Direct observation of stopped light in a whispering-gallery-mode microresonator

A. A. Savchenkov, A. B. Matsko, V. S. Ilchenko, D. Strekalov, and L. Maleki
Jet Propulsion Laboratory, California Institute of Technology, 4800 Oak Grove Drive, Pasadena, California 91109-8099, USA
(Received 7 August 2006; published 17 August 2007)

We introduce the definition of group velocity for a system with a discrete spectrum and apply it to a linear resonator. We show that a positive, negative, or zero group velocity can be obtained for light propagating in the whispering-gallery modes of a microspherical resonator. The associated group delay is practically independent of the ring-down time of the resonator. We demonstrate "stopped light" in an experiment with a fused silica microsphere.

DOI: 10.1103/PhysRevA.76.023816 PACS number(s): 42.60.Da, 42.25.Bs, 42.25.Gy, 42.25.Hz

The group velocity is usually introduced in a class of problems where responsivity of a material can be considered as a continuous function of frequency. For instance, problems of propagation of narrowband light in systems like atomic media belong to this class [1,2]. However, for the broad class of systems with discrete spectra, the common definition of the group velocity sometimes is not valid. An optical resonator is an example of such a system. We show here that the discreteness of the spectrum brings different features to the notion of the group velocity defined as the velocity of a train of optical pulses; namely, such a train can be delayed by a linear resonator for much longer than the ring-down time of the resonator. Such a delay is impossible for a single pulse interacting with a linear lossless resonator, even though linear resonators as well as their chains can introduce a significant group delay [3–6].

This peculiarity arises when a conceptual transition is made from the framework of a distributed resonator to that of a lumped resonator. While distributed resonators possess an infinite number of modes, only a finite number of modes are usually considered for their spectral studies, and often only a single mode is retained for the sake of simplicity. Such an approximation silently transforms the distributed object to a lumped one, discarding multiple phenomena, one of which is the subject of our study.

Propagation of slow light in a dispersive medium can be characterized by the propagation of a beat note envelope of two plain monochromatic electromagnetic waves [$E_1 = \tilde{E}\exp(-i\omega_1 t + ik_1 z) + \text{c.c.}$ and $E_2 = \tilde{E}\exp(-i\omega_2 t + ik_2 z) + \text{c.c.}$] in the medium [2]. The beat note of the waves is described by $|E_1+E_2|^2 = 2|\tilde{E}|^2\{1+\cos[(\omega_1-\omega_2)t-(k_1-k_2)z]\}$. The velocity of its propagation, $V_g=(\omega_1-\omega_2)/(k_1-k_2)$, corresponds to the conventional definition of the group velocity $\partial\omega/\partial k$ if $\omega_1\to\omega_2$ and the wave vector k is a continuous function of frequency ω.

Let us consider a lumped model of a resonator with a transfer function

$$H(\omega) = \frac{\gamma + i(\omega-\omega_0)}{\gamma - i(\omega-\omega_0)}, \tag{1}$$

where γ is the full width at half maximum and ω_0 is the frequency of the resonance. A monochromatic signal with frequency ω acquires a phase shift $\arg[H(\omega)]$ when passing through the resonator. The resonator delays the beat note of waves E_1 and E_2 by an amount of time $\tau_g(\omega_1,\omega_2) \leqslant 2/\gamma$. The maximum delay is achieved for $\omega_1 \to \omega_2$, and, as a general rule, $\tau_g(\omega_1,\omega_2)(\omega_1-\omega_2) \leqslant 2\pi$. As a result of the above conditions, there is a general belief that the group delay introduced by a linear resonator cannot exceed the ring-down time of the resonator.

In what follows we show that this conclusion is not valid for a distributed resonator such as a microsphere resonator that supports whispering-gallery modes (WGMs). The group delay in such a resonator can exceed its ring-down time significantly. This happens because a WGM resonator belongs to the class of systems with a discrete optical spectrum. Such a resonator can be described using a lumped model within each spectral line. However, the model is not valid if the light interacts with multiple modes. The usual definition of the group velocity $\partial\omega/\partial k$ does not hold in this case. For example, the expression $V_g=(\omega_1-\omega_2)/(k_1-k_2)$ is the only correct definition of the group velocity for a bichromatic field. A similar method should be applied to describe the propagation of a generalized optical field that has a discrete spectrum, e.g., a train of pulses, in a distributed resonator. The spectrum of the field consists of a series of arbitrarily narrowband (for an arbitrarily long train) lines enveloped by the Fourier transform of an individual pulse. The number of "significant" spectral lines that are not too strongly suppressed by the envelope is given by essentially the duty cycle of the pulse train. This number may be just a few for a dense series of smooth (e.g., Gaussian) pulses. The group velocity of the train can be extremely small if the spectrum of the resonator with which the pulses interact is properly engineered.

Let us now turn to a more formal discussion. We present the electric field inside the microsphere resonator as

$$E = \Psi e^{-i\omega t} + \text{c.c.}, \tag{2}$$

where the spatial field distribution has the general form

$$\Psi = \bar{\Psi} P_l^m(\cos\theta) J_{l+1/2}(k_{l,q} r) e^{im\phi}/\sqrt{r}. \tag{3}$$

θ, ϕ, r are the spherical coordinates; the indices l, m, and q determine the spatial distribution of the field, $m=0,1,2,\ldots$ and $q=1,2,\ldots$ are the azimuthal and radial quantization numbers, respectively, and $l=0,1,2,\ldots$ is the orbital mode number. $\Psi(\theta,\phi,r)$ is the mode spatial profile, and

 ©2007 The American Physical Society

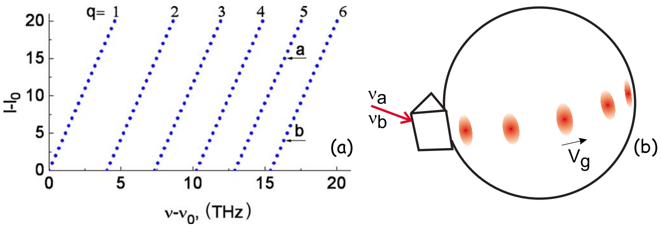

FIG. 1. (Color online) (a) WGM frequencies of a microsphere with radius $R=150$ μm and index of refraction $n=1.4$. The carrier frequency is equal to $\nu_0=478$ THz (wavelength $\lambda=635$ nm), $l_0=2078$. The frequency difference between modes a ($l_a=15$, $q_a=5$) and b ($l_b=4$, $q_b=6$) is equal to $\delta\nu_{ab}=48$ GHz. (b) An interference pattern on the surface of the microsphere is created by modes a and b. We have assumed that the modes are fundamental azimuthal, i.e., $m=l$ for each mode.

$$k_{l,q} \simeq \frac{1}{R}\left[l + \alpha_q\left(\frac{l}{2}\right)^{1/3}\right] \quad (4)$$

is the mode wave number, with $k_{l,q}=\omega n/c$, where n is the index of refraction of the resonator material, R the resonator radius, and α_q the qth root of the Airy function: $\mathrm{Ai}(-\alpha_q)=0$. An example of the spectrum of the resonator evaluated from Eq. (4) is shown in Fig. 1(a).

We now show that the group velocity of light propagating in the microsphere can have any desired value. We assume that a microsphere is excited with a bichromatic light by means of, e.g., a prism coupler [see Fig. 1(b)]. The frequencies of this light are resonant with two whispering-gallery modes, for instance, modes a and b in Fig. 1(a). Both modes have a nonzero electromagnetic field at the surface of the microsphere and, as a result, they interfere. One can observe this interference by covering the microsphere with a fluorescent substance and taking advantage of the interaction of the evanescent field of the modes with the substance. The surface distribution of the power P scattered by the substance is described by

$$P \simeq \widetilde{P}(\theta)[1+\cos(\Delta\omega_{ab}t - \Delta m_{ab}\phi)], \quad (5)$$

where $\widetilde{P}(\theta)$ is a normalization function, $\Delta\omega_{ab}=2\pi\Delta\nu_{ab}$, and $\Delta m_{ab}=l_a-l_b$ when the modes belong to the fundamental azimuthal family. According to Eq. (5), Δm_{ab} is the number of maxima of the interference pattern on the surface of the resonator [Fig. 1(b)]. The pattern moves with a velocity $V_g=R\Delta\omega_{ab}/\Delta m_{ab}$ along the surface of the microsphere. This is the velocity of the beat note of the bichromatic light propagating in the resonator, i.e., the group velocity. Its value is equal to $1.4\times10^{-2}c$ for the selected WGMs (a and b). The value of the group velocity is much smaller when the frequency difference between the modes is small. The group velocity goes to zero if $\omega_a-\omega_b\to 0$; i.e., in this case the light is "stopped" inside the resonator. Finally, the group velocity is negative when $\Delta m_{ab}<0$, or superluminal when $R/\Delta m_{ab}$ is large enough.

We have discussed the propagation of the light field (E) inside the resonator. Let us now find how the group velocity inside the resonator compares to the effective group velocity for the light that has passed through the resonator (the group delay of the field E_{out} with respect to E_{in}). Assuming that the resonator is lossless and that the entrance and exit coupling efficiencies are identical, we infer that each harmonic of the bichromatic light that travels through the resonator experiences a phase shift:

$$E_{out\ a} = E_{in\ a}\exp(i2\pi m_a), \quad (6)$$

$$E_{out\ b} = E_{in\ b}\exp(i2\pi m_b). \quad (7)$$

The beat note of the harmonics acquires a phase shift $\pi(m_a-m_b)$, which corresponds to $2\pi(l_a-l_b)$ for the main mode sequence. Keeping in mind that the light has traveled a distance $2\pi R$, we find the group velocity $V_g=R\Delta\omega_{ab}/(l_a-l_b)$.

The velocity can be either subluminal or superluminal depending on the WGMs that the light interacts with. The group delay $\tau_g=2\pi(l_a-l_b)/\Delta\omega_{ab}$ does not depend on the spectral width of the modes, and consequently their ring-down times, thus proving the assertion made above. It is easy to find now the value of the product $\tau_g\Delta\nu_{ab}=l_a-l_b$, where $2\pi\Delta\nu_{ab}=\Delta\omega_{ab}$. For the example presented in Fig. 1 this value is equal to 11. For the lumped model of the resonator this value is always less than 1.

It is possible to find three nearly equidistant modes in a sphere of a given radius suitable for the delay of a beat note formed by three monochromatic waves resonant with the modes. For instance, using Eq. (4) and assuming that $n=1.4$ and $R=150$ μm, we find that modes with (i) $l_a=1368$, $q_a=1$, (ii) $l_b=1319$, $q_b=5$, and (iii) $l_c=1270$, $q_c=11$ are nearly equidistant ($\omega_b-\omega_a\simeq 2\pi\times 100$ GHz, $|\omega_a+\omega_c-2\omega_b| < 2\pi\times 4$ GHz). Hence, if the quality factor of the modes is low enough (the spectral width of the modes exceeds 4 GHz), they can be considered as equidistant. The group velocity of three monochromatic waves resonant with the modes is $V_g\simeq 7\times10^{-3}c$.

We also find that modes of the same sphere with numbers (i) $l_a=1403$, $q_a=1$, (ii) $l_b=1354$, $q_b=5$, and (iii) $l_c=1305$, $q_c=11$ completely overlap if their quality factor is less than 10^5. Hence, a running monochromatic wave interacting with the modes will produce an interference pattern on the surface of the resonator with zero group velocity. In what follows we verify this particular case experimentally.

We have considered the possibility of the delay of a beat note of two and three monochromatic waves. In principle, it is possible to engineer the shape of the WGM resonator (not necessarily a sphere) as well as the evanescent field coupler in such a way that a beat note of more than two waves could be delayed. It is possible to construct a WGM-resonator-based delay line for a train of optical pulses, as well. The effective dispersion of this system is envisioned by overlaying Fig. 1(a) with an equidistant frequency spectrum of the pulse train. All frequency components that happen to couple into some mode produce a phase-shifted output similar to Eqs. (6) and (7): $E_{out\ j}=E_{in\ j}\exp(i2\pi m_j)$.

If the effective dispersion is linear, i.e., $m_j=\Delta m j$, the change will amount to an overall delay (or advance, for negative Δm) of the pulse train by $\tau=2\pi\Delta m/\Delta\omega$. Not surprisingly, this is the same result we obtained for the bichromatic field. Let us point out that our extension of the definition for group velocity to the discrete spectra is consistent with a very intuitive notion of group delay for a pulse train: the delay of the train as a whole, that is, the delay of each of its pulses.

Note that the effective dispersion can be nonlinear, in which case the resonator would change the pulse shape as well as the pulse train delay. Furthermore, some frequency components may not find a suitable WGM to couple to, in which case the resonator serves not only as a phase, but also as an amplitude mask. However, before these effects are considered for a practical use, e.g., pulse delay with dispersion compensation, etc., they need to be studied more thoroughly. In particular, the required degree of control over the WGM spectrum needs to be established and demonstrated. In what follows we describe an approach to such control for a cylindrical resonator with radius R and height L.

The field distribution of the basic sequence of high-order WGMs ($l=m \gg 1$) belonging to the cylinder can be described by an approximate equation in cylindric coordinates ρ, ϕ, z,

$$\frac{\partial^2 \Phi}{\partial \rho^2} + \frac{\partial^2 \Phi}{\partial z^2} + \left(k_{l,q}^2 - \frac{l^2}{R^2} - [\rho + A(z)] \frac{2l^2}{R^3} \right) \Phi = 0, \qquad (8)$$

where we have assumed that $\Phi = \Psi \exp(-il\phi)$, that the modes are localized in the vicinity of the equator of the resonator, and that the resonator radius changes as $r = R + A(z)$ $[R \gg |A(z)|]$ in the vicinity of the equator; and have introduced a new variable $\rho = r - R$ $(R \gg |\rho|)$. We separate variables, writing $\Phi = \Phi_\rho \Phi_z$, and write

$$\frac{\partial^2 \Phi_z}{\partial z^2} - A(z) \frac{2l^2}{R^3} \Phi_z = - k_h^2 \Phi_z, \qquad (9)$$

$$\frac{\partial^2 \Phi_\rho}{\partial \rho^2} + \left(k_{l,q}^2 - k_h^2 - \frac{l^2}{R^2} - \rho \frac{2l^2}{R^3} \right) \Phi_\rho = 0. \qquad (10)$$

Assuming that $A(z) = \Sigma_j A_j \cos(2\pi jz/L)$, where $(\pi R/Ll)^2 \gg |A_j/R|$, we find

$$k_{l,q,j}^2 \simeq \frac{1}{R^2} \left[l + \alpha_q \left(\frac{l}{2} \right)^{1/3} \right]^2 + \frac{\pi^2 j^2}{L^2} + \frac{A_j l^2}{R^3}. \qquad (11)$$

By changing L and A_j we are able to change the WGM spectrum and shift selected mode families, creating sequences of equidistant modes required for the slow light experiment.

To confirm this theoretical prediction, we have performed an experiment with a WGM resonator to demonstrate the case of stopped light, $\Delta\omega_{ab} = 0$. The resonator, a microsphere with radius $R = 150$ μm, is fabricated with optical grade fused silica obtained from a multimode fiber. A taper is manually pulled out from the fiber using a hydrogen-oxygen microtorch. The thin end of the taper, approximately 50 μm in diameter, is gradually heated in a hydrogen flame until a sphere of the required size appears.

We use a single unmodulated 635 nm diode laser to demonstrate zero group velocity of light, which should be seen as a stationary interference pattern generated on the surface of the microsphere by running monochromatic waves. The light is sent into the resonator with an angle-cut fiber attached to a micromanipulator. The frequency of the laser is swept in a 5 GHz frequency span at the rate of 20 scans per second. The output of the fiber coupler is directed to an optical detector (Thorlabs Det110), and the signal from the detector is observed on an oscilloscope screen. The setup allows exciting different WGMs and selecting them by frequency. The coupling efficiency is controlled by the gap between the coupler and the surface of the silica resonator. Dry resonators made in this way have an intrinsic quality factor $Q \approx 10^9$ limited by the dust contamination of their surface. Nanometer scale dust particles attach to the resonator surface, resulting in an easily recognizable surface glow.

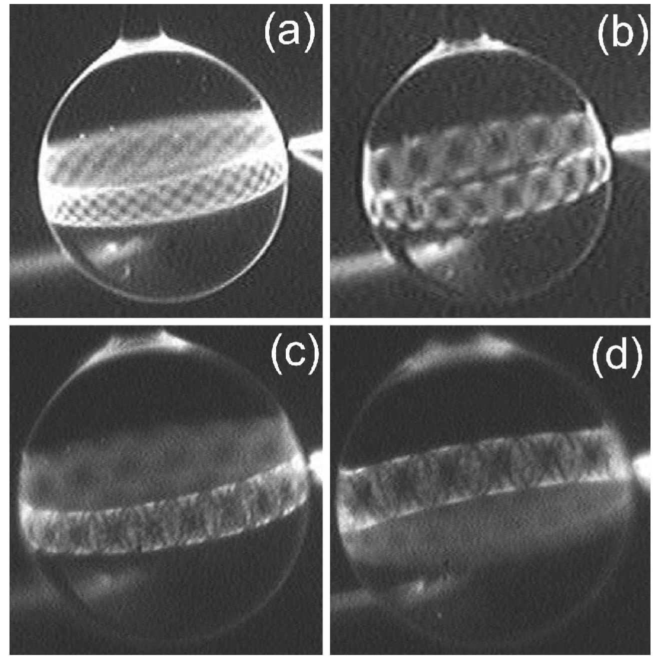

FIG. 2. (a)–(d) Interference patterns observed on the surface of a 300-μm-diameter optically pumped microsphere. Note the static interference pattern that exists despite the traveling-wave (clockwise) excitation. Light coupling is achieved with angle-cleaved fibers. The second output beam is due to precession of light in the WGM resonator. (c) and (d) are the same pattern observed by focusing the microscope on either the front or the back surface of the microsphere. The transitions between patterns (a), (b), and (c) were realized by tuning the coupling to allow the input light to interact with different mode families.

The fiber coupler and the resonator are placed inside a fluidic minicell in the focal plane of a microscope for visualization of the pattern. Visualization at 670 nm is made using a 1 μmol solution of the fluorescent dye Cy5 in methanol. Elastically scattered radiation at 635 nm is blocked by a thin film notch filter installed between the cell and the microscope. Spectral measurements are conducted while visually observing the fluorescence of the dissolved dye at the surface of the resonator.

Absorption of light by the dye solution leads to a reduction of the Q factor to $Q \approx 10^6$, which effectively eliminates the residual frequency difference of nearly degenerate WGMs belonging to different mode families. As a result, monochromatic light can simultaneously interact with sev-

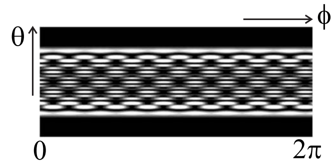

FIG. 3. Theoretical model of the pattern in Fig. 2(a). Six degenerate modes were taken into consideration.

eral WGMs. The reduction of the Q factor also ensures the absence of Rayleigh scattering on the resonator surface, preventing light reflection inside the resonator.

The interference of different groups of modes resulted in different *stationary* fluorescence patterns as seen in Fig. 2. The observed patterns are created by the interference of more than two spatially overlapping WGM frequencies, which are within their spectral widths. The stationarity of these patterns created by the *running* wave confirms the theoretical prediction. In particular, the experiment allows observing the case of stopped light. It is worth noting that the stopped light in the resonator does not carry any information, unlike the light stopped dynamically in atomic systems [7,8], because it represents a beat note of several frequency-degenerate running monochromatic waves. A numerical simulation of one of the patterns is depicted in Fig. 3.

Our observations are not in contradiction to the well-known fact that modes belonging to the same optical resonator are orthogonal. Put in mathematical terms, given the wave equation and the particular boundary conditions of the resonator shape, one obtains a set of eigenvalues and eigenvectors which by definition have zero overlap in time and space. Physically, this means that all resonator modes must differ by either frequency or spatial distribution. This works perfectly well for WGM resonators, and our observation does not contradict it. We are able to observe the interference pattern because modes overlap in space due to their finite bandwidth, so a monochromatic light source simultaneously excites several modes.

In conclusion, we have theoretically predicted the possibility of realizing a group delay of a train of pulses in a linear resonator that does not depend on the ring-down time of the modes. We have also experimentally demonstrated stopped light in a whispering-gallery-mode microsphere resonator using the results of that theory.

The research described in this paper was carried out at the Jet Propulsion Laboratory, California Institute of Technology, under a contract with the National Aeronautics and Space Administration and partial support from DARPA's AOSP Program.

[1] L. V. Hau, S. E. Harris, Z. Dutton, and C. H. Behroozi, Nature (London) **397**, 594 (1999).
[2] M. M. Kash, V. A. Sautenkov, A. S. Zibrov, L. Hollberg, G. R. Welch, M. D. Lukin, Y. Rostovtsev, E. S. Fry, and M. O. Scully, Phys. Rev. Lett. **82**, 5229 (1999).
[3] A. Yariv, Y. Xu, R. K. Lee, and A. Scherer, Opt. Lett. **24**, 711 (1999).
[4] J. E. Heebner, R. W. Boyd, and Q. Han Park, Phys. Rev. E **65**, 036619 (2002).
[5] A. Melloni, F. Morichetti, and M. Martinelli, Opt. Quantum Electron. **35**, 365 (2003).
[6] D. D. Smith, H. Chang, and K. A. Fuller, J. Opt. Soc. Am. B **20**, 1967 (2003).
[7] C. Liu, Z. Dutton, C. H. Behroozi, and L. V. Hau, Nature (London) **409**, 490 (2001).
[8] D. F. Phillips, A. Fleischhauer, A. Mair, R. L. Walsworth, and M. D. Lukin, Phys. Rev. Lett. **86**, 783 (2001).