Entangled light via nonlinear vacuum-multiparticle interactions

Mihai Macovei[1,*] and Gao-xiang Li[2,†]
[1]*Max-Planck Institute for Nuclear Physics, Saupfercheckweg 1, D-69117 Heidelberg, Germany*
[2]*Department of Physics, Huazhong Normal University, Wuhan 430079, China*
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We investigate the generation of a strongly entangled electromagnetic field through laser pumping a collection of N two-level atoms in a two-mode optical resonator. The vacuum-multiparticle interactions enhance the coupling of the atomic sample to the cavity modes facilitating the creation of entangled photons at higher frequencies. In the dispersive atom-cavity limit, one can obtain a steady-state output field consisting of tens of such photons per mode.

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I. INTRODUCTION

Nonlinear interactions play an essential role to entangle light or matter wave modes as, for instance, via spontaneous parametric down-conversion processes, multiwave mixing phenomena or very intense laser-vacuum interactions [1–9]. Potential applications of entanglement in quantum information processing make the topics attractive and widely investigated [10]. Therefore, various new alternative experiments are performed to generate highly entangled light. In particular, the electromagnetically induced transparency has been used to generate narrow-band entangled photons [11] while a four-wave parametric interaction in a two-level system was shown to generate highly nonclassical light [12]. An experimental demonstration of continuous variable entanglement using cold atoms in a high finesse optical cavity was presented as well [13]. The Kerr nonlinearity was exploited to generate two-mode entangled light in an optical fiber [14] or cavities [15], respectively. Note that earlier experimental demonstrations of nonclassical light properties include antibunched light [16], sub-Poissonian photon statistics [17], or squeezing [18].

Methods on how to entangle a pair of two-level emitters or more received much attention as well [19–26]. Entanglement of two particles inside an atomic cloud was demonstrated in Ref. [19] while an entangled two-atom system can be created via an overdamped optical cavity [20]. How to entangle two distant atomic ensembles was shown in Ref. [21]. The motion of $2N$ ions trapped in two separate single mode cavities can be entangled via a two-mode squeezed vacuum field [22]. Schemes to produce entangled motional states for the two trapped ions in one or two cavities were proposed too [23]. Interestingly, two initially entangled and afterward not interacting qubits can become completely disentangled in a finite time [24]. However, dark periods and revivals of entanglement in a two-qubit system was shown to occur via environmental vacuum modes [25]. In addition, long-time entanglement between two arbitrary qubits can be generated if they interact with a common thermal bath [26]. Note, however, that in spite of a large amount of contributions to the subject still there are discussions on quantifying entanglement. Therefore various criteria were formulated or used to deal with the problem. Continuous variable entanglement [27,28], violation of Bell or Cauchy-Schwartz inequalities [29,30] and entanglement of formation [31] are among them.

Here we investigate the possibility to generate continuous variable two-mode electromagnetic field (EMF) entanglement via laser-driving a collection of two-level atoms in a two-mode cavity. The atoms interact collectively via the surrounding electromagnetic vacuum modes. In the good-cavity limit the atomic subsystem reaches its steady-state faster than the field variables and it can be eliminated to obtain a master equation describing the cavity field modes only. Then the equations of motion for the field variables are obtained and, in the dispersive atom-cavity limit, the conditions to arrive at the cavity-field steady-state are discussed. Below the threshold the generated two-mode EMF consists of tens of highly correlated photons per mode. We demonstrate that the steady-state output two-mode EMF exhibits quantum features, i.e., continuous variable entanglement.

The paper is organized as follows. In Sec. II we introduce the model as well as the analytical formalism. Section III describes the two-mode entanglement while its existence is proved in Sec. IV. We finalize the article with Sec. V.

II. APPROACH

We consider a collection of laser-pumped two-level atoms, possessing the transition frequency ω_0, and interacting with a two-mode optical cavity (ω_1,ω_2) via a four-wave mixing process. The excited particles may decay spontaneously during the transitions $|2\rangle \leftrightarrow |1\rangle$, with a decay rate γ, due to interaction with the environmental vacuum modes. The collisional damping rate of atoms which alter the phase of the atomic state but not its population is given by γ_c. We shall describe the laser-atom system in the dressed-state representation of a single particle [32,33]: $|2\rangle = \cos\theta|+\rangle - \sin\theta|-\rangle$ and $|1\rangle = \cos\theta|-\rangle + \sin\theta|+\rangle$ where $\cot 2\theta = \Delta/(2\Omega_0)$. Here $\Delta = \omega_0 - \omega_L$ is the detuning of the laser frequency ω_L from the atomic transition frequency ω_0 while Ω_0 is the Rabi frequency describing the strength of the atom-laser interaction.

*On leave from Technical University of Moldova and Institute of Applied Physics, Chişinău, Moldova. mihai.macovei@mpi-hd.mpg.de
†gaox@phy.ccnu.edu.cn

In the mean-field, dipole, Born-Markov and secular approximations the system is characterized by the following master equation:

$$\dot{\rho} + \frac{i}{\hbar}[H,\rho] = -\gamma_0[R_z,R_z\rho] - \gamma_+[R_{+-},R_{-+}\rho] - \gamma_-[R_{-+},R_{+-}\rho] - \sum_{i\in\{1,2\}}\kappa_i[a_i^\dagger,a_i\rho] + \text{H.c.}, \quad (1)$$

where $H=H_f+H_a+H_{\rm int}$ with $H_f=\hbar\Delta_2 a_2^\dagger a_2-\hbar\Delta_1 a_1^\dagger a_1$ being the free electromagnetic field modes Hamiltonian, $H_a=\hbar\Omega R_z$ is the free Hamiltonian of the laser-dressed atoms, and

$$H_{\rm int} = \{(F\cos^2\theta - F^\dagger \sin^2\theta)R_{+-} + \text{H.c.}\} + (F+F^\dagger)R_z \sin 2\theta/2 \quad (2)$$

describes the interaction of the cavity modes with the dressed atomic sample. Here $\Delta_1=\omega_L-\omega_1$, $\Delta_2=\omega_2-\omega_L$, $\Omega=\sqrt{\Omega_0^2+(\Delta/2)^2}$, and $F=\hbar(g_1a_1+g_2a_2)$, where g_i are the atom-cavity couplings, while a_i $[a_i^\dagger]$ are the cavity photon annihilation [creation] operators, and $[a_i,a_l^\dagger]=\delta_{il}$, $[a_i,a_l]=[a_i^\dagger,a_l^\dagger]=0$, $\{i,l\in 1,2\}$. The collective dressed-state atomic operators $R_{\alpha\beta}=\sum_{j=1}^N R_{\alpha\beta}^{(j)}=\sum_{j=1}^N|\alpha_j\rangle\langle\beta_j|$ describe the transition between the dressed states $|\beta\rangle$ and $|\alpha\rangle$ for $\alpha\neq\beta$ and population for $\alpha=\beta$ $\{\alpha,\beta\in +,-\}$, and obey the standard commutation relations of su(2) algebra. $R_z=R_{++}-R_{--}$ is the dressed-state inversion operator. The quantum dissipations due to spontaneous emission into surrounding electromagnetic field modes as well as collisional damping are described by the terms proportional to $\gamma_0=(\gamma\sin^2 2\theta+\gamma_c\cos^2 2\theta)/4$, $\gamma_+=\gamma\cos^4\theta+\gamma_c\sin^2 2\theta/4$ and $\gamma_-=\gamma\sin^4\theta+\gamma_c\sin^2 2\theta/4$, respectively. The last term in Eq. (1) characterizes the damping of the cavity modes with κ_i being the field decay rates of the mode $i\in\{1,2\}$.

We assume an intense pumping field, i.e., $\Omega\gg\{N\gamma, g_{1,2}\sqrt{N}\}$, and a high quality cavity such that $\gamma N\gg\kappa_{1,2}$. In this case the atomic subsystem achieves its steady state on a time scale faster than the cavity field and, thus, the atomic variables can be eliminated to arrive at a master equation for the cavity field modes alone:

$$\dot{\rho} - i\delta\sum_{i\in\{1,2\}}[a_i^\dagger a_i,\rho] = -\sum_{i\in\{1,2\}}[A_i(\rho a_i a_i^\dagger - a_i^\dagger\rho a_i) + \bar{B}_i(a_i^\dagger a_i\rho - a_i\rho a_i^\dagger)] + \sum_{i\neq j\in\{1,2\}}[C_i(a_j^\dagger a_i^\dagger\rho - a_i^\dagger\rho a_j^\dagger) + D_i(\rho a_i^\dagger a_j^\dagger - a_j^\dagger\rho a_i^\dagger)] + \text{H.c.} \quad (3)$$

Here $\delta=(\Delta_2-\Delta_1)/2$, $\bar{B}_i=B_i+\kappa_i$ $\{i\in 1,2\}$ and

$$A_1 = \frac{Ng_1^2}{\gamma}[\chi_0(\bar{\Delta}_1) + \sin^4\theta\chi_-(\bar{\Delta}_1) + \cos^4\theta\chi_+(\bar{\Delta}_1)],$$

$$B_1 = \frac{Ng_1^2}{\gamma}[\chi_0(\bar{\Delta}_1) + \cos^4\theta\chi_-^*(-\bar{\Delta}_1) + \sin^4\theta\chi_+^*(-\bar{\Delta}_1)],$$

$$C_1 = \frac{Ng_1g_2}{4\gamma}\{\sin^2 2\theta[\chi_-^*(\bar{\Delta}_1) + \chi_+^*(\bar{\Delta}_1)] - 4\chi_0(-\bar{\Delta}_1)\},$$

$$D_1 = \frac{Ng_1g_2}{4\gamma}\{\sin^2 2\theta[\chi_-(-\bar{\Delta}_1) + \chi_+(-\bar{\Delta}_1)] - 4\chi_0(-\bar{\Delta}_1)\}. \quad (4)$$

Further, A_2, B_2, C_2, and D_2 can be obtained from A_1, B_1, C_1, and D_1 by replacing $\bar{\Delta}_1$ with $-\bar{\Delta}_2$, $\{\Delta_1,\Delta_2\neq 0\}$, and $g_1\leftrightarrow g_2$, respectively. Other parameters are

$$\chi_\mp(z) = \frac{\langle R_{\mp\pm}R_{\pm\mp}\rangle_s/N^2}{\bar{\Gamma}_\perp \mp i(2\bar{\Omega}\mp z)}, \quad \chi_0(z) = \frac{\langle R_z^2\rangle_s/4N^2}{\bar{\Gamma}_\parallel + iz}\sin^2 2\theta,$$

$\bar{\Gamma}_m=(\Gamma_m^{(s)}-\cos 2\theta\langle R_z\rangle_s)/N$ $\{m\in\parallel,\perp\}$, with $\Gamma_\parallel^{(s)}=1+\cos^2 2\theta+\gamma_c\sin^2 2\theta/\gamma$ and $\Gamma_\perp^{(s)}=1+\sin^2 2\theta/2+\gamma_c(1+\cos^2 2\theta)/(2\gamma)$ being the corresponding single-particle decay rates, and scaled parameters were introduced, i.e., $\bar{\Omega}=\Omega/(N\gamma)$, $\bar{\Delta}_i=\Delta_i/(N\gamma)$ $\{i\in 1,2\}$. Note that to obtain Eq. (3) we decoupled the involved multiparticle correlators—an approximation valid for larger N, i.e., $N\gg 1$ [33]. The corresponding equation for $N=1$ (or independent atoms) is identical to Eq. (3) but with single-atom decay rates $\{\Gamma_\parallel^{(s)},\Gamma_\perp^{(s)}\}$ instead of collective ones. The steady-state expectation values for the atomic correlators entering into the above expressions can be estimated from the steady-state solution of the master equation describing the strongly pumped atoms in the absence of the cavity, i.e.,

$$\rho_s^{(0)} = Z^{-1}\exp[-\xi R_z], \quad (5)$$

where $2\xi=\ln(\gamma_+/\gamma_-)$ and Z is chosen such that $\text{Tr}\{\rho_s^{(0)}\}=1$. Considering an atomic coherent state $|n\rangle$ denoting a symmetrized N-atom state in which $N-n$ particles are in the lower dressed state $|-\rangle$ and n atoms are excited to the upper dressed state $|+\rangle$, and that $R_{-+}|n\rangle=\sqrt{n(N-n+1)}|n-1\rangle$, $R_{+-}|n\rangle=\sqrt{(N-n)(n+1)}|n+1\rangle$, and $R_z|n\rangle=(2n-N)|n\rangle$ one can calculate the expectation values of any atomic correlators of interest [32,33] as soon as $\Omega\gg\{N\gamma,g_{1,2}\sqrt{N}\}$ and $N\gamma\gg\kappa_{1,2}$.

The physical meaning of the parameters in Eq. (3) is as follows. A $(\bar{B})$ describes the process of increasing (decreasing) of the photon number into cavity modes as well as the Stark shift. On the other hand, C and D characterize the creation and annihilation of a photon into each cavity mode as well as their correlations induced by the four-wave mixing effect.

Finally, our system can be implemented in principle using pumped atomic fluxes passing through a high-quality cavity or atomic ensembles trapped inside such resonators. We note the existence of a number of recent experiments dealing with many particles inside an optical cavity. For instance, generation of squeezed states of the electromagnetic field by a collection of atoms within a high-finesse cavity was reported in Ref.[34]. Observation of normal-mode splitting for smaller as well as larger two-state atomic ensembles was described in Refs. [35] and [36], respectively. Anharmonicities of the vacuum Rabi peaks in a many-atom system was experimentally observed as well [37]. Other systems could be driven solid state media inside optical cavities as the superradiant emission of a thin solid sample in an optical resonator was already observed [38].

In what follows, we shall analyze in detail the quantum nature of the cavity electromagnetic field generated by strongly pumping a collection of two-level emitters.

III. TWO-MODE ENTANGLEMENT

As was mentioned in the Introduction, the question whether a particular system is entangled or not is still open. However a complex system ($s1$ and $s2$) is entangled if the join density operator ρ can not be written as: $\rho = \Sigma_n p_n \rho_n^{(s1)} \otimes \rho_n^{(s2)}$ with $p_n \geq 0$ and $\Sigma_n p_n = 1$. In order to establish the existence of entanglement between the two cavity modes, both initially in the vacuum state, we shall consider a sufficient criteria for continuous variable entanglement as that proposed in Refs. [27] and [28] since the field inside the resonator is in a two-mode Gaussian state.

We define two field operators $\hat{u} = b\hat{x}_1 - (1/b)\hat{x}_2$ and $\hat{v} = b\hat{p}_1 + (1/b)\hat{p}_2$ with the quadratures $\hat{x}_j = (a_j + a_j^\dagger)/\sqrt{2}$, and $\hat{p}_j = -i(a_j - a_j^\dagger)/\sqrt{2}$ ($j \in \{1,2\}$), respectively. The sum of quantum fluctuations is denoted as: $\Sigma = \langle (\Delta\hat{u})^2 \rangle + \langle (\Delta\hat{v})^2 \rangle$. Then, one can show that $\Sigma = 2nb^2 + 2m/b^2 - 4|c|$ where b is a state dependent (nonzero) real-number while $n = \langle a_1^\dagger a_1 \rangle + 1/2$, $m = \langle a_2^\dagger a_2 \rangle + 1/2$, and $c = |\langle a_1 a_2 \rangle|$. The cavity modes are entangled if and only if the quantity Σ obeys

$$\Sigma < b^2 + 1/b^2, \tag{6}$$

and, thus, the entanglement condition of the cavity field can be represented as

$$E = 2nb^2 + 2m/b^2 - 4|c| - b^2 - 1/b^2 < 0, \tag{7}$$

where $b^2 = \sqrt{(2m-1)/(2n-1)}$. It is actually evident that entanglement occurs if

$$E = 4\left(\sqrt{\langle a_1^\dagger a_1 \rangle \langle a_2^\dagger a_2 \rangle} - |\langle a_1 a_2 \rangle|\right) < 0, \tag{8}$$

which indicates the quantum nature of the cavity electromagnetic field.

The equations of motion for the field correlators in Eq. (8) can be obtained with the help of Eq. (3) and using the property $\langle Q \rangle = \mathrm{Tr}\{Q\rho\}$:

$$\frac{d}{dt}\langle a_1^\dagger a_1 \rangle = (\alpha_1 + \alpha_1^*)\langle a_1^\dagger a_1 \rangle + (\eta_2 \langle a_1^\dagger a_2^\dagger \rangle + \text{c.c.}) + \beta_1,$$

$$\frac{d}{dt}\langle a_2^\dagger a_2 \rangle = (\alpha_2 + \alpha_2^*)\langle a_2^\dagger a_2 \rangle + (\eta_1 \langle a_1^\dagger a_2^\dagger \rangle + \text{c.c.}) + \beta_2,$$

$$\frac{d}{dt}\langle a_1^\dagger a_2^\dagger \rangle = (\alpha_1^* + \alpha_2^*)\langle a_1^\dagger a_2^\dagger \rangle + \eta_2^* \langle a_2^\dagger a_2 \rangle + \eta_1^* \langle a_1^\dagger a_1 \rangle + \beta_3^*,$$

$$\frac{d}{dt}\langle a_1 a_2 \rangle = (\alpha_1 + \alpha_2)\langle a_1 a_2 \rangle + \eta_2 \langle a_2^\dagger a_2 \rangle + \eta_1 \langle a_1^\dagger a_1 \rangle + \beta_3, \tag{9}$$

where $\alpha_i = A_i - \bar{B}_i - i\delta$, $\beta_i = A_i + A_i^*$, $\eta_i = C_i - D_i$ ($i \in 1,2$), and $\beta_3 = C_1 + C_2$.

In the next section we shall discuss the steady-state entanglement of the two cavity modes using Eqs. (5), (8), and (9).

IV. RESULTS AND DISCUSSIONS

To elucidate the role the collectivity plays to entangles the two field modes we shall consider further that $g_1 = g_2 \equiv g$, $\kappa_1 = \kappa_2 \equiv \kappa$ and $|\Delta_1| \approx |\Delta_2| \gg g\sqrt{N}$ so that $|\Delta_1 - \Delta_2| \sim \kappa$. We assume also that $|\bar{\Delta}_{1(2)}| \gg \bar{\Gamma}_\parallel$ and $|2\bar{\Omega} \pm \bar{\Delta}_{1(2)}| \gg \bar{\Gamma}_\perp$. Then the steady-state expectation values for the field correlators in Eqs. (9) are

$$\langle a_1^\dagger a_1 \rangle = \langle a_2^\dagger a_2 \rangle = \frac{(\bar{g}/\sqrt{2}\kappa)^2}{1 + [(\delta + \delta_s)/\kappa]^2 - (\bar{g}/\kappa)^2},$$

$$\langle a_1 a_2 \rangle = [\langle a_1^\dagger a_2^\dagger \rangle]^* = \frac{\bar{g}/2\kappa}{1 + [(\delta + \delta_s)/\kappa]^2 - (\bar{g}/\kappa)^2} \times [(\delta + \delta_s)/\kappa + i]. \tag{10}$$

Here the effective coupling of the dressed atomic system to the cavity modes, i.e., $\bar{g}$, and the frequency shift δ_s due to the dispersive atom-cavity interaction are represented as follows:

$$\bar{g} = \frac{g^2 \bar{\Omega} \langle R_z \rangle_s / (N\gamma)}{4\bar{\Omega}^2 - \bar{\Delta}_1^2} \sin^2 2\theta,$$

$$\delta_s = \frac{g^2 \bar{\Omega} \langle R_z \rangle_s / (N\gamma)}{4\bar{\Omega}^2 - \bar{\Delta}_1^2} (2 - \sin^2 2\theta). \tag{11}$$

To reach the cavity-field steady-state these parameters must satisfy the inequality $(\bar{g}/\kappa)^2 < 1 + [(\delta + \delta_s)/\kappa]^2$. The frequency shift δ_s can be compensated by slightly modifying δ, i.e., $(\delta + \delta_s)/\kappa = 0$, to arrive at $\bar{g}/\kappa < 1$ or

$$\frac{(g^2/\kappa\gamma)\langle R_z \rangle_s / N}{(\Omega_0/N\gamma)[3 + 4(\Delta/2\Omega_0)^2]\sqrt{1 + (\Delta/2\Omega_0)^2}} < 1, \tag{12}$$

if $|\Delta_1| = \Omega_0$ [see Eq. (11)]. This condition is satisfied for single-atom systems as well as for collectively interacting particles. However for single-particle systems a much higher coupling g of the atom to the cavity modes is required. The reason is that the effective coupling $\bar{g}$ is proportional to the dressed-state inversion operator $\langle R_z \rangle_s$ and decreases for larger values of $\Delta/(2\Omega_0)$. The dressed quasilevels are equally populated if $\Delta/(2\Omega_0) = 0$, that is $\langle R_z \rangle_s = 0$ and, thus, $\bar{g} = 0$. The population can be transferred between the dressed states by modifying $\Delta/(2\Omega_0) \neq 0$ leading to $\bar{g} \neq 0$ for strong couplings g only. For instance, in order to fulfill the condition $|\bar{g}/\kappa| < 1$, i.e., Eq. (12), for single-atom systems in the whole frequency range $-1 \leq \Delta/(2\Omega_0) \leq 1$ then one should approximately have $g^2/(\kappa\Omega_0) < 6.7$ with $\gamma_c/\gamma = 0.2$. The parameter $g^2/(\kappa\Omega_0)$ will be even higher for larger values of γ_c/γ. On the other hand, for $N = 100$, $|\bar{g}/\kappa| < 1$ if $Ng^2/(\kappa\Omega_0) < 3.2$ and this parameter $(Ng^2/\kappa\Omega_0)$ is almost insensitive on γ_c/γ for larger atomic samples. If, for example, one considers pumped small-sized multiparticle atomic ensembles with $N \gg 1$ and $\Omega_0 \gg N\gamma$ in the dispersive limit discussed above then the cooperativity parameter $C = Ng^2/\kappa\gamma$ will be well below 10^4. In principle, atom-cavity couplings with C ranging from few hundreds to several thousands can be achieved experimentally [34,36]. Thus multiparticle inter-

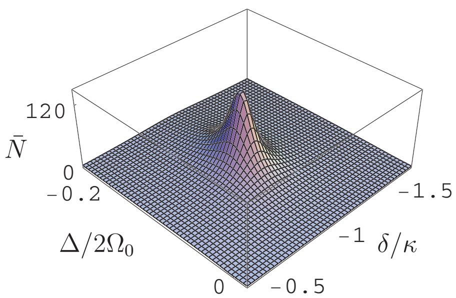

FIG. 1. (Color online) Mean number of entangled photons $\bar{N}$ as a function of δ/κ and $\Delta/2\Omega_0$. Here $|\Delta_1| \approx |\Delta_2| = \Omega_0$, $Ng^2/\kappa\Omega_0 = 3.2$, $\gamma_c/\gamma = 0.2$, and $N = 100$.

actions mediated by the surrounding vacuum reservoir contribute to an effective coupling that is enhanced by the collectivity, i.e., $\sqrt{N}g$. Increasing the effective atom-field coupling through many-atom interactions has a particular importance at optical frequencies to generate strongly entangled photons. Also, for many-particle samples the population can be effectively transferred between the dressed states because $\langle R_z \rangle_s / N \to 1$ $[\langle R_z \rangle_s / N \to -1]$ if $\Delta/(2\Omega_0) < \epsilon$ $[\Delta/(2\Omega_0) > \epsilon]$ with $\epsilon \ll 1$, and on a time scale proportional to N^{-1} for $\epsilon \neq 0$ [33].

Figure 1 shows the average generated cavity photon number, $\bar{N} = \langle a_1^\dagger a_1 \rangle + \langle a_2^\dagger a_2 \rangle$, as function of laser and cavity modes detunings, respectively, according to Eq. (10). Below threshold one has a steady-state output EMF with $\bar{N} \gg 1$. For instance, if $Ng^2/\kappa\Omega_0 = 3.2$, $\gamma_c/\gamma = 0.2$, $N = 100$ and $\delta/\kappa = -1$ then we obtain more than 60 photons per mode in the steady state (see Fig. 1).

Next we shall focus on the quantum properties of the cavity electromagnetic fields. The expression (8) characterizing the continuous variable field entanglement is given now by

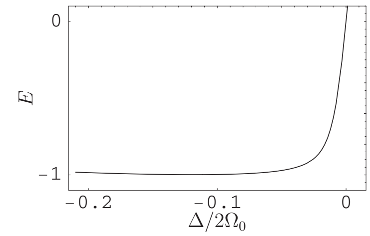

FIG. 2. Entanglement parameter E as a function of $\Delta/2\Omega_0$ for $Ng^2/\kappa\Omega_0 = 3.2$, $\gamma_c/\gamma = 0.2$, $N = 100$, $|\Delta_1| \approx |\Delta_2| = \Omega_0$, and $\delta/\kappa = -1$.

$$E = \frac{2(\bar{g}/\kappa)[\bar{g}/\kappa - \sqrt{1 + [(\delta + \delta_s)/\kappa]^2}]}{1 + [(\delta + \delta_s)/\kappa]^2 - (\bar{g}/\kappa)^2}. \quad (13)$$

If $(\delta + \delta_s)/\kappa = 0$, then $E \to -1$ when $|\bar{g}/\kappa|$ approaches unity. Figure 2 depicts the steady-state behavior of the continuous variable entanglement characteristics E as function of $\Delta/2\Omega_0$ when $\delta/\kappa = -1$ while other parameters are the same as in Fig. 1. One can see that the generated two-mode EMF is highly correlated, i.e., entangled.

V. CONCLUSION

We have investigated the interaction of an ensemble of two-level atoms with a two-mode cavity as well as with an intense driving coherent field. The coupling to the cavity modes is enhanced by the collective effects. In the dispersive atom-cavity limit the nonlinearity entangles the output field.

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