Direct space-time observation of pulse tunneling in an electromagnetic band gap

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We present space-time-resolved measurements of electromagnetic pulses tunneling through a coaxial electromagnetic band gap structure. The results show that during the tunneling process the field distribution inside the barrier is an exponentially decaying standing wave whose amplitude increases and decreases as it slowly follows the temporal evolution of the input pulse. At no time is a pulse maximum found inside the barrier, and hence the transmitted peak is not the incident peak that has propagated to the exit. The results support the quasistatic interpretation of tunneling dynamics and confirm that the group delay is not the traversal time of the input pulse peak.

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I. INTRODUCTION

How long does it take a particle to tunnel through a barrier? Seventy-five years after MacColl answered that it "takes no appreciable time" [1], the question continues to occupy physicists and generate controversy [2–8]. The controversy stems from the fact that several tunneling time definitions predict a group velocity that not only exceeds the vacuum speed of light but also increases with barrier length, ultimately becoming infinite. This increase of group velocity v_g with barrier length L follows from an application of the relation $v_g = L/\tau_g$ to Hartman's calculation of a tunneling time τ_g that becomes independent of barrier length [9]. To test these predictions of the quantum theory, a number of experiments have been reported using electromagnetic wave packets which can tunnel through regions of evanescence in a manner analogous to the tunneling of quantum particles [10–14]. These experiments have yielded apparent superluminal tunneling group velocities ranging from 1.7c to 5c and beyond. Some of these experiments have also confirmed the Hartman effect, the saturation of tunneling time with barrier length, thus reinforcing the view that the tunneling process is superluminal.

In recent papers, however, Winful has argued that tunneling is not a pulse propagation phenomenon that can be characterized with a group velocity [15–18]. It is a quasistatic phenomenon in which the barrier acts as a lumped element with respect to the wave packet, whose spatial extent greatly exceeds the barrier length. The barrier is essentially an evanescent mode resonator characterized by a cavity lifetime rather than by a transit time. The Hartman effect is explained by the fact that the cavity lifetime is proportional to the stored energy in the barrier, a quantity that saturates with barrier length because of the exponential decay of the energy density with distance. Numerical simulations have shown that, during the tunneling process, the entire evanescent mode rises and falls, adiabatically following the incident pulse with a small delay due to energy storage [16,17]. At no time is a pulse peak to be found inside the barrier, and hence the transmitted peak is not the incident peak that has propagated to the exit. To date, however, tunneling experiments have monitored only incident and transmitted pulses. The proceedings within the barrier itself have remained a dark and hidden secret from an observational standpoint.

In this paper we look inside a barrier and provide space-time-resolved measurements of electromagnetic wave packet tunneling which support the quasistatic interpretation of the tunneling process. The incident field is seen to modulate the amplitude and hence the stored energy of an exponentially decaying quasi-standing-wave within the barrier. During the evolution of the incident pulse the intrabarrier field is a monotonic function of distance and no propagating envelope or pulse peak is detected inside the barrier.

II. THE EXPERIMENT

A. The electromagnetic band gap structure

The barrier used in this work is a one-dimensional periodic structure made by alternating 5-m-long coaxial cable segments of characteristic impedance $Z_1 = 75\ \Omega$ (RG-59/U) with segments of characteristic impedance $Z_2 = 50\ \Omega$ (RG-58/U), similar to the configuration used in earlier studies [14,19,20]. The periodic structure formed by the alternating impedances operates on the same principle as the multilayer dielectric structures used in many tunneling time experiments [11–13]: waves whose frequencies lie within a certain stop band are severely attenuated as a result of coherent Bragg reflections. The stop band is centered at the Bragg condition where the period Λ of the structure equals one-half the electromagnetic wavelength within the medium. Here the 10-m period of our structure creates a stop band centered at 10 MHz with a width of 2.86 MHz. This makes it possible to study the spatiotemporal evolution of tunneling pulses on a meter length scale with microsecond time scale pulses. The 5-m segments were further subdivided into sections of 1.25 m joined by T connectors to enable measurement of the electric signal inside the barrier with increased spatial resolution. To simplify analysis the barriers were made symmetric so that each one started and ended with a 75 Ω segment. A 1.5-unit-cell barrier is therefore composed of a 75 Ω cable followed by a 50 and a 75 Ω segment. Measurements were carried out on uniform lengths of both types of cables to determine the phase velocity and spectral dependence of the attenuation. In the frequency range of interest, the phase velocity in both segments was 0.66c and the attenuation in the cables varied from 5.1×10^{-3} to 6.5×10^{-3} m^{-1}. Tunneling experiments were done with 10 MHz electric sinusoidal carrier waves with a 2 μs full width at half maximum (FWHM) Gaussian envelope produced by an HP33120A signal generator. The pulse duration was so chosen as to limit the spectral width of the wave to 0.5 MHz, well within the bandwidth of the stop band. This satisfies the slowly-varying-envelope approximation assumed throughout the rest of this paper and ensures that there is a negligible contribution from spectral components outside the stop band. The spatial extent of the pulse is thus $v\tau_p=396$ m, which greatly exceeds the maximum barrier length of 95 m used in these experiments, thus satisfying the conditions for a quasistatic interaction [15–18]. It should be noted that in all reported observations of distortionless, "superluminal" tunneling, the pulse lengths have exceeded the barrier length [10–14].

B. Input-output measurements

The system was first characterized by measuring the incident, transmitted, and reflected pulses with a detector and digital oscilloscope. The transmission and reflection delays of the 2 μs FWHM Gaussian pulses were obtained by using the center of mass definition of pulse delay [21]:

$$\tau_{c.m.} = \frac{\int t|E(t)|dt}{\int |E(t)|dt}, \qquad (1)$$

where $|E(t)|$ is the amplitude of the electric signal. For symmetric pulses such as the ones used in this experiment, the center of mass delay is the same as the group delay: the time at which the transmitted field reaches a peak, given that the incident field was a maximum at $t=0$. The delays obtained by this method agreed with delays calculated by taking the frequency derivative of the transmission and reflection phase shifts:

$$\tau_{gt} = \frac{\partial \phi_t}{\partial \omega}, \quad \tau_{gr} = \frac{\partial \phi_r}{\partial \omega}. \qquad (2)$$

Figure 1 shows the incident and transmitted pulses for a 9.5-unit-cell barrier. (The black bar in the figure indicates the transit time of a light front in a uniform waveguide and shows that the pulse is much longer than this transit time.) The measured group delay was 145 ns. It is seen that there is no distortion of the transmitted pulse. The field amplitude transmission was about 2.6% and was the same for the bulk of the incident pulse. In other words, for signal levels above the noise level every part of the input pulse experienced the same transmission. The transmission group delay is much smaller than the group delay of 480 ns that would be measured in a uniform 75 Ω waveguide. It is this small group delay that is often taken to imply that electromagnetic pulses travel with superluminal group velocity through the stop band of a periodic structure. This inference is made by taking the length of the structure and dividing by the delay: "v_g" $=L/\tau_g$. This quantity represents a meaningful velocity only if the group delay is an actual traversal time. For the particular structure used here, the observed delay, if taken as a propagation delay, would mean a "group velocity" of 2.2c. This classical notion of group velocity requires that an identifiable object actually propagates from A to B, passing through every point in between. The identifiable object in this case is the incident electromagnetic pulse and its spatial position is marked by its peak. The question then is whether an identifiable pulse peak actually propagates through the barrier.

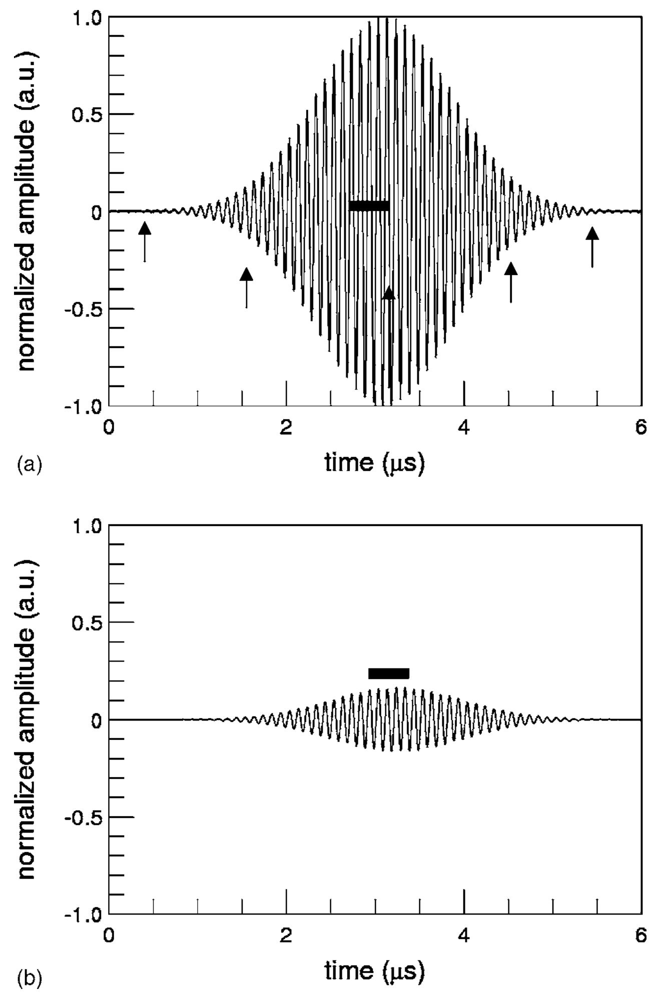

FIG. 1. (a) Incident Gaussian pulse. The arrows indicate points within the temporal profile at which snapshots are taken of the intrabarrier field distribution. The black horizontal bar in the middle of the pulse represents the transit time (0.48 μs) of a light front through a uniform cable. (b) Transmitted pulse. The transmitted field attains a peak 145 ns after the incident pulse reaches a peak. The transmitted pulse is undistorted and there is no observable reshaping or shortening.

C. Space-time-resolved measurements inside the barrier

To investigate this question, measurements were made inside the barrier (with a 1.25 m spatial resolution) by trans-

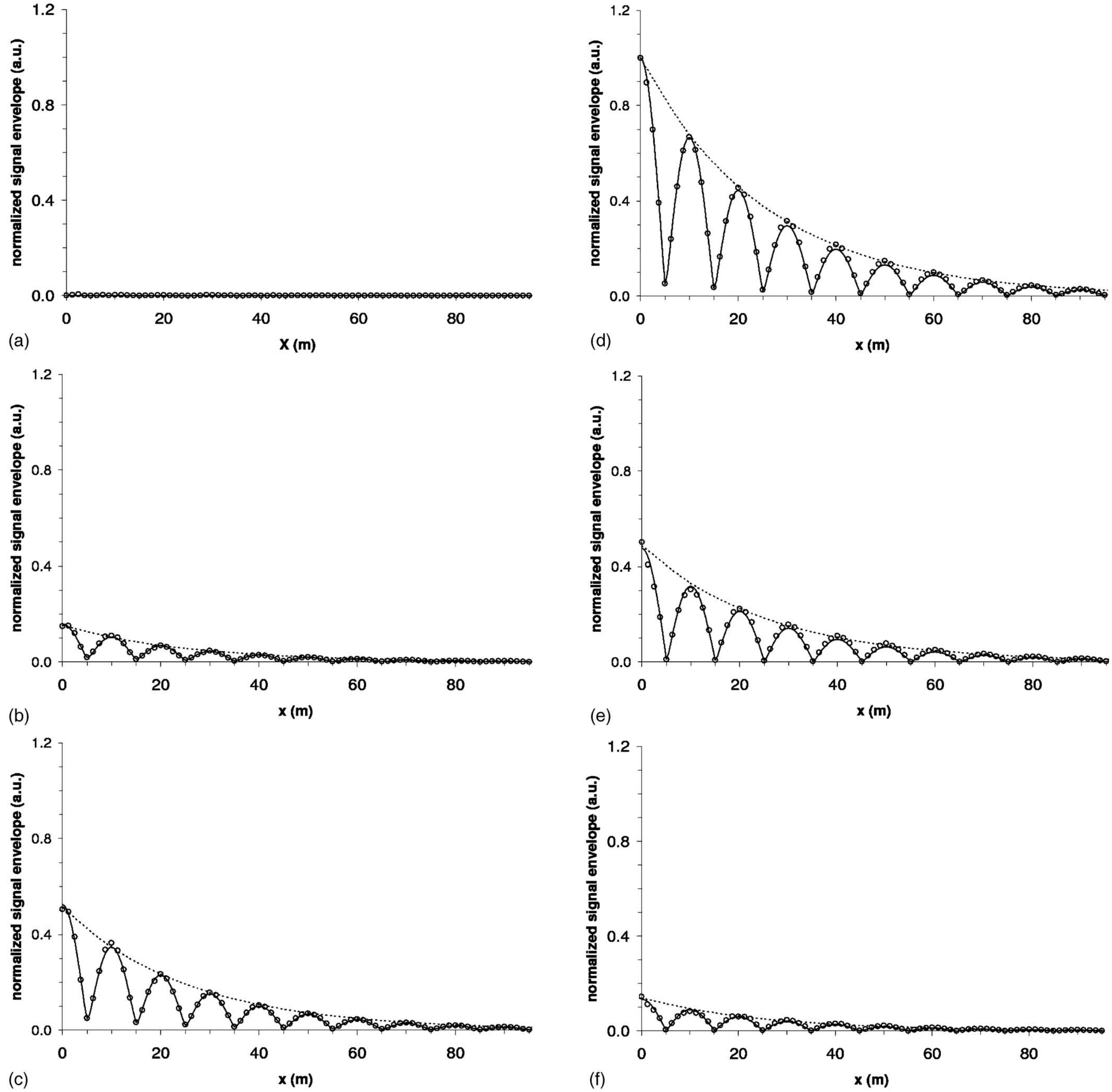

FIG. 2. Theoretical (solid line) and experimental (empty circles) time evolution of the signal envelope inside a 9.5-unit-cell barrier. The signal is composed of a 2 μs FWHM Gaussian envelope with a 10 MHz carrier wave. The snapshots are taken at the following time instants, measured from the peak of the incident pulse: (a) −5, (b) −2.5, (c) −1.5, (d) 0, (e) 1.5, and (f) 2.5 μs. The dashed lines are exponential curves that represent the evanescent behavior of the signal.

lating the detector along the waveguide segments for several time instants within the input pulse envelope. An external TTL trigger was used to guarantee that each measurement was within the same temporal window. These multiple traces at different points therefore allow the recreation of the signal profile inside the barrier as a function of time. Measurements were made for various time instants before the peak, at the peak, and after the peak of the incident pulse. Figure 2 shows the envelope dynamics of a single pulse tunneling through a 9.5-unit-cell barrier. The minima and maxima are the nodes and antinodes of the standing wave formed by forward- and backscattered waves, the period corresponding to one-half the wavelength of the carrier. The dots are the experimental points and the solid curve is the theoretical result obtained by numerically propagating the Fourier components of the input pulse through the periodic structure [19]. The agreement is excellent, with no adjustable parameters. The envelope of the standing wave is accurately fitted by an exponential function $\exp(-\kappa z)$ with $\kappa=0.039\ \text{m}^{-1}$. During the tunneling process, this envelope slowly rises and falls, following the temporal variations in the incident pulse envelope. It is clear that what is seen here has nothing to do with pulse "propagation," the transport of a spatially localized excitation. The pulse envelope does not propagate through the barrier. It simply modulates the amplitude of the quasi-standing-wave, thus determining how much of the carrier will be transmitted at the exit. There is no sense in which the incident pulse peak has propagated to the exit. (For short pulses whose bandwidth exceeded the band gap, pulse breakup occurred and multiple peaks were seen to propagate through the barrier. However, such propagating peaks do not represent tunneling pulses.)

A commonly used explanation for the short group delays observed in barrier tunneling is the "reshaping" argument, which claims that the early part of the pulse is transmitted while the later part is rejected, resulting in a forward shift of the peak of the pulse [3,7]. By that argument the barrier acts essentially as a time-dependent shutter. Our measurements do not support this view. We see here that the transmission is the same for all parts of the incident pulse, within the resolution of our detection system. Tunneling requires that the interference process that leads to Bragg reflection be established before the bulk of the pulse arrives. In our experiment we see that the standing wave due to Bragg reflection is already established as early as -2.5 μs before the peak of the incident pulse. The location of that point within the pulse is shown by the first arrow in Fig. 1(a) while the standing wave is displayed in Fig. 2(b). Obviously, the formation of the standing wave is already complete in the distant wings of the incident pulse. Theory shows that it occurs within one transit time (of a light front) after the turn on of a pulse [17]. Once this interference is set up, all parts of the delayed incoming pulse experience the same steady-state transmission. That is why the pulse is neither distorted nor shortened, as would have been expected if the barrier were acting as a time-dependent shutter.

A corollary of the "pulse reshaping" argument is that the entire transmitted pulse is carved out of a tiny leading edge of the incident pulse [3,7,11]. This is also not supported by our experimental results. The group delay is a very small fraction of the pulse length. In our case the group delay is of order 145 ns while the incident and transmitted pulses have a width of 2 μs, 10 times the delay. Because of this very short delay, incident and transmitted pulses practically overlap in time. There is no way that the entire transmitted pulse could have been created by just the leading edge, given that the $1/e$ lifetime of any energy from the leading edge that is stored in the barrier is only 145 ns, while the transmitted pulse has a width (FWHM) of 2 μs. It is clear then that the entire incident pulse, not just its leading edge, contributes to the transmitted pulse.

Although numerical solution of the propagation equations yields exact results for the electric field spatial evolution, an approximate approach based on coupled-mode theory [15] provides substantial physical insight and gives results that are in reasonable agreement with the experimental data. A very important parameter in the coupled-mode theory is the coupling constant κ. Within the stop band the field decays approximately as $\exp(-\kappa z)$ except near the exit, where it deviates from the simple exponential behavior in order to match the boundary conditions. From coupled-mode theory

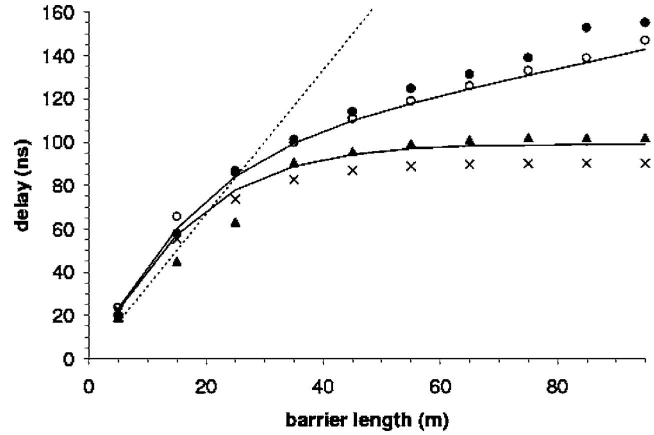

FIG. 3. Experimental dwell time (crosses), reflection group delay (triangles), and transmission group delay (circles). The filled circles were obtained by computing the center of mass of the signals while the empty circles were obtained from the frequency derivative of the measured transmission phase shift. The solid curves represent the theoretical results obtained from the stepped-impedance transmission line model with loss included. The dashed line represents the delay of a signal traveling at the nominal phase velocity of 0.66c on a uniform transmission line. In the absence of loss all the curves collapse into one curve that saturates with distance.

the half-width of the stop band in a lossless periodic structure is $\Omega_c = \kappa v$ rad/s, where v is the phase velocity in the unperturbed waveguides. Using the phase velocity of 0.66c we find a half bandwidth of $f_c = 1.26$ MHz. This is in good agreement with the measured half bandwidth of 1.43 MHz. The limiting group delay for a lossless barrier is given by $1/\Omega_c = 126$ ns, also in reasonable agreement with experiment. The coupling constant can, of course, be calculated directly from the impedances of the transmission line segments. The procedure is exactly the same as for multilayer dielectric structures [22] and we find the result

$$\kappa = \frac{2}{\Lambda} \frac{|Z_2 - Z_1|}{Z_2 + Z_1}. \quad (3)$$

Using the cable impedances and period we find $\kappa = 0.04$ m^{-1}, again in agreement with the experimental decay rate of the field envelope.

D. Direct measurement of dwell time

The ability to measure the spatial distribution of the field in the barrier means that we can directly measure the energy stored and determine how it varies with barrier length. This makes it possible to measure directly the dwell time in tunneling, a quantity defined as [2,3,15]

$$\tau_d = \frac{\langle U \rangle}{P_{in}}, \quad (4)$$

where $\langle U \rangle$ is the stored energy and P_{in} is the input power. By integrating under the measured profiles we obtained the stored energy and hence the dwell time for several barrier lengths. The results shown in Fig. 3 (crosses) represent our actual measurement of the dwell time in barrier tunneling. Experimentally, we find that the stored energy and the dwell time saturate with barrier length. The reflection group delay also saturates as shown. The transmission group delay, however, does not quite saturate with barrier length, a circumstance attributable to the presence of loss in the barrier. It increases with barrier length but at a slower rate than the rate of increase in a uniform waveguide. Inclusion of loss in the theory yields excellent agreement with the experimental results. The presence of absorption reduces the effect of the interferences that lead to Bragg reflection and the establishment of a barrier. With increased loss, the group delay should approach that of a uniform waveguide and thus increase with length. A similar effect attributable to loss has been seen in the tunneling of acoustic waves through phononic band gaps [23]. The reflection group delay, however, will still saturate since the contributions to the reflected wave vanish after a certain decay distance. The periodic structure with absorption can thus be seen as a combination of a shortened perfect Bragg reflector and a uniform but absorptive waveguide without reflections. The portion of the input that enters the uniform waveguide makes no contribution to the reflected wave. The dwell time still saturates in the presence of a moderate amount of loss since the barrier can only store a finite amount of energy. It is surprising that even though there have been numerous measurements of group delay in barrier tunneling, the dwell time had never been measured (to the best of our knowledge) until the work reported here.

III. DISCUSSION

In the tunneling process an exponentially decaying quasistanding-wave (with imperfect nodes) is set up. Our experiment permits us to directly observe the spatiotemporal evolution of these standing waves. When a structure (such as a cavity resonator) supports a standing wave, the entire structure pulsates and throbs as a single unit. When this standing wave is modulated, it can follow the modulation adiabatically so long as the time scale of the modulation is long compared to the transit time of a light front within the structure. In tunneling, the slow envelope of the incident pulse modulates this standing wave. There is a small time lag in the modulation response because the standing wave is in contact with boundaries that permit energy to leak out of the structure, which energy must then be replenished. The group delay measures the $1/e$ lifetime of this stored energy escaping through both ends [18]. This delay is not a transit time.

It is widely believed that wave packets tunnel with superluminal group velocity through photonic and quantum barriers. The evidence for these rather large velocities is based on measurements of *group delay*. These group delays are then used to calculate a group velocity through $v_g = L/\tau_g$. However, to properly speak of a group velocity, the wave packet must be localized with respect to the spatial extent of the medium within which it propagates. This is *never* the case in tunneling, which has been shown to be a quasistatic process requiring wave packets long compared to the barrier's characteristic length [16,17]. Because the barrier is short compared to the wave packet, it acts as a lumped element with respect to the envelope. In that regard it is very much like a lossy capacitor characterized by an *RC* time constant. This *RC* time constant is of course not a traversal time and one would not use it to calculate a "group velocity" by dividing the capacitor length by this time constant. By the same token, while group delay is a perfectly valid concept for a barrier as a measure of the lifetime of stored energy, "group velocity" has no meaning since the delay is not a traversal time. Our experiment confirms that the group delay in tunneling is not a traversal time. In the absence of loss, the group delay for our photonic barrier is identical to the dwell time, which is the lifetime of stored energy and is certainly not a transit time. Thus, tunneling-time experiments measure photon lifetimes and not transit times. In all cases the wave packet is much longer than the reflective barrier and hence we are dealing with the quasistatic excitation and decay of a cavity rather than the propagation of a localized (with respect to the barrier extent) pulse.

IV. CONCLUSION

In conclusion, we have presented space-time-resolved measurements of pulse tunneling through an electromagnetic band gap. The results confirm the quasistatic interpretation of tunneling dynamics, in which the output envelope adiabatically follows the input with a small delay due to energy storage. The incident peak does not actually propagate to the exit which means that the tunneling group delay is not a traversal time and cannot be used to assign a group velocity. We have also presented a direct measurement of dwell time in barrier tunneling, something made possible by our ability to explore directly the intrabarrier field and stored energy. Finally, we find no evidence for the pulse reshaping which has been suggested as a mechanism for superluminal tunneling. Our results represent a contribution to the understanding of tunneling time for evanescent waves in optics, electromagnetics, and acoustics, as well as for matter waves in quantum barriers.

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