω_m with frequencies ω_m and two quantized fields with frequencies ω_{f_m}, where $m=1,2$. The last channel is excited by two classical external fields Ω_3 and Ω_4 with frequencies ω_3 and ω_4, respectively. This channel is assumed to satisfy the Raman resonance $\omega_3 - \omega_4 = \omega_0$, where ω_0 is the energy difference between levels $|e\rangle$ and $|g\rangle$, while the other two channels have small detunings Δ_1 and Δ_2 from the Raman resonance. The Hamiltonian of the system can be written as

$$H = \hbar\omega_g \sum_{j=1}^{2} |g\rangle_{jj}\langle g| + \hbar\omega_e \sum_{j=1}^{2} |e\rangle_{jj}\langle e| + \hbar\omega_c \sum_{j=1}^{2} |c\rangle_{jj}\langle c| + \hbar \sum_{m=1}^{2} \omega_{f_m} a_m^\dagger a_m + \left(\hbar \sum_{m=1}^{2} g_m^* a_m^\dagger \sum_{j=1}^{2} |e\rangle_{jj}\langle c| + \text{H.c.}\right) + \left(\hbar \sum_{m=1}^{3} \Omega_m^* e^{-i\omega_m t} \sum_{j=1}^{2} |c\rangle_{jj}\langle g| + \text{H.c.}\right) + \left(\hbar\Omega_4 e^{-i\omega_4 t} \sum_{j=1}^{2} |c\rangle_{jj}\langle e| + \text{H.c.}\right). \quad (1)$$

Here, a_m $(m=1,2)$ is the annihilation operator for each cavity mode, ω_i $(i \in e,c,g)$ is the energy of the atomic level i, g_m $(m=1,2)$ is the coupling constant of the cavity mode with the atom, and $\Omega_{1,2,3,4}$ are the coupling strengths of the atom with the laser fields.

To eliminate spontaneous emission, it is required that

$$\left\{\frac{g_1}{\delta_1}, \frac{g_2}{\delta_2}, \frac{\Omega_1}{\delta_1}, \frac{\Omega_2}{\delta_2}, \frac{\Omega_3}{\delta_3}, \frac{\Omega_4}{\delta_3}\right\} \ll 1. \quad (2)$$

The detunings δ_l $(l=1,2,3)$ are assumed to be sufficiently large so that the top level $|c\rangle$ can be removed adiabatically. The approximation leads to an effective atomic system with two energy levels $|e\rangle$ and $|g\rangle$. In addition, in order to avoid undesired atomic transitions, we consider the rotating-wave-approximation conditions

$$\delta_1 - \delta_2 \gg \left\{\Delta_1, \Delta_2, \frac{|g_1\Omega_2|}{\delta_2}, \frac{|g_2\Omega_1|}{\delta_2}, \frac{|g_1\Omega_3|}{\delta_3}, \frac{|g_2\Omega_3|}{\delta_3}, \frac{|\Omega_1\Omega_4|}{\delta_3}, \frac{|\Omega_2\Omega_4|}{\delta_3}\right\},$$
$$\delta_2 - \delta_3 \gg \left\{\Delta_1, \Delta_2, \frac{|g_1\Omega_2|}{\delta_2}, \frac{|g_2\Omega_1|}{\delta_2}, \frac{|g_1\Omega_3|}{\delta_3}, \frac{|g_2\Omega_3|}{\delta_3}, \frac{|\Omega_1\Omega_4|}{\delta_3}, \frac{|\Omega_2\Omega_4|}{\delta_3}\right\}.$$

We can obtain an effective Hamiltonian with small detunings $\Delta_{1,2}$,

$$H'(t) = \hbar \sum_{j=1}^{2} \{[h_1(t)a_1 + h_2(t)a_2]\sigma_j^- + [h_1^*(t)a_1^\dagger + h_2^*(t)a_2^\dagger]\sigma_j^+\} + \hbar \sum_{j=1}^{2} [r(t)\sigma_j^+ + r^*(t)\sigma_j^-], \quad (3)$$

where $\sigma^+ = |e\rangle\langle g|$ and $\sigma^- = |g\rangle\langle e|$, and the effective couplings $r(t)$, $h_1(t)$ and $h_2(t)$ are

$$r(t) = -\frac{2\Omega_3^*\Omega_4}{\delta_3},$$
$$h_1(t) = -g_1\Omega_1 e^{i\Delta_1 t}\left(\frac{1}{\delta_1} + \frac{1}{\delta_1 + \Delta_1}\right) = h_1 e^{i\Delta_1 t},$$
$$h_2(t) = -g_2\Omega_2 e^{i\Delta_2 t}\left(\frac{1}{\delta_2} + \frac{1}{\delta_2 + \Delta_2}\right) = h_2 e^{i\Delta_2 t}. \quad (4)$$

For simplicity, we assume $r(t)$ is real. In the strong effective classical driving regime $r \gg |h_1| \,\&\, |h_2|$, Eq. (3) in the interaction picture can be written as

$$H_{\text{eff}} \approx (\hbar/2) \sum_{i=1}^{2} [h_i(t)a_i + h_i^*(t)a_i^\dagger](\sigma_1^x + \sigma_2^x), \quad (5)$$

where $\sigma_j^x = \sigma_j^+ + \sigma_j^-$.

In cavity QED quantum computation, cavity decay due to coupling to the environment is the main source of decoherence. Through the interaction with the environment, the system evolves as a mixed state. The effect of the cavity decay should be considered in our quantum computation scheme. We here use the quantum trajectory method [11] to evaluate the geometric phase in the case of weak cavity decay. Although this method has been shown to be unsuitable for a general situation with stochastic unravelings [12,13], it can still be used to associate a geometric phase to *an individual quantum trajectory* through measurement of the environment [9,13].

We investigate a no-jump trajectory corresponding to the situation that no leaky photons from the cavity are detected. When no photon is detected, the wave function of the system, $|\Psi^0(t)\rangle$, evolves according to an effective non-Hermitian Hamiltonian due to its coupling to the reservoir:

$$H_{\text{non}} = H_{\text{eff}} - i\hbar\kappa_1 a_1^\dagger a_1 - i\hbar\kappa_2 a_2^\dagger a_2, \quad (6)$$

where κ_i $(i=1,2)$ is the cavity decay rate for mode a_i. By considering very weak decay rates, the system evolves according to the non-Hermitian Hamiltonian [14] so that we have the wave function of the system $|\Psi^0(t)\rangle = e^{-\kappa_1 t a_1^\dagger a_1 - \kappa_2 t a_2^\dagger a_2}|\Phi(t)\rangle$, and

$$i\hbar \frac{d}{dt}|\Phi(t)\rangle = \tilde{H}|\Phi(t)\rangle, \quad (7)$$

where $\tilde{H} = (\hbar/2)[\Sigma_{i=1}^2 h_i(t)e^{-\kappa_i t}a_i + h_i^*(t)e^{\kappa_i t}a_i^\dagger](\sigma_1^x + \sigma_2^x)$ and $|\Phi(t)\rangle = \tilde{U}(t)|\Phi(0)\rangle$ with $\tilde{U}(t)$ is the nonunitary evolution op-

erator, $|\Phi(0)\rangle=|0\rangle_1|0\rangle_2|kl\rangle$ is the initial state of the system with the subscripts 1 and 2 representing cavity modes, and we assume that the cavity modes are initially in vacuum states. The state $|kl\rangle$ represents the atomic state with $k,l=+,-$. We choose the eigenstates of σ^x, $|\pm\rangle=(|g\rangle\pm|e\rangle)/\sqrt{2}$, as the computation basis, such that the evolution governed by the Hamiltonian (6) will not give rise to any population changes. In the computational basis $\{|++\rangle,|+-\rangle,|-+\rangle,|--\rangle\}$ (with corresponding eigenvalues λ_{kl}), $\tilde{U}(t)$ is of the form $\tilde{U}(t)=\mathrm{diag}[\tilde{U}_{++}(t),1,1,\tilde{U}_{--}(t)]$ with elements given by

$$\tilde{U}_{kl}(t)=\exp\sum_{i=1}^{2}\left(\alpha_{i,kl}^{(1)}(t)a_i+\alpha_{i,kl}^{(2)}(t)a_i^{\dagger}+\frac{\mu_{i,kl}(t)}{2}\right), \tag{8}$$

where $\mu_{i,kl}(t)=\int_0^t\alpha_{i,kl}^{(2)}(\tau)d\alpha_{i,kl}^{(1)}(\tau)-\int_0^t\alpha_{i,kl}^{(1)}(\tau)d\alpha_{i,kl}^{(2)}(\tau)$ with $d\alpha_{i,kl}^{(1)}(\tau)=-i(\lambda_{kl}/2)h_i(\tau)e^{-\kappa_i\tau}d\tau$, $d\alpha_{i,kl}^{(2)}(\tau)=-i(\lambda_{kl}/2)h_i^*(\tau)\times e^{\kappa_i\tau}d\tau$.

In the case of the no-jump trajectory [11], the total and dynamic phases acquired from $t=0$ to $t=T$ can be found as follows:

$$\gamma_{kl}^{t,0}=-\frac{\lambda_{kl}}{4}\int_0^T\sum_{i=1}^{2}e^{-\kappa_i t}\{h_i(t)\alpha_{i,kl}^{(2)}(t)+[h_i(t)\alpha_{i,kl}^{(2)}(t)]^*\}dt,$$

$$\gamma_{kl}^{d,0}=-\frac{\lambda_{kl}}{2}\int_0^T\sum_{i=1}^{2}e^{-\kappa_i t}\{h_i(t)\alpha_{i,kl}^{(2)}(t)+[h_i(t)\alpha_{i,kl}^{(2)}(t)]^*\}dt. \tag{9}$$

By comparing the above equations, we have $\gamma_{kl}^{g,0}=\gamma_{kl}^{t,0}-\gamma_{kl}^{d,0}=-\gamma_{kl}^{t,0}$, indicating that the total phase and the dynamic phase are both proportional to the geometric phase.

For further analysis, let us work out the wave function

$$|\Psi_{kl}^0(t)\rangle=\exp(R_{kl}+i\gamma_{kl}^{t,0})|\alpha_{1,kl}^{(2)}e^{-\kappa_1 t}\rangle_1|\alpha_{2,kl}^{(2)}e^{-\kappa_2 t}\rangle_2|kl\rangle,$$

where $|\alpha_{i,kl}^{(2)}e^{-\kappa_i t}\rangle$ $(i=1,2)$ is a coherent state with

$$\alpha_{i,kl}^{(2)}e^{-\kappa_i t}=\frac{\lambda_{kl}h_i(\Delta_i-i\kappa_i)}{2(\kappa_i^2+\Delta_i^2)}(e^{-i\Delta_i t}-e^{-\kappa_i t}), \tag{10}$$

R_{kl} represents amplitude damping due to decay, which is found to be (with $t=T=2m\pi/\Delta_1=2n\pi/\Delta_2$)

$$R_{kl}=\sum_{i=1}^{2}\frac{\lambda_{kl}^2|h_i|^2}{8(\kappa_i^2+\Delta_i^2)^2}[\kappa_i^2(3-2\kappa_i T-4e^{-\kappa_i T}+e^{-2\kappa_i T})-\Delta_i^2(1+2\kappa_i T-e^{-2\kappa_i T})], \tag{11}$$

and

$$\gamma_{kl}^{t,0}=\sum_{i=1}^{2}\frac{\lambda_{kl}^2|h_i|^2\Delta_i}{4(\kappa_i^2+\Delta_i^2)^2}[2\kappa_i(1-e^{-\kappa_i T})-(\kappa_i^2+\Delta_i^2)T]. \tag{12}$$

In the computational basis, the evolution is described by a diagonal matrix, $\mathrm{diag}[e^{R_{++}}e^{i\gamma_{++}^{t,0}},1,1,e^{R_{--}}e^{i\gamma_{--}^{t,0}}]$, which possesses the global geometric feature.

Cavity decay in general disallows the execution of cyclic evolution. However, when κ_i $(i=1,2)$ is very small in comparison with Δ_i $(i=1,2)$, h_1, and h_2, we can choose $t=T=2m\pi/\Delta_1=2n\pi/\Delta_2$ (m,n being positive integers) as the time interval for an approximate closed path with small coherent amplitudes. We should choose m,n as small as possible (e.g., $m=n=1$), such that the coherent amplitudes are small enough. The presence of the cavity decay affects the fidelity F_{kl} of the quantum gate operation. F_{kl} is defined as

$$F_{kl}=|\langle\Psi_{kl}^{(i)}(T)|\Psi_{kl}^0(T)\rangle|^2, \tag{13}$$

where $|\Psi_{kl}^{(i)}(T)\rangle=|\Psi_{kl}^0(T)\rangle|_{\kappa_1,\kappa_2=0}$ corresponds to the state of the system at time T in the ideal case. In Fig. 2, we plot the fidelity F as a function of $x=\kappa_1/|h_1|=\kappa_2/|h_2|$ when $\Delta_1/|h_1|=\Delta_2/|h_2|$. In Fig. 2 the curve 1 is for $m=2$, $n=1$ and the curve 2 is for $m=n=1$. We find that the fidelity F decreases when x increases. The result shows that x should be sufficiently small in order to keep a reasonable fidelity. For the purpose of comparison, we also show the result given in Ref. [9] when $m=1$ (Fig. 2, curve 3). It is clear that the fidelity of the gate proposed here attenuates more slowly with increasing decay than the fidelity of the one in Ref. [9]. That is to say that our gate is more tolerant to decay than the gate in Ref. [9].

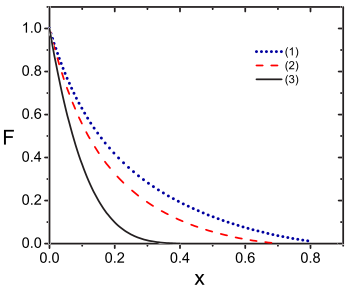

FIG. 2. (Color online) Fidelity $F_{++/--}$ of the phase gate after operating time $T=2m\pi/\Delta_1=2n\pi/\Delta_2$ versus $x=\kappa_1/|h_1|=\kappa_2/|h_2|$ when $\Delta_1/|h_1|=\Delta_2/|h_2|$.

In the following, let us give a brief explanation for the decay tolerance of our scheme. In our scheme the phases acquired by the atomic states to realize a certain phase gate are due to the cyclic evolution of the cavity modes. There are two cavity modes contributing to the phase accumulation, and the time consumed for realizing a certain phase gate is thus doubly reduced. From Eq. (8), it can be seen that the phase $\gamma_{kl}^{t,0}$ consists of two parts, one from each modes [see Eq. (9)]. For example, in an ideal case, Eq. (12) can be written as

$$\gamma_{kl}^{t,0}=-\frac{2m\pi\lambda_{kl}^2|h_1|^2}{4\Delta_1^2}-\frac{2n\pi\lambda_{kl}^2|h_2|^2}{4\Delta_2^2}. \tag{14}$$

By assuming the simple case that $\Delta_1=\Delta_2=\Delta$ and $m=n=1$, we have

$$\gamma_{kl}^{t,0}=-\frac{\pi\lambda_{kl}^2(|h_1|^2+|h_2|^2)}{2\Delta^2}. \tag{15}$$

In order to realize a π-phase gate, we need $\gamma_{++/--}^{t,0}=-\pi$ and hence $2\pi(|h_1|^2+|h_2|^2)/\Delta^2=\pi$. The evolution time is found to be $T_{\text{two mode}}=2\pi/\Delta=2\pi/\sqrt{2(|h_1|^2+|h_2|^2)}$. Let us recall that the evolution time is $T_{\text{one mode}}=2\pi/\Delta=2\pi/\sqrt{2|h_1|^2}$ in the single-mode case [9]. For the gate proposed in Ref. [9], one needs to increase the value of $|h_1|$ in order to reduce the evolution time such that the evolution time is less than the

photon lifetime. However, as a result of the rotating-wave approximation and the physical constraints in practical experimental techniques needed to achieve strong coupling in cavity QED, $|h_1|$ cannot take sufficiently high values. Consequently, one needs to maintain a balance between the two requirements. In the two-mode case the situation can be modulated because there are two parts $|h_1|$ and $|h_2|$ contributing to the total phase and hence the evolution time. The use of a two-mode cavity allows us to include the term $|h_2|$ to offset the value of $|h_1|$. The evolution time of the gate discussed here can be less than that of the gate proposed in Ref. [9]. This tells us that the present gate is more tolerant to cavity decay than the one given in Ref. [9] due to the reduction of evolution time.

The scheme can also be generalized to a system comprised of two atoms interacting with multimode fields. The Hamiltonian is given by

$$\tilde{H}_N = \hbar/2\left(\sum_{i=1}^{N} h_i(t)e^{-\kappa_i t}a_i + h_i^*(t)e^{\kappa_i t}a_i^\dagger\right)(\sigma_1^x + \sigma_2^x),$$

where a_i is the annihilation operator of cavity mode i, κ_i represents the decay rate of the corresponding cavity. In the computational basis $\{|++\rangle,|+-\rangle,|-+\rangle,|--\rangle\}$, the phase accumulated during the evolution can be found to be

$$\gamma_N^{i,0} = -\frac{\lambda_{kl}}{4}\int_0^T \sum_{i=1}^{N} e^{-\kappa_i t}\{h_i(t)\alpha_{i,kl}^{(2)}(t) + [h_i(t)\alpha_{i,kl}^{(2)}(t)]^*\}dt,$$

$$\gamma_N^{d,0} = -\frac{\lambda_{kl}}{2}\int_0^T \sum_{i=1}^{N} e^{-\kappa_i t}\{h_i(t)\alpha_{i,kl}^{(2)}(t) + [h_i(t)\alpha_{i,kl}^{(2)}(t)]^*\}dt,$$

where λ_{kl} is the eigenvalue of $(\sigma_1^x+\sigma_2^x)$, $\alpha_{i,kl}^{(2)}(t) = \int_0^t(-i\lambda_{kl}/2)h_i^*(\tau)e^{\kappa_i\tau}d\tau$. There are N parts in $\gamma_N^{i,0}$ due to the N-mode cavity fields. It is reasonable to say that the more modes there are in the cavity, the more tolerant the gate can be to cavity decay.

We next assess the feasibility of our scheme under current experimental techniques. Let us recall that the detunings δ_1 and δ_2 (see Fig. 1) should be sufficiently large compared with g_i $(i=1,2)$ and Ω_j $(j=1,2,3,4)$ such that the atomic excited state can be adiabatically eliminated. By taking $g_1 = g_2 = g$, $\delta_1 = 120|g|$, $\delta_2 = 80|g|$, $\delta_3 = 40|g|$ and $|\Omega_1| = |g|$, $|\Omega_2| = \frac{2}{3}|g|$, $|\Omega_3| = |\Omega_4| = 2|g|$ such that $|h_1| = |h_2| = |h| \cong |g|/60$, it is easy to check that the conditions defined in Eqs. (2) and (3) and $|r| \gg |h_{1,2}|$ are satisfied. Suppose we wish to achieve a total phase $-\pi$ with $m=n=1$ and $\Delta_1 = \Delta_2 = \Delta$. According to Eq. (12), we can find the relation between Δ and $|h|$: $\Delta = 2|h|$ in the ideal case. Furthermore, the gate time is approximately $T_{\min} = 2\pi/\Delta = \pi/|h|$. In the microwave cavity QED experiment of Walther and co-workers [15], the coupling constant of the cavity mode and the atom is $|g| = 41$ kHz and the photon lifetime achieved is $T_c = 0.3$ s. It can be seen that the gate time $T_{\min}$ is approximately 0.0046 s, which is much shorter than the photon lifetime T_c. So it is possible to experimentally realize the proposed scheme.

Of course in order to achieve a two- or more-mode interaction with the atoms, our scheme requires more laser beams, which obviously increases the experimental difficulties; therefore it is interesting to explore how to further minimize the experimental parameters. One possible way is to control the shape of the laser beams so that the evolution paths with multiple cavity modes can be achieved by using fewer laser beams in one gate operation. This method has been developed by Zhu *et al.* [16] in a trapped-ion quantum computation model. In future work, we will examine the feasibility of such an idea in our scheme.

In summary, we present a scheme for unconventional geometric quantum computation. The scheme is proposed to be realized in a system that consists of two atoms interacting with a two-mode cavity field. We include the effect of cavity decay on the gate in the investigation. The time evolution of the system results in a two-qubit phase gate. It is found that the total phase possesses global geometric features so that the phase gate is robust to some types of noise. One advantage of the proposed phase gate is that it is not affected by spontaneous emission because it involves only metastable states of the atoms. It is worth noting that the evolution time of the gate is less than that of the gate proposed in Ref. [9]. This tells us that the gate can be more tolerant to cavity decay than the previous one. This is another advantage of the gate. The scheme can also be generalized to a system consisting of two atoms interacting with an N-mode cavity field.

This work was supported by NUS Research Grant No. WBS: R-144-000-189-305. C.F.W. acknowledges financial support from Singapore Millennium Foundation. H.S.G. acknowledges support from the National Science Council, Taiwan under Grants No. NSC95-2112-M-002-018 and No. NSC95-2112-M-002-054, and support from the National Taiwan University under Grant No. 95R0034-02. C.H.O. thanks the National Taiwan University and National Center for Theoretical Sciences, Taiwan, for hospitality and support.

[1] M. V. Berry, Proc. R. Soc. London, Ser. A **392**, 45 (1984).
[2] L. M. Duan *et al.*, Science **292**, 1695 (2001).
[3] J. A. Jones *et al.*, Nature (London) **403**, 869 (2000).
[4] G. Falci *et al.*, Nature (London) **407**, 355 (2000).
[5] S. L. Zhu and Z. D. Wang, Phys. Rev. A **67**, 022319 (2003).
[6] S. L. Zhu and Z. D. Wang, Phys. Rev. Lett. **91**, 187902 (2003).
[7] S. B. Zheng, Phys. Rev. A **70**, 052320 (2004).
[8] C. Y. Chen *et al.*, Phys. Rev. A **73**, 032344 (2006); C. Y. Chen *et al.*, *ibid.* **74**, 032328 (2006).
[9] X. L. Feng *et al.*, Phys. Rev. A **75**, 052312 (2007).
[10] D. Leibfried *et al.*, Nature (London) **422**, 412 (2003).
[11] A. Carollo *et al.*, Phys. Rev. Lett. **90**, 160402 (2003).
[12] A. Bassi and E. Ippoliti, Phys. Rev. A **73**, 062104 (2006).
[13] E. Sjöqvist, Acta Phys. Hung. B: Quantum Electron. **26**, 195 (2006).
[14] A. Barchielli and V. Belavkin, J. Phys. A **24**, 1495 (1991); G. Hegerfeldt and D. Sondermann, Quantum Semiclassic. Opt. **8**, 121 (1996); H. J. Carmichael, Phys. Rev. Lett. **70**, 2273 (1993).
[15] S. Brattke *et al.*, Phys. Rev. Lett. **86**, 3534 (2001).
[16] S. L. Zhu *et al.*, Europhys. Lett. **73**, 485 (2006).