Quantum-mechanical formulation of light propagation: A multiple-scattering approach

Wei Guo*

Department of Physics and Optical Science, University of North Carolina at Charlotte, Charlotte, North Carolina 28223, USA

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Since in quantum optics light is represented in terms of photons, light propagation through a linear medium is discussed quantum mechanically in this paper by following the multiple-scattering process of one incident photon from the medium. To treat the photon and the medium on the same quantum footing, the medium is assumed to be an ensemble of uniformly distributed identical two-level atoms. It is found that inside the medium the incident photon follows the same propagation rules as a plane wave does in the classical domain, and has a possibility to become entangled with the atoms. It is also found that when interacting with a two-level test atom outside the medium, the output photon appears to be formally in a single mode identical to that of the incident photon.

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I. INTRODUCTION

Although the theory of quantum optics is successful in explaining those time-dependent phenomena related to light-atom interactions [1], it is believed to become cumbersome when used to formulate light propagation [2]. To discuss light propagation in the quantum framework, different approaches have been developed; one example is the momentum-operator method [2–4]. All these approaches have one point in common, that is, they all require quantization of electromagnetic fields inside the medium through which light travels, and consider only the spatial progression of electromagnetic wave operators. See, in addition, Refs. [5–8]. In contrast, in the theory of classical optics, light propagation is treated in a simple manner [9]. For instance, it is well known that one way to formulate light propagation is to formulate it as a multiple-scattering process: Light propagation from one point to another is represented by all scattering events connecting these points [10–12]. Since light is expressed in terms of photons in quantum optics, light propagation through a medium should also allow a similar treatment as a multiple-scattering process of individual photons from the particles that constitute the medium, without quantizing at all the electromagnetic fields inside the medium. This viewpoint is pursued in this paper via specifically examining every order of scattering experienced by one single incident photon. The reason one photon is sufficient is that the present discussion is limited to the linear domain, in which photons do not combine nonlinearly to create new photons.

Outside the medium, the incident photon is certainly scattered into various modes. But photons cannot simply be added as numbers and do not have an unambiguously defined phase [1,13]. To measure cumulative effects of and possible correlation among the output photons in different modes, also discussed in this paper is the transition prompted by the output photon of a two-level test atom from its ground state to an excited state.

The medium is assumed to be an ensemble of N identical two-level atoms that are uniformly distributed in the region $0 \leqslant z \leqslant d$ and have an excited state $|E\rangle$ and a ground state $|G\rangle$, so that the atoms and photons are treated on the same quantum footing. The energies of states $|E\rangle$ and $|G\rangle$ are $\hbar\omega_E$ and $\hbar\omega_G$, respectively ($\omega_0 = \omega_E - \omega_G$). The test atom's two states are similarly denoted as $|E^T\rangle$, the excited state with energy $\hbar\omega_T^E$, and $|G^T\rangle$, the ground state with energy $\hbar\omega_G^T$ ($\omega_T = \omega_E^T - \omega_G^T$). In this paper, the system under investigation is composed of the atoms and photons, and has a Hamiltonian H that is decomposed into an unperturbed component H_0 and an interaction component V in the minimal-coupling form [14]:

$$H_0 = \sum_j (\hbar\omega_E|E^j\rangle\langle E^j| + \hbar\omega_G|G^j\rangle\langle G^j|) + \sum_\alpha \hbar\omega_\alpha a_\alpha^\dagger a_\alpha + \hbar\omega_E^T|E^T\rangle\langle E^T| + \hbar\omega_G^T|G^T\rangle\langle G^T|, \tag{1}$$

$$V = \sum_{j,\alpha} (\vec{\mu}_{GE}^{(j)} \cdot \vec{g}_\alpha |G^j\rangle\langle E^j| e^{-i\vec{k}_\alpha\cdot\vec{R}_j} a_\alpha^\dagger + \vec{\mu}_{EG}^{(j)} \cdot \vec{g}_\alpha^* |E^j\rangle\langle G^j| e^{i\vec{k}_\alpha\cdot\vec{R}_j} a_\alpha) + \sum_\alpha (\vec{\mu}_{GE}^{(T)} \cdot \vec{g}_\alpha^{(T)} |G^T\rangle\langle E^T| e^{-i\vec{k}_\alpha\cdot\vec{R}_T} a_\alpha^\dagger + \vec{\mu}_{EG}^{(T)} \cdot \vec{g}_\alpha^{(T)*} |E^T\rangle\langle G^T| \times e^{i\vec{k}_\alpha\cdot\vec{R}_T} a_\alpha) \equiv V_1 + V_2. \tag{2}$$

In the preceding expressions, superscripts j are used to denote that $|G^j\rangle$ and $|E^j\rangle$ are, respectively, the ground and excited states of the jth atom at $\vec{R}_j$ in the ensemble, and $\vec{g}_\alpha = i\sqrt{2\pi\hbar\omega_0^2/(L^3\omega_\alpha)}\vec{\epsilon}_\alpha$ and $\vec{g}_\alpha^{(T)} = i\sqrt{2\pi\hbar\omega_T^2/(L^3\omega_\alpha)}\vec{\epsilon}_\alpha$ are the coupling constants of the αth quantized electromagnetic mode ω_α (whose unit vector of polarization is $\vec{\epsilon}_\alpha$) with the atoms in the ensemble and the test atom located at $\vec{R}_T$ in the region $z > d$, respectively ($|\vec{k}_\alpha| = \omega_\alpha/c$, where c is the speed of light in vacuum). The quantization volume is L^3. While $\vec{\mu}_{GE}^{(j)}$, whose complex conjugate is $\vec{\mu}_{EG}^{(j)}$, represents the matrix element between $|G^j\rangle$ and $|E^j\rangle$ of the electric dipole moment $\vec{\mu}^{(j)}$ belonging to the jth atom inside the ensemble, the symbol $\vec{\mu}_{GE}^{(T)}$ is defined as the matrix element between $|G^T\rangle$ and $|E^T\rangle$ of the electric dipole moment $\vec{\mu}^{(T)}$ belonging to the test atom. Throughout the present paper, Greek letters are reserved for the field modes. In Eq. (2), the interaction between the fields and the atoms in the ensemble is denoted as V_1, and the

*wguo2@email.uncc.edu

interaction between the fields and the test atom as V_2. The quantities $a_\alpha^\dagger$ and a_α are, respectively, the creation and annihilation operators for mode α. As usual, the asterisk denotes the conjugate of a complex quantity.

In quantum optics, a scattering process involves the following steps. An incident photon is absorbed by a ground-state atom, causing the atom to jump to an excited state, and, when the atom returns to its ground state, a photon is released to go to other atoms [13]. To describe these steps, the counter-rotating terms are not needed (and thus ignored) in the interaction Hamiltonian V, because these terms, if included, would be largely responsible for the creation of virtual photons and would have little connection to the scattering of the incident photon. See Appendix A for a more detailed discussion.

The discussion to follow is presented in the Schrödinger picture, so that the state $|\psi(t)\rangle$ of the system at time t is related to the system's initial state $|\psi(0)\rangle$ through the following integral relation:

$$|\psi(t)\rangle = -\frac{1}{2\pi i}\int_{-\infty}^{\infty} dq \frac{e^{-iqt/\hbar}}{q-H}|\psi(0)\rangle, \tag{3}$$

where it is understood that the denominator in the integrand contains an imaginary component $i\eta(\eta\to 0^+)$. For simplicity, all the atoms are assumed to be in the ground state when $t=0$. As a result, the initial state of the system reads $|\psi(0)\rangle=(\Pi_{j=1}^N|G^j\rangle)|G^T\rangle|1_I\rangle$, where $|1_I\rangle$ represents the state of the single incident photon. This photon is incident on the atomic ensemble from the $z<0$ region and has a wave vector $\vec{k}_I$ ($|k_I|=\omega_I/c$) and a polarization vector $\vec{\epsilon}_I$ along $\hat{x}$.

As in Ref. [15], the Green function $1/(q-H)$ in Eq. (3) is expanded into ascending powers of V_2:

$$\begin{aligned}\frac{1}{q-H} &= \frac{1}{q-H_0-V_1} + \frac{1}{q-H_0-V_1}V_2\frac{1}{q-H_0-V_1} \\ &\quad + \frac{1}{q-H_0-V_1}V_2\frac{1}{q-H_0-V_1}V_2 \\ &\quad \times \frac{1}{q-H_0-V_1} + \cdots .\end{aligned} \tag{4}$$

Since the test atom is needed in the present discussion merely for the purpose of measuring the output photon, radiation from the test atom should not be allowed to interact with the atoms in the ensemble. This requirement clearly means that the expression in Eq. (4) is in need of modification, because, in its present form, the Green function describes not only the interactions between the incident photon and the atomic ensemble [see the operator $1/(q-H_0-V_1)$], but also those processes in which the photon first interacts with the atoms in the ensemble, then with the test atom, and finally with the atoms in the ensemble again (see the second term on the right-hand side of the same equation). All the following terms in Eq. (4) mean that the photon is scattered back and forth between the ensemble and the test atom even more times. After being tailored to the present problem, the Green function reduces to

$$\begin{aligned}\frac{1}{q-H} &= \frac{1}{q-H_0} \\ &\quad \times\left(V_2 + V_2\frac{1}{q-H_0}V_2\frac{1}{q-H_0}V_2+\cdots\right)\frac{1}{q-H_0-V_1} \\ &= \frac{1}{q-H_0}\left(V_2 + V_2\frac{1}{q-H_0}V_2\frac{1}{q-H_0}V_2+\cdots\right) \\ &\quad \times\left(\frac{1}{q-H_0}+\frac{1}{q-H_0}V_1\frac{1}{q-H_0}\right. \\ &\quad \left. +\frac{1}{q-H_0}V_1\frac{1}{q-H_0}V_1\frac{1}{q-H_0}+\cdots\right),\end{aligned} \tag{5}$$

where it is recognized that the series of V_1 operators is obtained through the expansion of $1/(q-H_0-V_1)$, and describes the multiple-scattering events the incident photon experiences when propagating through the atomic ensemble. The series of operators V_2, on the other hand, illustrates that, after interacting with the output photon, the test atom still needs to conduct radiation reactions before finally settling into its excited state. The terms containing even orders of V_2 are excluded in Eq. (5), because they would instead force the test atom to settle into its ground state and, thus, would represent scattering of the output photon from the test atom, a situation that is clearly not needed in the present discussion.

In Sec. II, propagation of the incident photon through the atomic ensemble is analyzed. Then, in the following section, calculation of the test atom's transition probability amplitude from ground state to excited state is presented. The paper is finally summarized in Sec. IV.

II. PHOTON PROPAGATION THROUGH THE ATOMIC ENSEMBLE

To describe accurately how the incident photon is multiply scattered from the atoms in the ensemble, every term in the serial expansion of $1/(q-H_0-V_1)$ [see Eq. (5)] has to be examined. The first term in the expansion, $1/(q-H_0)$, contains no interaction Hamiltonian V_1 and thus can represent only the one possibility that the incident photon approaches the test atom directly without interacting with the atoms in the ensemble at all,

$$\begin{aligned}\frac{1}{q-H_0}|\psi(0)\rangle &= \frac{1}{q-N\hbar\omega_G-\hbar\omega_G^T-\hbar\omega_I}|\psi(0)\rangle \\ &\equiv \frac{1}{q-E_0}|\psi(0)\rangle,\end{aligned} \tag{6}$$

where $\hbar\omega_I$ denotes the energy carried by the incident photon. From the viewpoint of classical optics [11], Eq. (6) represents the zeroth-order scattering of light, which certainly leaves the state $|\psi(0)\rangle$ of the system unchanged.

In the second term, one V_1 exists. For the present initial state and the structure of V_1, this operator allows each atom in the ensemble to have a chance to transit to its excited state by absorbing the incident photon:

$$\frac{1}{q-H_0}V_1\frac{1}{q-H_0}|\psi(0)\rangle = \frac{1}{q-E_0}\sum_j \frac{\vec{\mu}_{EG}^{(j)}\cdot\vec{g}_I^*}{q-E_1}e^{i\vec{k}_I\cdot\vec{R}_j}|E^j\rangle\prod_{j\neq l}|G^l\rangle|G^T\rangle|0\rangle, \quad (7)$$

where $E_1=(N-1)\hbar\omega_G+\hbar\omega_E+\hbar\omega_G^T$, and $|0\rangle$ denotes the vacuum state created after the incident photon is absorbed. Any atom that does not absorb the incident photon remains in the ground state; see $|G^l\rangle$ and $|G^T\rangle$ in the preceding equation. While in the excited state, an atom [atom j in Eq. (7) for example] can emit one photon and return to the ground state, with the help of the third term in the serial expansion of $1/(q-H_0-V_1)$:

$$\frac{1}{q-H_0}V_1\frac{1}{q-H_0}V_1\frac{1}{q-H_0}|\psi(0)\rangle = \frac{1}{q-E_0}\sum_{j,\alpha}\frac{(\vec{\mu}_{GE}^{(j)}\cdot\vec{g}_\alpha)(\vec{\mu}_{EG}^{(j)}\cdot\vec{g}_I^*)}{(q-E_1)(q-E_\alpha)}e^{i(\vec{k}_I-\vec{k}_\alpha)\cdot\vec{R}_j} \times|G^j\rangle\prod_{j\neq l}|G^l\rangle|G^T\rangle|1_\alpha\rangle, \quad (8)$$

where $E_\alpha=N\hbar\omega_G+\hbar\omega_G^T+\hbar\omega_\alpha$, and the emitted photon $|1_\alpha\rangle$ can be in any mode. Equations (7) and (8) describe a complete scattering cycle: One atom first absorbs the incident photon to go to the excited state and then returns to its ground state through emitting a different photon $|1_\alpha\rangle$. This cycle, however, does not exhaust all the transitions the atom can make, because the atom can, in addition, repeatedly absorb and release the same photon it already created before the photon goes to a different atom. Such repeated absorption and emission of photons by the same atom is called a radiation reaction and was demonstrated to be the physical origin of spontaneous emission in Ref. [15]. In the present discussion, the radiation reactions are generated by those terms in the expansion of $1/(q-H_0-V_1)$ that contain even orders of V_1. It will become clear in the following section that it is through the consideration of the radiation reactions that the atomic spontaneous emission rate is naturally introduced into the atomic polarizability. After all the corrections from the radiation reactions are added to the scattering cycle in Eqs. (7) and (8), the state S_1 of the system, which represents the first-order scattering of the incident photon from the atomic ensemble, is obtained:

$$S_1=\frac{1}{(q-E_0)(q-E_1-E_\Delta)}\sum_{j,\alpha}\frac{(\vec{\mu}_{GE}^{(j)}\cdot\vec{g}_\alpha)(\vec{\mu}_{EG}^{(j)}\cdot\vec{g}_I^*)}{q-E_\alpha}e^{i(\vec{k}_I-\vec{k}_\alpha)\cdot\vec{R}_j} \times|G^j\rangle\prod_{l\neq j}|G^l\rangle|G^T\rangle|1_\alpha\rangle, \quad (9)$$

which shows, as expected, that, after the scattering, all the atoms are again in their ground states. In Eq. (9), $E_\Delta=\Sigma_\alpha[|\vec{\mu}_{GE}^{(j)}\cdot\vec{g}_\alpha|^2/(q-E_\alpha)]$ is the change brought to the atomic excited state by the radiation reactions and can be evaluated by using the usual mode-continuum-limit approximation and requiring $q=E_1$ (see Ref. [15]),

$$E_\Delta=-\frac{\Gamma_0\hbar}{2\pi\omega_0}\left[\Omega+\omega_0\ln\left(\frac{\Omega-\omega_0}{\omega_0}\right)\right]-i\frac{\Gamma_0\hbar}{2}.$$

In the preceding relation, $\Gamma_0=4|\vec{\mu}_{GE}|^2\omega_0^3/(3\hbar c^3)$ is the spontaneous emission rate of any excited atom in the ensemble, and Ω the cutoff frequency needed to make the nonrelativistic Hamiltonian H valid [16] in the present discussion.

Note that in the serial expansion of $1/(q-H_0-V_1)$ odd powers of V_1 also exist. What the odd powers of V_1 achieve is to leave the system in such states that each atom in the ensemble has a chance to settle into its excited state through absorbing the incident photon; see Eq. (7) for an example. Although not needed in the present discussion of photon scattering, these states clearly show, along with S_1, that, as the incident photon propagates through the atomic ensemble, entanglement between the atoms and the photon is inevitably established. Entanglement of atomic ensembles is believed to be a promising way to realize quantum-information processing [17].

The emitted photon from one atom will eventually go to different atoms and be repeatedly absorbed and released by these atoms as well. Radiation reactions conducted by different atoms can still be taken into account by the operator $1/(q-H_0-V_1)$. To understand this point, one needs to note that, if organized into different groups, the operators V_1 in the expansion of $1/(q-H_0-V_1)$ are able to regulate the transitions of more than one atom. For example, in

$$\frac{1}{q-H_0}V_1\frac{1}{q-H_0}V_1\frac{1}{q-H_0}V_1\frac{1}{q-H_0}V_1\frac{1}{q-H_0},$$

while the right two V_1's can be used to force one atom to first absorb the incident photon and then emit another photon, the left two enable another atom to carry out the same absorption and emission transitions as soon as the photon created by the first atom arrives. After the relevant atoms at two different locations all complete their radiation reactions, the system develops into the following state:

$$S_{1\to2}=\frac{1}{(q-E_0)(q-E_1-E_\Delta)^2} \times\sum_{j,\alpha}\frac{(\vec{\mu}_{GE}^{(j)}\cdot\vec{g}_\alpha)(\vec{\mu}_{EG}^{(j)}\cdot\vec{g}_I^*)}{q-E_\alpha}e^{i(\vec{k}_I-\vec{k}_\alpha)\cdot\vec{R}_j} \times\sum_{l\neq j,\beta}\frac{(\vec{\mu}_{GE}^{(l)}\cdot\vec{g}_\beta)(\vec{\mu}_{EG}^{(l)}\cdot\vec{g}_\alpha^*)}{q-E_\beta}e^{i(\vec{k}_\alpha-\vec{k}_\beta)\cdot\vec{R}_l} \times|G^l\rangle\prod_{m\neq l}|G^m\rangle|G^T\rangle|1_\beta\rangle, \quad (10)$$

which is a representation of the second-order scattering of the incident photon from the atoms. At the end of the second-order scattering, all the atoms are again in their ground states. Similarly, after the third-order photon scattering (which involves atoms at three different locations), the state of the system reads

$$S_{1\to2\to3} = \frac{1}{(q-E_0)(q-E_1-E_\Delta)^3} \times \sum_{j,\alpha} \frac{(\vec{\mu}_{GE}^{(j)}\cdot\vec{g}_\alpha)(\vec{\mu}_{EG}^{(j)}\cdot\vec{g}_I^*)}{q-E_\alpha} e^{i(\vec{k}_I-\vec{k}_\alpha)\cdot\vec{R}_j} \times \sum_{l\neq j,\beta} \frac{(\vec{\mu}_{GE}^{(l)}\cdot\vec{g}_\beta)(\vec{\mu}_{EG}^{(l)}\cdot\vec{g}_\alpha^*)}{q-E_\beta} e^{i(\vec{k}_\alpha-\vec{k}_\beta)\cdot\vec{R}_l} \times \sum_{m\neq l,\gamma} \frac{(\vec{\mu}_{GE}^{(m)}\cdot\vec{g}_\gamma)(\vec{\mu}_{EG}^{(m)}\cdot\vec{g}_\beta^*)}{q-E_\gamma} \times e^{i(\vec{k}_\beta-\vec{k}_\gamma)\cdot\vec{R}_m}|G^m\rangle\prod_{n\neq m}|G^n\rangle|G^T\rangle|1_\gamma\rangle. \tag{11}$$

In Eqs. (10) and (11), the emitted photons can be in any mode, such as $|1_\beta\rangle$ and $|1_\gamma\rangle$.

Propagation of the incident photon through the atomic ensemble is completely described by the multiple-scattering series formed by adding to the first three terms in Eqs. (6) and (9)–(11) all higher-order terms. But, since photons do not have an unambiguously defined phase [1,13], the resultant multiple-scattering series cannot be expressed in closed form, and can shed only limited light on what can be learned from the output photon. One way to study the output photon requires that the photon be measured. In the following section, the interaction between the output photon and the test atom is analyzed, and the test atom's transition amplitude A from ground state to excited state as a result of such interaction is calculated. It is then demonstrated that A not only can be expressed in closed form but also, more importantly, contains specific information about the propagation of the incident photon through the atomic ensemble. It is additionally shown that, as far as light detection is concerned, A is equivalent to the amplitude of a transmitted classical wave. In passing, the present formulation of photon scattering is based on mode expansion, an approach also used in Ref. [18] in the description of nonlinear wave interactions.

III. EXCITATION OF THE TEST ATOM

At the location of the test atom, the incident photon is scattered from the atomic ensemble into all the modes allowed by the theory of quantum optics [see, for example, $|1_\beta\rangle$ and $|1_\gamma\rangle$ in Eqs. (10) and (11)], and the test atom needs to respond to the output photon in all these modes before transiting to its excited state $|E^T\rangle$. With the help of Eq. (5), one finds that the interaction between the test atom and the output photon changes the state of the system from that represented by the multiple-scattering series in Sec. II to the following state:

$$\begin{aligned}\frac{1}{q-H}|\psi(0)\rangle &= \frac{1}{q-H_0}\left(V_2+V_2\frac{1}{q-H_0}V_2\frac{1}{q-H_0}V_2+\cdots\right)\left(\frac{1}{q-E_0}\prod_j|G^j\rangle|G^T\rangle|1_I\rangle+\frac{1}{(q-E_0)(q-E_1-E_\Delta)}\right.\\
&\quad\times\sum_{j,\alpha}\frac{(\vec{\mu}_{GE}^{(j)}\cdot\vec{g}_\alpha)(\vec{\mu}_{EG}^{(j)}\cdot\vec{g}_I^*)}{q-E_\alpha}e^{i(\vec{k}_I-\vec{k}_\alpha)\cdot\vec{R}_j}|G^j\rangle\prod_{l\neq j}|G^l\rangle|G^T\rangle|1_\alpha\rangle+\frac{1}{(q-E_0)(q-E_1-E_\Delta)^2}\\
&\quad\left.\times\sum_{j,\alpha}\frac{(\vec{\mu}_{GE}^{(j)}\cdot\vec{g}_\alpha)(\vec{\mu}_{EG}^{(j)}\cdot\vec{g}_I^*)}{q-E_\alpha}e^{i(\vec{k}_I-\vec{k}_\alpha)\cdot\vec{R}_j}\sum_{l\neq j,\beta}\frac{(\vec{\mu}_{GE}^{(l)}\cdot\vec{g}_\beta)(\vec{\mu}_{EG}^{(l)}\cdot\vec{g}_\alpha^*)}{q-E_\beta}e^{i(\vec{k}_\alpha-\vec{k}_\beta)\cdot\vec{R}_l}|G^l\rangle\prod_{m\neq l}|G^m\rangle|G^T\rangle|1_\beta\rangle+\cdots\right)\\
&=\left((\vec{\mu}_{EG}^{(T)}\cdot\vec{g}_I^{(T)*})e^{i\vec{k}_I\cdot\vec{R}_T}\prod_j|G^j\rangle+\frac{1}{q-E_1-E_\Delta}\sum_{j,\alpha}\frac{(\vec{\mu}_{GE}^{(j)}\cdot\vec{g}_\alpha)(\vec{\mu}_{EG}^{(j)}\cdot\vec{g}_I^*)}{q-E_\alpha}\right.\\
&\quad\times e^{i(\vec{k}_I-\vec{k}_\alpha)\cdot\vec{R}_j}(\vec{\mu}_{EG}^{(T)}\cdot\vec{g}_\alpha^{(T)*})e^{i\vec{k}_\alpha\cdot\vec{R}_T}|G^j\rangle\prod_{l\neq j}|G^l\rangle+\frac{1}{(q-E_1-E_\Delta)^2}\sum_{j,\alpha}\frac{(\vec{\mu}_{GE}^{(j)}\cdot\vec{g}_\alpha)(\vec{\mu}_{EG}^{(j)}\cdot\vec{g}_I^*)}{q-E_\alpha}e^{i(\vec{k}_I-\vec{k}_\alpha)\cdot\vec{R}_j}\\
&\quad\left.\times\sum_{l\neq j,\beta}\frac{(\vec{\mu}_{GE}^{(l)}\cdot\vec{g}_\beta)(\vec{\mu}_{EG}^{(l)}\cdot\vec{g}_\alpha^*)}{q-E_\beta}e^{i(\vec{k}_\alpha-\vec{k}_\beta)\cdot\vec{R}_l}(\vec{\mu}_{EG}^{(T)}\cdot\vec{g}_\beta^{(T)*})e^{i\vec{k}_\beta\cdot\vec{R}_T}|G^l\rangle\prod_{m\neq l}|G^m\rangle+\cdots\right)\frac{|E^T\rangle|0\rangle}{(q-E_2-E_\Delta^T)(q-E_0)}.\end{aligned}\tag{12}$$

New quantities introduced in the preceding equation are the energy of the system when the test atom is in its excited state, $E_2=N\hbar\omega_G+\hbar\omega_E^T$, and the change to the atom's excited state level due to the radiation reactions $E_\Delta^T=-(\Gamma_0^T\hbar/2\omega_T\pi)\{\Omega+\omega_T\ln[(\Omega-\omega_T)/\omega_T]\}-\Gamma_0^T\hbar/2$, where $\Gamma_0^T=4|\vec{\mu}_{GE}^{(T)}|^2(\omega_T)^3/(3\hbar c^3)$ is the spontaneous emission rate of the test atom once in the excited state. The state in Eq. (12) is obtained after each term of the photon multiple-scattering series discussed in Sec. II is considered for its contribution to the excitation of the test atom. For example, in the large parentheses after the second equality in Eq. (12), the first term arises from the zeroth-order scattering of the incident photon, the second term from the first-order scattering, and all the other remaining terms from higher-order scattering events. Thus, all the information regarding the propagation of the incident photon through the atomic ensemble is now contained in the state in Eq. (12).

The time-dependent probability amplitude A for the test atom to transit from its ground state to an excited state under the influence of the output photon is obtained after the scalar product between the state in Eq. (12) and $|\psi_f\rangle = \Sigma_{j=1}^{N}|G^j\rangle|E^T\rangle|0\rangle$ is found, and after the residue theorem is subsequently applied to the scalar product; see Eq. (3). Note that only the pole around $E_0 - i\eta$ is required by the residue theorem. The pole from E_Δ, on the other hand, should be ignored, because this pole would result in the generation of an exponentially decaying wave, which will diminish A as $t\to\infty$. In classical optics, a dipole driven by an external electromagnetic field is also known to create an evanescent wave that exponentially decays due to the radiation reactions, and a propagating wave bearing the same frequency as that of the external field. The pole associated with E_Δ^T exhibits—aside from the fact that due to spontaneous emission the test atom can never stay in its excited state forever—little about the propagation of the incident photon, and, thus, should be ignored too. Specifically, A is found to be

$$
\begin{aligned}
A = \frac{e^{-iE_0t/\hbar}}{E_0 - E_2 - E_\Delta^T}\Bigg(& (\vec{\mu}_{EG}^{(T)}\cdot\vec{g}_I^{(T)*})e^{i\vec{k}_I\cdot\vec{R}_T} + \frac{1}{E_0 - E_1 - E_\Delta} \\
& \times\sum_{j,\alpha}\frac{(\vec{\mu}_{GE}^{(j)}\cdot\vec{g}_\alpha)(\vec{\mu}_{EG}^{(j)}\cdot\vec{g}_I^{*})}{E_0 - E_\alpha}e^{i(\vec{k}_I-\vec{k}_\alpha)\cdot\vec{R}_j}(\vec{\mu}_{EG}^{(T)}\cdot\vec{g}_\alpha^{(T)*})e^{i\vec{k}_\alpha\cdot\vec{R}_T} \\
& + \frac{1}{(E_0 - E_1 - E_\Delta)^2}\sum_{j,\alpha}\frac{(\vec{\mu}_{GE}^{(j)}\cdot\vec{g}_\alpha)(\vec{\mu}_{EG}^{(j)}\cdot\vec{g}_I^{*})}{E_0 - E_\alpha}e^{i(\vec{k}_I-\vec{k}_\alpha)\cdot\vec{R}_j} \\
& \times\sum_{l\neq j,\beta}\frac{(\vec{\mu}_{GE}^{(l)}\cdot\vec{g}_\beta)(\vec{\mu}_{EG}^{(l)}\cdot\vec{g}_\alpha^{*})}{E_0 - E_\beta} \\
& \times e^{i(\vec{k}_\alpha-\vec{k}_\beta)\cdot\vec{R}_l}(\vec{\mu}_{EG}^{(T)}\cdot\vec{g}_\beta^{(T)*})e^{i\vec{k}_\beta\cdot\vec{R}_T} + \cdots\Bigg).
\end{aligned}
\tag{13}
$$

When the summation over individual atoms in Eq. (13) is replaced by integration, the expression of A is simplified. Note that the atoms in the ensemble are already assumed to have a uniform distribution (characterized from now on by a constant density n):

$$
\begin{aligned}
A = \frac{e^{-iE_0t/\hbar}}{E_0 - E_2 - E_\Delta^T}\Bigg(& (\vec{\mu}_{EG}^{(T)}\cdot\vec{g}_I^{(T)*})e^{i\vec{k}_I\cdot\vec{R}_T} + \frac{n(2\pi)^2}{E_0 - E_1 - E_\Delta} \\
& \times\int_0^d dz_j e^{i(k_{Iz}-k_{\alpha z})z_j}\sum_\alpha\frac{(\vec{\mu}_{GE}^{(j)}\cdot\vec{g}_\alpha)(\vec{\mu}_{EG}^{(j)}\cdot\vec{g}_I^{*})}{E_0 - E_\alpha} \\
& \times(\vec{\mu}_{EG}^{(T)}\cdot\vec{g}_\alpha^{(T)*})e^{i\vec{k}_\alpha\cdot\vec{R}_T}\delta(\vec{k}_{I\perp}-\vec{k}_{\alpha\perp}) + \frac{n^2(2\pi)^4}{(E_0 - E_1 - E_\Delta)^2} \\
& \times\int_0^d dz_j e^{i(k_{Iz}-k_{\alpha z})z_j}\sum_\alpha\frac{(\vec{\mu}_{GE}^{(j)}\cdot\vec{g}_\alpha)(\vec{\mu}_{EG}^{(j)}\cdot\vec{g}_I^{*})}{E_0 - E_\alpha}\delta(\vec{k}_{I\perp}-\vec{k}_{\alpha\perp}) \\
& \times\int_0^d dz_l e^{i(k_{\alpha z}-k_{\beta z})z_l}\sum_\beta\frac{(\vec{\mu}_{GE}^{(l)}\cdot\vec{g}_\beta)(\vec{\mu}_{EG}^{(l)}\cdot\vec{g}_\alpha^{*})}{E_0 - E_\beta} \\
& \times\delta(\vec{k}_{\alpha\perp}-\vec{k}_{\beta\perp})\times(\vec{\mu}_{EG}^{(T)}\cdot\vec{g}_\gamma^{(T)*})e^{i\vec{k}_\beta\cdot\vec{R}_T} + \cdots\Bigg),
\end{aligned}
\tag{14}
$$

where integration over the $x-y$ plane is responsible for the formation of two-dimensional δ functions. Also, in the preceding equation, $\vec{B}_\perp$ represents the component on the $x-y$ plane of an arbitrary vector $\vec{B}$. The orientation of any atom's dipole moment is unmeasurable and, thus, has to be averaged over [13]. It is found that after this orientation averaging the only surviving component of all $\vec{g}$ factors in Eq. (14) is along a direction parallel to $\hat{x}$, the same direction as the polarization of the incident photon. This conclusion reflects a familiar result in classical optics: The polarization of an incident TE wave remains unchanged as the wave travels into a uniform dielectric medium [9,12]. An application of the mode-continuum-limit approximation, aided by the δ functions, then shows that each sum over the field modes in Eq. (14) reduces to $G_z(z_1-z_2)$, the one-dimensional classical Green function $ie^{ik_{Iz}|z_1-z_2|}/(2k_{Iz})$, where the parameter k_{Iz} denotes the z component of the incident wave vector $\vec{k}_I$. [In this work, $G(r)=e^{ik_Ir}/4\pi r$ is referred to as the classical Green function in order to distinguish it from $1/(q-H)$.] As a result, A becomes

$$
\begin{aligned}
A = & \frac{e^{-iE_0t/\hbar}}{E_0 - E_2 - E_\Delta^T}\frac{\Delta}{\sqrt{\omega_I}}e^{i\vec{k}_{I\perp}\cdot\vec{R}_T} \\
& \times\Bigg[e^{ik_{Iz}R_{Tz}} + \left(-\frac{4\pi n|\vec{\mu}_{GE}|^2\omega_0^2}{3(E_0-E_1-E_\Delta)c^2}\right) \\
& \times\int_0^d dz_j e^{ik_{Iz}z_j}G_z(R_{Tz}-z_j) + \left(\frac{4\pi n|\vec{\mu}_{GE}|^2\omega_0^2}{3(E_0-E_1-E_\Delta)c^2}\right)^2 \\
& \times\int_0^d dz_j e^{ik_{Iz}z_j}G_z(z_j-z_l)\int_0^d dz_l G_z(R_{Tz}-z_l) + \cdots\Bigg],
\end{aligned}
\tag{15}
$$

where $\Delta = -i\mu_{EGx}^{(T)}\sqrt{2\pi\hbar\omega_T^2/L^3}$, and $\mu_{EGx}^{(T)}$ is the $\hat{x}$ component of $\vec{\mu}_{EG}^{(T)}$. The coefficient $-4\pi n|\vec{\mu}_{GE}|^2\omega_0^2/3c^2(E_0-E_1-E_\Delta)$ in Eq. (15) can be expressed in terms of Γ_0 as

$$
\begin{aligned}
& 4\pi nk_0^2\frac{\Gamma_0}{4k_0^3\left[\omega_0-\omega_I-\dfrac{\Gamma_0}{2\pi\omega_0}\left(\Omega+\omega_0\ln\dfrac{\Omega-\omega_0}{\omega_0}\right)-i\dfrac{\Gamma_0}{2}\right]} \\
& \equiv 4\pi nk_0^2P_{atom},
\end{aligned}
\tag{16}
$$

where $k_0=\omega_0/c$, and P_{atom} is the resonant component of the polarizability of the atoms in the ensemble [13,19]. Although a photon released from each atom in the ensemble can be in any mode, the symmetry of the ensemble in the $x-y$ plane (see the δ functions) and the sums over the modes [see, for example, Eq. (14)] practically dictate that any photon propagating from one atom to another must have the same wave number as the incident photon. It is then natural to find that the series in Eq. (15) resembles those in Ref. [11], where a classical multiple-scattering description of light propagation is presented. Nevertheless, it is necessary to note that, in Eq. (15), coefficients like $4\pi n|\vec{\mu}_{GE}|^2\omega_0^2/[3c^2(E_0-E_1-E_\Delta)]$ and its higher powers are in fact related to quantum probabilities.

Since the test atom's transition is caused by the output photon (which is in fact nothing other than the incident photon passing through the atomic ensemble), specific information about the spatial and temporal dependence of the incident photon's propagation inside the medium must be recorded in A [as well as the state in Eq. (12)]. Such information is demonstrated in Appendix B to be indeed in A, in particular, in the series composed of the classical Green function. Due to the symmetry of the atomic ensemble along the $x-y$ plane, however, only G_z appears in the serial representation A in Eq. (15).

The results reported in Ref. [11] are then used to put the series in Eq. (15) into a closed-form expression:

$$A = \frac{2}{E_0 - E_2 - E_\Delta^T}\frac{\Delta}{\sqrt{\omega_I}}D_0 k_z' e^{i(k_z'-k_{Iz})d}e^{i\vec{k}_I\cdot\vec{R}_T - iE_0 t/\hbar}, \quad (17)$$

where $k_z' = \sqrt{k_{Iz}^2 + 4\pi n k_0^2 P_{atom}}$, and $D_0 = 2k_{Iz}[(k_z'+k_{Iz})^2 - (k_z'-k_{Iz})^2 e^{2ik_z'd}]^{-1}$.

IV. CONCLUSION AND DISCUSSION

Linear propagation of light through an ensemble of identical two-level atoms is formulated through following the course of multiple scattering of one incident photon from the atoms, rather than through studying the spatial progression of the quantized electromagnetic wave operators. Since the multiple-scattering series obtained in Eq. (15) is practically the same as that describing the propagation of a classical wave through a dielectric slab formed by classical dipoles [11] (see also Appendix B), it is evident that inside the ensemble the incident photon actually travels abiding by the same rules as does a classical wave. This familiar result was also reached by other authors [2,4,5] using different approaches that all require quantization of the fields inside the medium.

Since the present formulation is a fully quantum formulation, it yields more information about light propagation. For example, since the atoms are treated on the same quantum footing as are photons, it is found in Sec. II that as the incident photon travels through the atomic ensemble it has a chance to become entangled with the atoms. This quantum phenomenon can never be exhibited if the medium is treated as a classical medium [2,4,5,7]. Another feature of this paper is that the discussion is presented in the Schrödinger picture and takes into consideration all atomic transitions relevant to photon propagation. As a result, the dispersion properties of light propagation are analyzed accurately without assuming that the susceptibility of the medium is constant [2,7] or employing approximation methods to consider possible losses in the medium [5]. In particular, this paper shows specifically through Eq. (16) that energy is taken out of the light through spontaneous emission. The Schrödinger-picture approach also allows one to argue in Appendix A that exclusion of the counter-rotating terms from H does not prevent the light propagation from being formulated properly.

For the purpose of measuring the output photon, also discussed in the paper is the excitation of a test atom by the output photon through calculating the transition amplitude A of the atom. It turns out that A can be put into closed form [see Eq. (17)], implying that, although the output photons in different modes do not have specific relative phases, they do correlate with each other when interacting with the test atom. More specifically, the quantity $2D_0 k_z' e^{i(k_z'-k_{Iz})d}e^{i\vec{k}_I\cdot\vec{R}_T}$ in Eq. (17) demonstrates that, when interacting with the test atom, the output photon formally appears to be in a mode identical to that of the incident photon. To understand this observation, it is necessary to note that photon propagation through the slab follows the classical laws, so that the output photon, which is the transmitted incident photon through the slab, must have the same wave vector or be in the same mode as the incident photon. Note that the quantity $2D_0 k_z' e^{i(k_z'-k_{Iz})d}e^{i\vec{k}_I\cdot\vec{R}_T}$ is in fact the transmitted plane wave $e^{i\vec{k}_I\cdot\vec{r}}$ through the atoms [11]. This analogy not only confirms that the present quantum formulation of light propagation is consistent with the classical formulation, but also, more importantly, demonstrates that when light detection is concerned the amplitude of a transmitted wave in classical optics is equivalent to the transition probability amplitude of the test atom (placed outside the medium) from the ground state to the excited state in quantum optics. The reason is simple: While light detection is described by the transition probability of a detector (the test atom in the present discussion) in quantum optics [1], it is related to light intensity in classical optics [20]. Light detection is a topic seldom discussed when light propagation is addressed through the spatial progression of the quantized field operators [2,4,5]. It is shown in Appendix B that the spatial and temporal dependence of light propagation is actually represented by the classical Green function, that is, information about photon propagation can be studied through A.

It is worthwhile to note that, in principle, the relations in Eqs. (13)–(15) and (17) are valid only as $t\to\infty$, because in these relations every order of photon scattering is included, and every order of photon scattering, as shown in Appendix B, takes time to complete.

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APPENDIX A: JUSTIFICATION OF THE ROTATING-WAVE APPROXIMATION IN V

In principle, the interaction Hamiltonian V should include both the rotating and counter-rotating components in order to describe accurately light-atom interactions. When photon propagation through (two-level) atoms is considered, however, the counter-rotating components can be excluded from V. For illustration, consider a case in which the incident photon is scattered only from atom A to atom B, all initially in the ground state. If the radiation reactions are additionally ignored for simplicity, the Green function in Eq. (3) contains only one term:

$$\frac{1}{q-H} = \frac{1}{q-H_0}V_1\frac{1}{q-H_0}V_1\frac{1}{q-H_0}V_1\frac{1}{q-H_0}, \quad (A1)$$

which evidently is sufficient for the following scattering process. Atom A first absorbs the incident photon (through the rightmost V_1) to go to the excited state and then releases a different photon (through the middle V_1) to return to the ground state. The released photon, when reaching atom B, causes the latter atom (through the leftmost V_1) to enter its excited state.

Since the rightmost V_1 addresses the interaction between the incident photon and atom A, it should only include those atomic operators that belong to atom A:

$$V_1 = \sum_\alpha (\vec{\mu}_{GE}^{(A)} \cdot \vec{g}_\alpha |G^A\rangle\langle E^A| e^{-i\vec{k}_\alpha \cdot \vec{R}_A} a_\alpha^\dagger + \vec{\mu}_{EG}^{(A)} \cdot \vec{g}_\alpha^* |E^A\rangle\langle G^A| \times e^{i\vec{k}_\alpha \cdot \vec{R}_A} a_\alpha) + \sum_\alpha (\vec{\mu}_{EG}^{(A)} \cdot \vec{g}_\alpha |E^A\rangle\langle G^A| e^{-i\vec{k}_\alpha \cdot \vec{R}_A} a_\alpha^\dagger + \vec{\mu}_{GE}^{(A)} \cdot \vec{g}_\alpha^* |G^A\rangle\langle E^A| e^{i\vec{k}_\alpha \cdot \vec{R}_A} a_\alpha), \tag{A2}$$

where the counter-rotating components are given in the second term, and $\vec{R}_A$ and $\vec{R}_B$ denote, respectively, the locations of atoms A and B. For the initial state $|G^A\rangle|G^B\rangle|1_I\rangle$, one finds that

$$\begin{aligned}\frac{1}{q-H_0}V_1\frac{1}{q-H_0}V_1\frac{1}{q-H_0}V_1\frac{1}{q-H_0}|G^A\rangle|G^B\rangle|1_I\rangle &= \frac{1}{q-2\hbar\omega_G-\hbar\omega_I}\frac{1}{q-H_0}V_1\frac{1}{q-H_0}V_1 \\ &\times\Bigg(\frac{\vec{\mu}_{EG}^{(A)}\cdot\vec{g}_I^*}{q-\hbar\omega_E-\hbar\omega_G}e^{i\vec{k}_I\cdot\vec{R}_A}|E^A\rangle|G^B\rangle|0\rangle \\ &+\sum_\alpha\frac{\vec{\mu}_{EG}^{(A)}\cdot\vec{g}_\alpha}{q-\hbar\omega_E-\hbar\omega_G-\hbar\omega_I-\hbar\omega_\alpha} \\ &\times e^{-i\vec{k}_\alpha\cdot\vec{R}_A}|E^A\rangle|G^B\rangle|1_I\rangle|1_\alpha\rangle\Bigg),\end{aligned}\tag{A3}$$

where the second term in the large parentheses is due to the counter-rotating component $\sum_\alpha \vec{\mu}_{EG}^{(A)} \cdot \vec{g}_\alpha |E^A\rangle\langle G^A| e^{-i\vec{k}_\alpha \cdot \vec{R}_A} a_\alpha^\dagger$, and represents such a transition that atom A spontaneously jumps (without the aid of the incident photon $|1_I\rangle$ at all) to the excited state and emits in the meantime one virtual photon [15] that can be in any mode $|1_\alpha\rangle$. Clearly this transition does not fit into the present discussion of photon scattering, because the incident photon, the photon to be scattered from atom A, does not even interact with atom A. As a result, the counter-rotating components in the rightmost V_1, as well as the term in Eq. (A3) created by these components, are not needed. The first term in the large parentheses is due to the rotating components and is a representation that atom A absorbs the incident photon to transit into the excited state.

The middle V_1 in Eq. (A1) is the same as that in Eq. (A2), because it is still used to describe the interaction between atom A and the fields. From Eq. (A3), it is found that the scattering process continued by the middle V_1 changes the state of the system to

$$\begin{aligned}&\frac{1}{q-2\hbar\omega_G-\hbar\omega_I}\frac{1}{q-H_0}V_1\frac{1}{q-H_0}V_1 \times\frac{\vec{\mu}_{EG}^{(A)}\cdot\vec{g}_I^* e^{i\vec{k}_I\cdot\vec{R}_A}}{q-\hbar\omega_E-\hbar\omega_G}|E^A\rangle|G^B\rangle|0\rangle \\ &= \frac{1}{q-2\hbar\omega_G-\hbar\omega_I}\frac{1}{q-H_0}V_1 \times\Bigg(\sum_\alpha\frac{\vec{\mu}_{GE}^{(A)}\cdot\vec{g}_\alpha}{q-2\hbar\omega_G-\hbar\omega_\alpha}\frac{\vec{\mu}_{EG}^{(A)}\cdot\vec{g}_I^*}{q-\hbar\omega_E-\hbar\omega_G}e^{i(\vec{k}_I-\vec{k}_\alpha)\cdot\vec{R}_A} \times|G^A\rangle|G^B\rangle|1_\alpha\rangle+0\Bigg),\end{aligned}\tag{A4}$$

where the null state in the large parentheses, a state equal to zero, is noted to be created by the counter-rotating components $\sum_\alpha \vec{\mu}_{GE}^{(A)} \cdot \vec{g}_\alpha^* |G^A\rangle\langle E^A| e^{i\vec{k}_\alpha \cdot \vec{R}_A} a_\alpha$. Thus, the counter-rotating components actually would stop the scattering process—the middle V_1 can be treated under the rotating-wave approximation. The nonzero state in the large parentheses can be understood like this: Under the action of the rotating components, atom A returns to its ground state and emits one photon $|1_\alpha\rangle$.

The leftmost V_1 is obtained from the expression in Eq. (A2) by replacing all the indices A by B, because this operator now determines the interaction between atom B and the fields. When the leftmost V_1 acts on the state in Eq. (A4), the following states are obtained:

$$\begin{aligned}&\frac{1}{q-2\hbar\omega_G-\hbar\omega_I}\frac{1}{q-H_0}V_1\sum_\alpha\frac{\vec{\mu}_{GE}^{(A)}\cdot\vec{g}_\alpha}{q-2\hbar\omega_G-\hbar\omega_\alpha}\frac{\vec{\mu}_{EG}^{(A)}\cdot\vec{g}_I^*}{q-\hbar\omega_E-\hbar\omega_G}e^{i(\vec{k}_I-\vec{k}_\alpha)\cdot\vec{R}_A}\times|G^A\rangle|G^B\rangle|1_\alpha\rangle \\ &= \frac{1}{q-2\hbar\omega_G-\hbar\omega_I}\Bigg(\sum_\alpha\frac{(\vec{\mu}_{EG}^{(B)}\cdot\vec{g}_\alpha^*)(\vec{\mu}_{GE}^{(A)}\cdot\vec{g}_\alpha)}{q-2\hbar\omega_G-\hbar\omega_\alpha}\frac{\vec{\mu}_{EG}^{(A)}\cdot\vec{g}_I^*}{(q-\hbar\omega_E-\hbar\omega_G)^2}e^{i(\vec{k}_I-\vec{k}_\alpha)\cdot\vec{R}_A+i\vec{k}_\alpha\cdot\vec{R}_B}\times|G^A\rangle|E^B\rangle|0\rangle \\ &+\sum_\beta\frac{\vec{\mu}_{EG}^{(B)}\cdot\vec{g}_\beta}{q-\hbar\omega_G-\hbar\omega_E-\hbar\omega_I-\hbar\omega_\alpha}e^{-i\vec{k}_\beta\cdot\vec{R}_B}\sum_\alpha\frac{\vec{\mu}_{EG}^{(A)}\cdot\vec{g}_\alpha}{q-2\hbar\omega_G-\hbar\omega_\alpha}\times\frac{\vec{\mu}_{EG}^{(A)}\cdot\vec{g}_I^*}{q-\hbar\omega_E-\hbar\omega_G}e^{i(\vec{k}_I-\vec{k}_\alpha)\cdot\vec{R}_A}|G^A\rangle|E^B\rangle|1_\alpha\rangle|1_\beta\rangle\Bigg),\end{aligned}\tag{A5}$$

where the first term in the large parentheses is due to the rotating components and shows that atom B absorbs one photon released by atom A and jumps into the excited state; all these transitions are consistent with the photon scattering process. The second term, on the other hand, is generated by the counter-rotating components, more specifically by $\Sigma_\alpha(\vec{\mu}_{EG}^{(B)}\cdot\vec{g}_\alpha|E^B\rangle\langle G^B|e^{-i\vec{k}_\alpha\cdot\vec{R}_B}a_\alpha^\dagger)$, and represents such a transition that atom B spontaneously creates a photon (which is still virtual and can be in any mode $|1_\beta\rangle$), and enters the excited state. This latter transition is evidently not caused by the photon from atom A and is, thus, not even part of the scattering process intended to be discussed here. Therefore, the counter-rotating components can be ignored in the leftmost V_1 too.

From the preceding discussion, it is evident that the counter-rotating components, although they might be important in other situations, are unnecessary for the formulation of light propagation, because they are mainly responsible for the creation of virtual photons and contribute little to the scattering of the incident photon. The Schrödinger-picture approach used in the present work requires every atomic transition to be tracked and consequently enables an exhibition of different roles played by the rotating and counter-rotating components in V.

APPENDIX B: CLASSICAL GREEN FUNCTION AND LIGHT PROPAGATION

The spatial and temporal dependence of photon propagation through the atomic ensemble is stored in the classical Green function G. To explain this point, consider an analogous multiple-scattering formulation of light propagation in the classical domain by computing the total field E at $\vec{r}$ after an incident plane wave $e^{i\vec{k}\cdot\vec{r}-i\omega t}$ is scattered from N atoms. Such a treatment is relevant, because it is already shown in Sec. III that propagation of the incident photon through the atomic ensemble follows the same classical laws. The density n of the atoms is

$$n(\vec{r})=\sum_{j=1}^{N}\delta(\vec{r}-\vec{r}_j). \tag{B1}$$

For simplicity, all the waves are simplified as scalar waves. The zeroth-order component of E certainly contains the incident plane wave alone. The first-order component (denoted as E_1) results from the first-order scattering of the incident wave from the atoms [9]:

$$\begin{aligned}E_1(\vec{r},t)&=\alpha_{atom}k^2\int\int\frac{\delta(t_1-[t-|\vec{r}-\vec{r}_1|c^{-1}])}{|\vec{r}-\vec{r}_1|}\\&\quad\times e^{i\vec{k}\cdot\vec{r}_1-i\omega t_1}n(\vec{r}_1)d\vec{r}_1dt_1\\&=\alpha_{atom}k^2\sum_{j=1}^{N}\frac{e^{i\vec{k}\cdot\vec{r}_j-i\omega(t-|\vec{r}-\vec{r}_j|c^{-1})}}{|\vec{r}-\vec{r}_j|}\\&=\alpha_{atom}k^2\sum_{j=1}^{N}\frac{e^{ik|\vec{r}-\vec{r}_j|+i\vec{k}\cdot\vec{r}_j-i\omega t}}{|\vec{r}-\vec{r}_j|},\end{aligned} \tag{B2}$$

where it is clear that the spatial and temporal dependence of light propagation from a particular atom, the jth atom for example, to $\vec{r}$, is contained in $ik|\vec{r}-\vec{r}_j|$, that is, in the classical Green function $e^{ik|\vec{r}-\vec{r}_1|}/(4\pi|\vec{r}-\vec{r}_j|)$. In the preceding equation, α_{atom} denotes the atomic polarizability, and $k=\omega/c$. When all higher-order components are included in E, one has

$$\begin{aligned}E(\vec{r},t)=e^{-i\omega t}\Bigg(&e^{i\vec{k}\cdot\vec{r}}+\alpha_{atom}k^2\sum_{j=1}^{N}\frac{e^{ik|\vec{r}-\vec{r}_j|}}{|\vec{r}-\vec{r}_j|}e^{i\vec{k}\cdot\vec{r}_j}\\&+\alpha_{atom}^2k^4\sum_{j\neq l}^{N}\frac{e^{ik|\vec{r}-\vec{r}_l|+ik|\vec{r}_l-\vec{r}_j|}}{|\vec{r}-\vec{r}_l|\cdot|\vec{r}_l-\vec{r}_j|}e^{i\vec{k}\cdot\vec{r}_j}+\cdots\Bigg).\end{aligned} \tag{B3}$$

If the atoms are again assumed to have a uniform distribution n_0 between $z=0$ and d, and if the results in Ref. [11] are used, E reduces to a series of the one-dimensional Green function:

$$\begin{aligned}E(\vec{r},t)=e^{i\vec{k}\cdot\vec{r}_\perp-i\omega t}\Bigg(&e^{ik_zz}+4\pi\alpha_{atom}k^2n_0\\&\times\int_0^d dz_jG_z(z-z_j)e^{ik_zz_j}+(4\pi\alpha_{atom}k^2n_0)^2\\&\times\int_0^d dz_lG_z(z-z_l)\int_0^d dz_jG_z(z_l-z_j)e^{ik_zz_j}+\cdots\Bigg),\end{aligned} \tag{B4}$$

which is formally identical to the quantum series in Eq. (15) and confirms that A contains all the information in its G_z series about the propagation of the incident photon.

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